Interpolation in 2D

Theorem 14.2: There exist interpolation error constants C_i, depending only on the minimum angle in the mesh $\mathcal{T}_h$ and the order of the estimate m, such that the piecewise linear interpolant $\pi_h w \in V_h$ of a function w satisfies

$$\| D^m (w - \pi_h w) \| \le C_i \| h^{2-m} D^2 w \|$$

for $m = 0, 1$, where $D^1 w = Dw = \nabla w$, and $D^2 w = \left(\sum_{i,j=1}^{2} \left(\frac{\partial^2 v}{\partial x_i \partial x_j} \right)^2 \right)^{1/2}$, and

$$\| h^{-2+m} D^m (w - \pi_h w) \| + \left(\sum_{K \in \mathcal{T}_h} h_K^{-3} \| w - \pi_h w \|_{\partial K}^2 \right)^{1/2} \le C_i \| D^2 w \|$$

Further, there exist interpolant $\tilde{\pi}_h w_h \in V_h$ s.t.

$$\| h^{-1+m} D^m (w - \tilde{\pi}_h w) \| + \left(\sum_{K \in \mathcal{T}_h} h_K^{-1} \| w - \tilde{\pi}_h w \|_{\partial K}^2 \right)^{1/2} \le C_i \| D w \|$$

$\tilde{\pi}_h w$ defined using suitable averages of w around the nodal points. ($\| D w \| < \infty$ does not guarentee ~~[illegible]~~ well defined nodal values; but $\| D^2 w \| < \infty$ does).