Lecture 5

$$\begin{cases} -(au')' + cu = f & \text{in } (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

with $a(x) > 0$ & $c(x) \geq 0$

Variational formulation: Find $u \in V$ s.t.

$$\int_0^1 (au'v' + cuv)\,dx = \int_0^1 fv\,dx \quad \forall v \in V$$

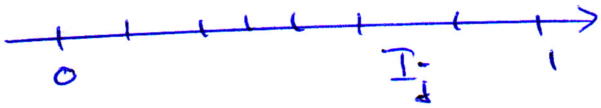
$$V = \{v : \|v\|^2 + \|v'\|^2 < \infty,\ v(0) = v(1) = 0\}$$

Galerkin FEM (cG(1)): Find $U \in V_h$ s.t.

$$\int_0^1 (aU'v' + cUv)\,dx = \int_0^1 fv\,dx \quad \forall v \in V_h$$

$V_h = \{v : v$ cont. p.w. linear on I_j, $v(0) = v(1) = 0\}$

$I_j = (x_{j-1}, x_j)$



We want to estimate the L_2-norm of the error $e = u - U$. To do this we introduce the dual problem:

$$\begin{cases} -(a\varphi')' + c\varphi = e & \text{in } (0,1) \\ \varphi(0) = \varphi(1) = 0 \end{cases}$$