A priori error estimate for L_2-norm

Assume $c=0$ & constant mesh size h.

Theorem 15.5: $\|u-U\| \le C_i S_a \|h(u-U)'\|_a \le C_i^2 S_a \|h^2 u''\|_a$

with $S_a = \max\limits_{\xi \neq 0} \dfrac{\|\varphi''\|_a}{\|\xi\|}$ with φ dual sol. with ξ as data.

Proof: $\|e\|^2 = \int_0^1 a e' \varphi' \, dx = \int_0^1 a e' (\varphi - \pi_h \varphi)' \, dx$

$\le \|he'\|_a \|h^{-1}(\varphi - \pi_h \varphi)'\|_a \le C_i \|he'\|_a \|\varphi''\|_a$

And note that $\|he'\|_a \le C_i \|h^2 u''\|_a$ $\square$

2D error estimates:

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma \end{cases}$$

Variational formulation: Find $u \in V$ s.t.

$(\nabla u, \nabla v) = (f, v) \quad \forall v \in V$

cG(1): Find $U \in V_h$ s.t.

$(\nabla U, \nabla v) = (f, v) \quad \forall v \in V_h$

Dual problem:

$$\begin{cases} -\Delta \varphi = e & \text{in } \Omega \\ \varphi = 0 & \text{on } \Gamma \end{cases}$$