Thm 15.6 (Strong stability):

If Ω convex with polygonal boundary, or if Ω is general with smooth boundary, then there exist a constant S independent of f such that

$$\|D^2 u\| \le S\|\Delta u\| = S\|f\|$$

(If Ω convex $\Rightarrow S = 1$)

A priori error estimate:

$$\|e\|^2 = (u-U, u-U) = (\nabla(u-U), \nabla\varphi)$$

$$= (\nabla(u-U), \nabla(\varphi - \pi_h\varphi)) \le C_i \|h\nabla(u-U)\| \, \|D^2\varphi\|$$

$$\le S C_i \|h\nabla(u-U)\| \cdot \|e\|$$

$$\Rightarrow \|e\| \le S C_i \|h\nabla(u-U)\| \le S C_i^2 \|h^2 D^2 u\|$$

A posteriori error estimate:

$$\|e\|^2 = (\nabla(u-U), \nabla\varphi) = (f,\varphi) - (\nabla U, \nabla\varphi)$$

$$= (f, \varphi - \pi_h\varphi) - (\nabla U, \nabla(\varphi - \pi_h\varphi))$$

Partial integration, interpolation error est., strong stability $\Rightarrow \|u-U\| \le S C_i \|h^2 R(U)\|$

$$R(U) = R_1(U) + R_2(U)$$

$$R_1(U) = |f + \Delta U| \quad \text{on } K \in \mathcal{T}_h$$

$$R_2(U) = \frac{1}{2} \max_{S \subset \partial K} h_K^{-1} |[\partial_S U]| \quad \text{on } K \in \mathcal{T}_h$$