Adaptive error control

A posteriori error estimates only involves the computed solution, and can thus be used in an adaptive algorithm.

Ex. $\|\nabla u - \nabla U\| \leq C_i \|h R(U)\|$

$\|u - U\| \leq S C_i \|h^2 R(U)\|$

To guarantee that $\|\nabla u - \nabla U\| < TOL$,

choose $h = h(x)$ such that

$C_i \|h R(U)\| < TOL$.

Adaptive alg.

1. Choose initial coarse mesh $\mathcal{T}_h^{(0)}$
2. Compute FEM solution $U \in V_h$
3. Compute residual $R(U)$ & evaluate $C_i \|h R(U)\|$
   If smaller than TOL stop, else
4. Refine the mesh where $\int_K (h_K R(U))^2 dx$
   is large.

Further reading:

CDE 15.3, 15.5