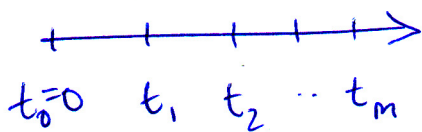


# Lecture 7: Initial value problems (IVP) ①

$$\begin{cases} \dot{u}(x,t) + A(u(x,t)) = f(x,t) & \forall (x,t) \in \mathbb{R} \times [0,T] \\ u(x,t) = w(x,t) & \forall (x,t) \in \Gamma \times [0,T] \\ u(x,0) = u_0(x) \end{cases}$$

Time stepping: Discretize in time  $[0,T]$



solution at  $t=t_m$  is given by solution & data at earlier time steps  $t_n$ ,  $n < m$ .

Stability: growth/decay of perturbations of a solution with time.

(Time & space) Discretization introduces perturbations.

In general: error accumulate (grow with time)

Parabolic problem:  $(Av, v) \geq 0$ ,  $(Aw, v) = (w, Av)$   
for all  $v, w$ .

Parabolic problems are dissipative: errors do not accumulate!

(Elliptic problems:  $(Av, v) = a(u, v) \geq \alpha \|v\|_V^2$   
 $\forall v \in V$ )