Set $v=U$ in (G): $(AU,U)=(f,U)$

(7)

$\Rightarrow c\|U\|^2 \le (f,U) \le \frac{c}{2}\|U\|^2+\frac{1}{2c}\|f\|^2$

$\Rightarrow c\|U\|^2 \le \frac{1}{c}\|f\|^2 \Rightarrow \boxed{\|U\| \le C\|f\|}$

For (G) we only have control of $\|U\|$, whereas for (Sd) we have control of: $\|U\|, \|\sqrt{\delta}AU\|, \|\sqrt{\hat{\varepsilon}}\nabla U\|$

Ex: $Au = \beta\cdot\nabla u + \alpha u$

Add small diffusion: $Au - \nabla\cdot(\varepsilon\nabla u) = f$

$\Rightarrow \beta\cdot\nabla u + \alpha u - \nabla\cdot(\varepsilon\nabla u) = f$

(G) Find $U\in V_h$: $(AU,v)+(\varepsilon\nabla U,\nabla v)=(f,v)\ \forall v\in V_h$

$\ldots \Rightarrow (v=U)$ $\boxed{\|\sqrt{\varepsilon}\nabla U\| + \|U\| \le C\|f\|}$

(Sd) Find $U\in V_h$: $(AU,v)+(\delta AU,Av)+(\hat{\varepsilon}\nabla U,\nabla v) = (f,v)\ \forall v\in V_h$

$\ldots \Rightarrow$ $\boxed{\|\sqrt{\hat{\varepsilon}}\nabla U\| + \|\sqrt{\delta}AU\| + \|U\| \le C\|f\|}$

with $\hat{\varepsilon} = \max\{\varepsilon, \gamma, h^2|f-AU|\}$

- For small ε we have no control of derivatives in (G) $\Rightarrow$ oscillations may occur.
- In (Sd) we always have controll of $\|\sqrt{\delta}AU\|$ & $\|\sqrt{\hat{\varepsilon}}\nabla U\|$.