MATEMATIK
Chalmers tekniska högskola

Hjälpmedel: Inga hjälpmedel är tillåtna
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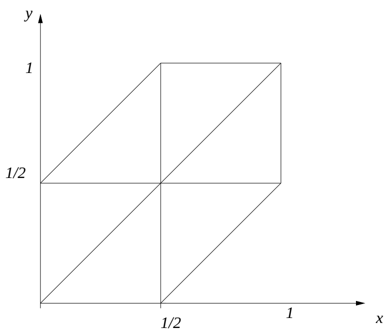
2002–12–17 kl. 14.15-19.15

TMA372/MAN660 Partiella differentialekvationer TM, IMP, E3, GU

OBS! Skriv namn och personnummer på samtliga inlämnade papper.

1. Let Ω be the triangulated domain below. Compute the cG(1) solution of

$$\begin{cases} -\Delta u = 1, & \text{on } \Omega \\ u_x(1,y) = 0,\ 1/2 \le y \le 1, & u(x,y) = 0, \quad \text{on the rest of boundary.} \end{cases}$$



2. Consider the initial value problem $\dot{u}(t) + a(t)u(t) = f(t)$, for $0 < t < T$, and $u(0) = u_0$. Prove the stability estimates

$$|u(t)| \le e^{-\alpha t}|u_0| + \frac{1}{\alpha}(1 - e^{-\alpha t}) \max_{0 \le s \le t} |f(s)|, \quad a(t) \ge \alpha > 0, \quad \text{and}$$

$$|u(t)| \le |u_0| + \int_0^t |f(s)|\, ds, \quad a(t) \ge 0.$$

3. Prove that if $u = 0$ on the boundary of the unit square Ω, then

$$\Big(\int_\Omega |u|^2\, dx\Big)^{1/2} \le \Big(\int_\Omega |\nabla u|^2\, dx\Big)^{1/2}.$$

4. Prove an a priori and an a posteriori error estimate (in the energy norm: $\|u\|_E^2 := \|u'\|^2 + \|u\|^2$) for the cG(1) finite element method for the problem

$$\begin{cases} -u'' + \alpha u' + u = f, & 0 < x < 1, \\ u(0) = u(1) = 0. \end{cases}$$

where $\alpha \ge 0$. For which value of α is the a priori error estimate optimal?

5. Consider the boundary value problem

$$\begin{cases} -\Delta u = 0, & \text{in a bounded domain } \Omega \subset \mathbb{R}^d,\ d = 2, 3. \\ \frac{\partial u}{\partial n} + u = g, & \text{on } \Gamma = \partial\Omega. \end{cases}$$

a) Prove the L_2 stability estimate

$$||\nabla u||_{L_2(\Omega)} + \frac{1}{2}||u||_{L_2(\Gamma)} \le \frac{1}{2}||g||_{L_2(\Gamma)}.$$

b) Verify the conditions on Riesz/Lax-Milgram theorems for this problem.