DN2260, Laboration

September 11, 2011

Formulation

Given the 1D ordinary differential equation

$$-\frac{d}{dx}\left(k\,\frac{du}{dx}\right) = f$$

with boundary conditions

$$u(2) = 3 \textit{ and } u(4) = 7$$

and $k(x) = x$ and $f(x) = 4 - 4x$.

Task 1

Solve the problem (by pen and paper, and in computer) by deriving the weak formulation and using 3 linear finite elements with nodes/endpoints in $x_1 = 2$, $x_2 = 3$, $x_3 = 3.5$ and $x_4 = 4$. Here, you may use any method of integration: analytical or numerical for instance. Construct the local element matrix and load vector and combine them into the global matrix and vector.

Task 2

Refine the grid once and find the new linear finite element solution. Refine the grid once more and find the new linear finite element solution. Here, each refinement divides each old element into two halfsized new ones. Use the same solution method as in **Task 1**.

To think about: Do you *think* you have 2 correct decimals? How do you reason?

(NB: Only free thinking! Error estimation comes later in the course!).

Task 3

Derive the analytical solution to the differential equation and compare the results.

Task 4

Is the Euclidian norm of the residual, $||u - u_{anal}||_{l_2} = \left(\sum(u_i - u_{anal,i})^2\right)^{1/2}$, a good error indicator? Give at least one argument for and one against the use of the norm.

Note! This is a simple differential problem which can be solved analytically. In fact, if you use equal lengths on the linear elements and exact integration, the "FEM method" will give the exact solution in the nodes, regardless of steplength!

This is meant as an computer assignment! Even though the integrals and the systems of equation could be solved "by hand", it would require a lot of work. Work which is easily done with a computer program! You may here choose any language, but Matlab is recommended. (For the 2D-FEM-assignment, all the provided code in the course library is written in Matlab).

Good luck! Murtazo Nazarov