

Mathematical Models, Analysis and Simulation, Part I

Fall 2009

August 21, 2009

Homework 4, Strang Ch. 2.4-5, circuit analysis, Max. score: 6.0, Deadline Sun Oct. 25

Strang Ch. 2.4 (Score: 2.0)

To be handed in

Problems 2.4.1, 2.4.4, 2.4.18

Recommended

Problems 2.4.9, 2.4.17

2. Linear Electrical Networks

Read Strang's book, Chapter 2.4 and 2.5 (until p. 157) as well as the handout *Introduction to the modified nodal analysis*.

DC Analysis (Score: 2.0)

1. Write a MATLAB function `dc_anal` which implements the MNA analysis! Consult sections 5.1 and 5.4 of the handout.

It is convenient to use the following calling sequence:

$$[\mathbf{e}, \mathbf{iV}] = \text{dc_anal}(\mathbf{AR}, \mathbf{AV}, \mathbf{AI}, \mathbf{R}, \mathbf{V}, \mathbf{I}).$$

The meaning of the parameters is as follows:

- \mathbf{AR} , \mathbf{AV} , \mathbf{AI} are the incidence matrices of the resistive, voltage source, and current source branches, respectively.
- \mathbf{R} , \mathbf{V} , \mathbf{I} are the vectors of the values of the resistors, voltage sources, and current sources, respectively.
- \mathbf{e} is the vector of node voltages (excluding the ground node), and \mathbf{iV} is the vector of currents through the voltage source branches.

Let $n + 1$ be the number of nodes in the circuit. Then number the nodes from 1 to $n + 1$ with the ground node being node number $n + 1$. If the number of resistive branches is b_R , the number of voltage source branches b_V , and the number of current source branches b_I , then the matrices and vectors have the following dimensions

matrix	dimension
AR	$n \times b_R$
AV	$n \times b_V$
AI	$n \times b_I$
R	b_R
V	b_V
I	b_I
e	n
iV	b_V

What happens if some of the branches are missing? Assume that our circuit does not include current sources. Then, $b_I = 0$. Hence, the matrix AI has dimension $n \times 0$. In the MATLAB language, this represents a (dimensioned) empty matrix. This is completely consistent with MATLAB's interpretation of matrices. Similarly, the vector I has dimension 0×1 . They can be dimensioned in MATLAB using $\text{AI} = \text{zeros}(n, b_I)$ and $\text{I} = \text{zeros}(b_I, 1)$.

2. Consider the electric circuit given in Figure 1. The given quantities are the voltage of the source E , as well as the values of the resistors R_s, R_a, R_b, R_c, R_d and R_g . The problem consists of computing the current through the resistor R_g as a function of R_g .

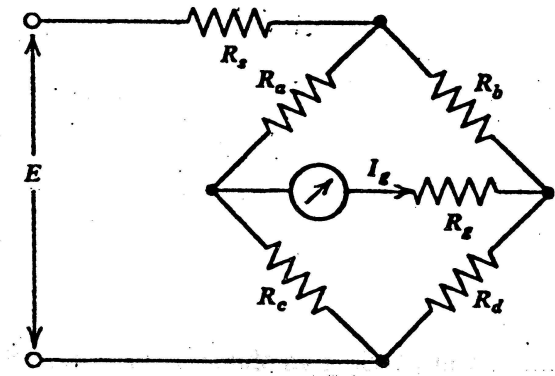


Figure 1: Circuit for the DC analysis problem

- (a) The values of the elements are as follows:

$$E = 10V, R_s = 0.001\Omega, R_a = 100\Omega, R_b = R_c = R_d = 10\Omega.$$

The resistor R_g shall vary between 10Ω and 200Ω .

- (b) Derive the necessary incidence matrices and vectors describing the circuit and compute the current I_g through R_g for sufficiently many values of that resistor by your function `dc_anal`. Plot a graph presenting I_g as a function of R_g . Do not forget to add correct denotations to the axis (values, units etc.)!

3. What is the resistance of a cube?

We consider a circuit which is constructed as follows:

- (a) Draw the lattice of a cube (that is, all 12 edges).
 (b) Every edge is replaced by a resistor with a resistance of 1Ω .
 (c) The nodes of the circuit are represented by the corners of the cube.

How large is the resistance between two corners which are located diagonally opposite to each other in the cube lattice?

4. (optional) Another example of this type is provided in Strang, p. 155. Solve problems 2.4.21, 2.4.22 there.

AC Analysis (Score: 0.5)

In this problem, we are interested in the computation of transfer curves of RLC circuits. The method of choice is the $j\omega$ -method. This approach makes the analysis mathematically similar to the DC case. Consult sections 5.2 and 5.4 of the handout.

1. Write a MATLAB function `ac_anal` which implements the $j\omega$ -method!

The appropriate calling sequence for the function might be:

$$[e, iV] = \text{ac_anal}(AR, AC, AL, AV, AI, R, C, L, V, I, \omega).$$

2. In earlier days of the music business, the black vinyl disks played the role of what the CD is today. For mechanical reasons, the low-frequency electrical signal was not modulated linearly to mechanical vibrations on the vinyl record. Instead, higher frequencies were amplified while lower frequencies were damped. Waves at 1 kHz were neither amplified nor damped. The exact amplification curve (RIAA curve) is defined by a reference circuit. The opposite effect in a phono player is called RIAA equalization. The respective circuit is sketched in Figure 2. The physical parameters are as follows:

$$R1 = 1\Omega, R1A = 212.8m\Omega, C1 = 3.528mF, C1A = 352.8\mu F.$$

The circuit is valid for a frequency f of $E1$ between 20Hz and 20kHz. For simplicity choose an effective voltage of 1V.

Compute and plot the equalization curve! The x -axis shall be the frequency f , while the y -axis is the quotient of the effective voltage $v_5(f)$ at node 5 and $v_5(1kHz)$ (measured in dB, i.e., $y = 20 \log_{10} |v_5(f)/v_5(1kHz)|$)! Do not forget to add correct denotations to the plot!

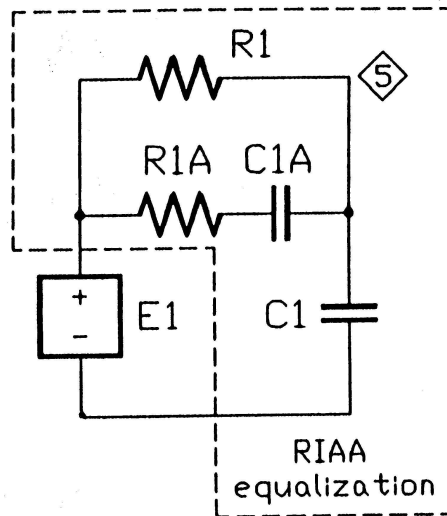


Figure 2: RIAA equalization circuit

Transient analysis: Rectifier Circuits (Score: 1.5)

Most electronic equipments at home need a direct current for proper work. The current provided at the wall outlet is an alternating one. This is very convenient for transforming the current to the necessary voltage inside the equipment. The next step is necessarily a rectifier circuit. Nowadays, integrated circuits are used almost exclusively for that purpose. In the very old days, a discrete diode circuitry was used instead. In this problem, we want to compute the transient behavior of two simple circuits, namely a one-way rectifier and a bridge rectifier.

1. We start with a one-way rectifier. The circuit is sketched in Figure 3

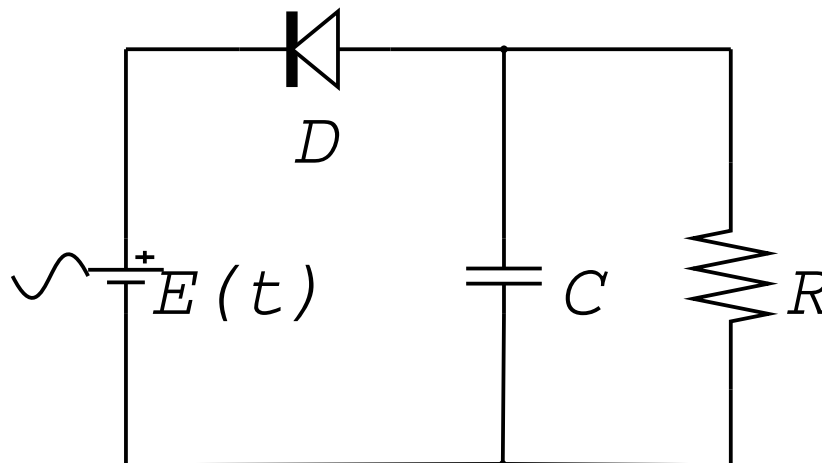


Figure 3: A simple one-way rectifier

The characteristics for the devices are as follows:

Voltage source $E(t) = E_0 \sin(2\pi ft)$ with a (peak) voltage of $E_0 = 5\text{V}$ and a frequency $f = 50\text{Hz}$.

Capacitor Here, we take a simple linear capacitor such that $i = C \frac{dv}{dt}$ with $C = 1000\mu\text{F}$.

Load resistor This resistor is governed by Ohm's law: $v = Ri$ with $R = 100\Omega$.

Diode The characteristics of the diode is highly nonlinear,

$$i = F_D(v) = I_D \left(\exp\left(\frac{v}{V_T}\right) - 1 \right)$$

with the temperature voltage $V_T = 0.0259\text{V}$ and $I_D = 1 \cdot 10^{-14}\text{A}$.

Since the characteristic of the diode is nonlinear, the simple linear MNA must be modified. We can consider the diode equation as a nonlinear conductance. In our circuit, we have two of them. Assuming that the ground node is between E, C and R , node 1 is between E and D , and node 2 is the remaining one, the term GA'_R becomes

$$GA'_R \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} e_2/R \\ F_D(e_1 - e_2) \end{pmatrix},$$

where the signs in front of the node potentials may vary according to the chosen branch orientation.

- (a) Solve the equation with a matlab solver of your choice for five periods of the input voltage. Obviously, the solver must be capable of solving differential-algebraic equations! Use the trivial initial value (all voltages and currents are 0). Plot the solution trajectories. Report about your experiences with different solvers. Try to change the load resistance R such that no waves are seen in the plot.
 - (b) The characteristics of the diode are idealized. A more realistic model is obtained if the capacitance of the pn-junction is taken into account. Let's do it by putting a capacitor with $C_D = 1\text{pF}$ in parallel to the diode. Make the same experiments as before! What do you observe?
2. (optional, score 2.0) A one-way rectifier is of not much use. A more practical circuit is the Graetz bridge rectifier given in Figure 4.

The solution of this problem by the tools of MATLAB is nontrivial. In order to simplify things, I propose to use some unphysical parameters for the elements:

Resistors The resistors have values of $R_i = 10^{-3}\Omega$, $R_V = 10^{-3}\Omega$, and $R_L = 1.5\Omega$.

Capacitor Choose $C = 100\mu\text{F}$.

Diode The diode characteristics should be as above with the exception of $I_D = 10^{-8}\text{A}$.

Voltage source As above.

Derive the equations and integrate them with a solver of your choice! The initial value should be zero for all node voltages. I recommend to use `ode15s`.

G O O D L U C K !

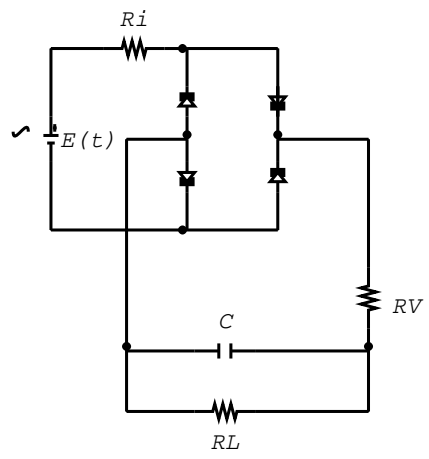


Figure 4: Graetz bridge rectifier