2D1290, NADA, KTH, Ninni Carlsund, 2006-06-07

2D1290 Advanced Numerical Methods: Written Examination

Wednesday 2006-06-07, kl 8-11

Aids: None. **Time:** 3 hours.

Answers may be given in English or Swedish.

Please note that answers should be explained and calculations shown unless the question states otherwise. A correct answer without explanation can thus be left without points.
A minimum of 15 points are required to pass the exam. (Maximum points are 30).

Using a desk calculator is not allowed. It is thus allowed to leave simple expressions unsimplified which could easily be calculated on a **simple** desk calculator.
For example $\alpha = 0.3 \cdot 0.15^3 \cdot \cos(\pi/4)$ or $\beta = 0.3 \left\{ \frac{0.15^3}{3} \cdot 0.7 + 2 \right\}$
Do not leave integrals, derivations, matrix operations or system of equations unsolved unless explicitly allowed.

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(3) **1.** Let a discretized problem be given, whose solution x is characterized by n (real or complex) numbers.

a) What do you understand by the notion of computational complexity of an algorithm for solving the discretized problem?

b) Why is linear complexity a requirement in (serial) large-scale computation?

c) Is linear complexity possible?

(5) **2.** We are aiming to calculate a matrix-vector product, ie y as in $y = Ax$.

a) Let B be a full $n \times n$-matrix with $\text{rank}(B) = k \ll n$ Propose and describe an efficient algorithm for computing the matrix-vector product $y = Bx$ and estimate its (time) computational complexity.

b) In general, the matrix A, obtained from eg a discretized partial differential equation, has full rank. How can the method in exercise **a** above be applied anyway?

(3) **3.** The Conjugate Gradient Method, CG, is a method to solve a linear system $Ax = b$.

a) What are the requirements on A and b for CG to work?

b) What is the computational complexity of CG?

c) What "optimizing" conditions does CG fulfill?

N.B. *The exam continues on the next page.*

(6) **4.** Assume A and/or b does not fulfill the requirements for CG.

a) Describe which requirement that is not fulfilled. Name and describe shortly one method for such problems. Describe the disadvantage with this method compared to CG (if it could have been applied). What is the computational complexity of the described method?

b) Describe an A and/or b with a requirement for CG that is not fulfilled, another than that described in exercise **a** above. Name and describe shortly one method for such problems. Describe the disadvantage with this method compared to CG (if it could have been applied). What is the computational complexity of the described method?

(2) **5.** Many of the methods are based on Krylov space sequences. Let A be a general matrix and u a given vector. Why is, in general, $u, Au, \ldots, A^{k-1}u$ *not* a nice basis for the Krylov space $U_k(A, u)$ (with respect to numerical computations)?

(5) **6.** Consider the fixpoint iteration

$$u^{n+1} = Au^n \quad \text{where} \quad A = \begin{pmatrix} 0.8 & 0.1 \\ 0 & 0.4 \end{pmatrix}$$

a) What is the convergence rate of the fixpoint iteration?

b) Let $V_p = (0, 1)^T$ be a vector belonging to an invariant subspace of A. Describe the RPM iteration method for the above example using V_p.

c) What is the convergence rate of the RPM method constructed in exercise **b** above?

(6) **7.** Describe the multigrid method. Describe in particular the requirements on the grids, prolongation, restriction, V- and W-cycle.

Good Luck & Happy Summer Holiday

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