Study of Motion of a Bowed Violin String

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The velocity of a bowed violin string was measured at various points along its length and with various bow positions by placing a magnetic field perpendicular to the string and observing the voltage induced in the string. The string was found to oscillate in a simple triangular pattern described by Helmholtz. At all points on the string, the ratio of flyback time to period of the oscillation equaled the distance from bridge to observation point divided by string length. This equality implies that larger-amplitude vibrations for a given bow speed can be achieved by bowing closer to the bridge. It also implies that the waveform at the bridge has an exceedingly rapid flyback time and hence is rich in high-frequency components which are transmitted to the body of the instrument. Deviations from a triangular waveform were observed. These consist of slight rounding of the corners of the triangle and a superimposed sinusoidal oscillation of small amplitude.

This paper reports a study of the motion of a violin string that was made by an electrodynamic technique. The method simplifies the observation of rapid vibrations on small strings. An earlier study using high-speed photographs of a cello string indicated the form of the motion was as predicted by Helmholtz, but the flyback time was longer than expected. The present study shows the violin string motion is essentially as predicted by the Helmholtz theory with certain exceptions that will be pointed out. We begin by describing the theoretical motion; then we describe the experimental results; and finally we point out certain acoustical implications of the results.

I. THEORY OF VIBRATIONS

The Helmholtz theory predicts the motion of a bowed string shown in Fig. 1. Figure 1(a) shows snapshots of the string at successive instants in time. At all times it is formed of two straight-line segments. The breakpoint between the segments propagates along the string at a uniform velocity and is reflected from the ends. The envelope of the breakpoint is a parabola.

Figure 1(b) shows the string motion as a function of time at three points on the string, denoted A, B, and C in Fig. 1(a). At all points, the time motion is a simple sawtooth triangle, but the ratio of rise time to fall time differs for the three points. At A, we have a rapid fall time and a slow rise time. At B, which is further along the string, we have a somewhat longer fall time and a correspondingly shorter rise time. In the exact center of the string, the fall time and rise time are precisely equal.

Helmholtz's analysis shows that the ratio of fall time to the period exactly equals the distance from the bridge to the point of observation divided by the length of the string. In other words:

\[ \frac{T_f}{P} = \frac{D}{L}, \]

where \( T_f \) is fall time, \( P \) the period of oscillation, \( D \) the distance from bridge to point of observation, and \( L \) the string length.

The experimental part of this paper presents evidence that the motion of an actual violin string is a reasonable approximation to the triangular waveforms shown in Fig. 1 at all points along the string and that \( T_f/P \) closely approximates \( D/L \) for all values of \( D \).

There are certain departures observed in the motions of actual strings from the ideal motion of Fig. 1. We discuss these after considering the motion of an actual string. In addition to departures due to the characteristics of actual strings, if an ideal or actual string is bowed at a node of one of its natural modes of vibration (bowed at distance \( m/n \cdot L \) from the bridge where \( m \) and \( n \) are integers), a different waveshape of vibration occurs. This effect was noted by Helmholtz and has been extensively studied by Raman. The difference arises because components at the nodal frequency are not excited. The resulting waveshape has a number of ripples in the basically triangular waveform shown in Fig. 1 at all positions except where the bow is located. If the bow is close to the bridge, as in normal violin technique, so that \( n \) is greater than about 6, then the ripples are so very small they can scarcely be seen. Con-
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Fig. 1. Ideal motion of a bowed string showing (a) snapshots of the entire string at successive instants in time and (b) the time motion at various places along the string.

 subsequently, we have not further considered these deviations in the remainder of this paper.

One may ask why the vibration waveshape described by Helmholtz is appropriate to excitation by the bow. A string can vibrate with many other waveshapes. The bow and string together form an oscillator. An explanation of the mechanism of an oscillator, which must be nonlinear in nature, is beyond our knowledge and our intentions in this paper. Schelleng has made a more complete analysis of the string and bow considered as an oscillator. We will merely point out that the string does oscillate, and that during the rise of the waveform, the string and the bow are stuck together and hence move at the velocity of the bow, which is constant. Thus a constant velocity during the risetime is appropriate.

The string slips along the bow hair during the fall time and thus moves at a different velocity than the bow. Although it does not seem essential that the string moves with constant velocity during the fall, this is a perfectly acceptable motion.

II. EXAMINATION OF ACTUAL STRINGS

The measurement technique was suggested to the authors by Rose. It is very simple and is shown in Fig. 2(a). Rose noted that almost all violin strings have a metal component and hence are conductors. If one simply puts a magnetic field across the string, the voltage at the endpoints will measure the velocity of the string at the position of the magnet. Figure 2(b) shows the velocity waveform to be expected from the ideal vibration shown in Fig. 1. It is simply a square wave that is positive during the rise time of the string motion and negative during the fall time. We are now in a position to see if the square wave is actually observed and if \( T_F/P = D/L \), where \( D = M \), the distance from the bridge to the magnet. The equality should hold for all positions of the magnet and for all distances \( B \) from the bridge to the bow.

A series of measurements was made with a violin and with a test string consisting of a piano wire 0.010 in. in diameter and 13.7 in. long, strung between two stops on a solid board and tuned to 360 Hz. The strings were instrumented as shown in Fig. 2. A photograph of the magnet clamped to the fingerboard of the violin is shown in Fig. 3. The length of the magnetic field along the string is 0.75 in. except for the data recorded in Fig. 9, where the magnetic field was shortened to 0.25 in. The strings were excited by a hand-held violin bow. Oscillograms of the string velocity were obtained by photographing the oscilloscope with a Polaroid camera. All measurements were made on the photographs.

Figure 4 shows some typical waveforms from the test string and from both open and stopped violin strings for various bow and magnet positions. With one important exception, which we discuss subsequently, all the waveforms we have observed are of the general form shown in Fig. 4. It is a good approximation to a square wave, indicating that the string does vibrate with a triangular waveform. Two deviations from rectangularity are visually most prominent:

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Fig. 2. Measurement technique suggested by Rose to observe the velocity of the motion of a violin string at point \( M \) by means of a magnetic field inducing voltage in the moving string. The figure shows (a) the equipment arrangement and (b) the expected waveforms.
(1) The sides of the pulse are not vertical. This could be caused by the finite length of the magnetic field, which would introduce a rise time of 0.03 period. It could be due to the stiffness of the string. The G, D, and A strings are larger in diameter and tend to be stiffer than the E string, which exhibits a noticeably faster rise time in Fig. 4. According to Lazarus, it could be due to the motion of the bridge (finite impedance of the bridge). We have not considered this latter effect.

(2) A small, almost sinusoidal oscillation is superimposed on the square wave. This oscillation is discussed in Sec. IV.

Having ascertained that the string waveform is approximately triangular, we are now in a position to see if \( T_f/P = D/L \). In the photographs, \( T_f \) was measured as the width of the pulse at its half-amplitude point. The magnet position, \( D \), was measured to the center of the magnet.

Figure 5 shows the relationship between \( T_f/P \) and \( D/L \) for the test string and for one bow position. Except for the second point, which may be an error, all observations fall close to the theoretical line of unity slope.

Figure 6 shows the lack of effect of bow position on \( T_f/P \) for the test string.

In Fig. 7, we have plotted the ratio of \( T_f/P \) divided by \( D/L \) for a violin. Figure 7(a) shows the effect of measurement position on the open D string; Fig. 7(b) shows the effect of bow position on the open D string; Fig. 7(c) shows various stopped and open strings for a fixed magnet position (2 in. from bridge) and bowing position (1 in. from bridge). These data show that the ratio \( (T_f/P)/(D/L) \) is close to unity independent of where the motion is measured, of where the string is bowed, of which string is sounded, and whether or not the string is stopped. All ratios are slightly larger than unity. A possible explanation is the stiffness of the string. One
would expect unity value only for a perfectly flexible string. Again, the E string is noticeably closer to unity than the other strings. The stopped note for which the string is about three-quarters of its open length seems to exhibit an unusually high ratio of \((T_f/P)/(D/L)\). We can think of no explanation.

### III. DOUBLE SLIP NOTES

One of the important exceptions to the triangular waveform exhibited in the preceding data is the multiple (usually double) slip waveform. Such a form is obtained when too light a bow pressure is used or when the bow position is far from the bridge. In practice, we have observed double slips in the tones produced by beginners, in rapid loud strokes (martele), and in sul tasto playing.

The tones studied in Sec. II were all produced by controlling the bow so as to achieve a tone judged by the ear to be of normal violin quality. For large distances from bridge to bowing point, such a tone was hard to achieve and the range of bow position studied was limited by the requirement of achieving normal quality.

Figure 8(b) shows an oscillogram of a normal *mezzo forte* sustained tone played by an experienced violin teacher (Mrs. D. F. Kautzmann, Summit, New Jersey). Figure 8(a) shows the oscillogram obtained when she reduced the bow pressure, thus producing an inferior tone, which she described as a "surface tone" and which she taught her students to avoid. Instead of having a single flyback or fall time, the surface tone exhibits two flyback cycles per period, one being slightly larger than the other.

The surface tone has a markedly different sound than the normal tone, and no experienced violinist would tolerate such quality in sustained tones. However, the
bow cannot be so well controlled in rapid passages. Figure 8(c) shows an oscillogram from a fortissimo note obtained from a martelé bow stroke. This oscillogram also exhibits a double flyback.

The sul tasto style of bowing also produces markedly different waveforms, which probably cause the different tone quality thus achieved. Figures 8(d), 8(e), and 8(f) give examples of oscillograms obtained by bowing over the fingerboard (3 in. from bridge) with various speeds and pressures. In only one of these, Fig. 8(d), is there an approximation to a single slip. In Fig. 8(e), the string motion appears to be mostly a fundamental sinusoid plus some second harmonic. In Fig. 8(f), the waveform contains much fourth harmonic. Clearly, the violinist can strongly influence the tone quality by his manner of bowing, at least for the extraordinary tones such as sul tasto.

IV. MINOR OSCILLATIONS OF THE STRING

As we noted in Sec. II, one of the most prominent visual deviations from a triangular waveshape is an almost sinusoidal oscillation in the velocity oscillogram. Figure 9 shows a sequence of oscillograms taken to study this oscillation. The tones are all played messa forte on the open D string. A series of bow positions was used to examine the effect of bow position on the oscillation.

The vibration is unusual in that its amplitude is zero (or very small) at the end of the flyback time and its amplitude increases during both the rise time and the flyback time. We would normally expect an oscillation to be started by one of the discontinuities in the string velocity at the beginning or end of the flyback time and subsequently that the oscillation would decay during the relatively stable motion of the rise time. Since the amplitude of this oscillation increases, it must be absorbing energy from the bow in some way not clear to us.

The period of the oscillation increases as the bow is positioned further from the bridge as shown in Fig. 10. (Data for the plot were read from the Fig. 9 oscillograms.) However, the rate of increase is not a linear function or some simple function for which we have a

![Fig. 10](image)

**Fig. 10.** Period of minor oscillation of violin string divided by period of string as a function of the bow position.

![Fig. 11](image)

**Fig. 11.** Waveshape at the violin bridge that would be observed if the motion of the string were an ideal triangular form.
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simple explanation. It does not result from a wave being reflected back and forth between the bridge and bow. Consequently, at the moment, we simply note the existence of the oscillation without being able to explain it.

V. CONCLUSIONS

The principal result of this study of the bowed string is a confirmation that its motion is a close approximation to the simple triangular motion predicted by Helmholtz and shown in Fig. 1. The main deviations from this form are (1) a small but unexplained sinusoidal oscillation and (2) some rounding of the corners of the triangle probably due to the stiffness of the string and the motion of the bridge. In addition to these deviations, the string occasionally vibrates in one of several modes that are grossly different from triangular, such as a double-slip mode. These modes can either be obtained accidently by using too light a bow pressure or intentionally as in the sul tasto style of playing. In either case, the sound is markedly changed.

In the triangular mode of vibration, the string and bow are stuck together during the rise time and hence move at the same velocity. This constraint has implications for the violinist. In order to achieve a loud tone, a large velocity must be produced. This can be done in two ways; either by moving the bow faster or by bowing closer to the bridge. The latter possibility works because, as is shown in Fig. 1, the envelope of the string motion is parabolic and a small motion near the bridge produces a much larger motion toward the center of the string. Although we have not analyzed bow pressure, bowing near the bridge probably requires higher pressures.

The final conclusion concerns acoustics. On the assumption that most of the sound of a violin comes from the body, the most important waveform of the string motion is that at the bridge, since the bridge transmits the vibrations to the body. This waveform for the ideal triangular motion is shown in Fig. 11 and consists of a slow rise and an instantaneous flyback. In an actual instrument, the flyback cannot be instantaneous because of string stiffness and other factors. However, if it were, the Fourier spectrum of the Fig. 11 waveform would be

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(n\omega t).$$

This is a very effective spectrum because it contains all the harmonics and the amplitude of successive higher harmonics decreases 6 dB/oct. The presence of all harmonics is a good way of exciting the resonances of the violin. The rate of decrease of 6 dB/oct is a good compromise between having weak high-frequency sound (as would be the case for a 12-dB/oct falloff) and a rate of falloff less than 6 dB/oct, which tends to give a harsh tone in many sounds.