I. INTRODUCTION

A. General background

The violin is by many considered as one of the most expressive musical instruments, offering a rich variety of tone color and allowing for an almost unlimited degree of expression and virtuosity in the hands of an achieved performer. At the same time, the violin is known to be one of the most difficult instruments to master, and it takes some skill even to produce a simple tone.

Tone production in violin playing involves many constraints. From an acoustical perspective the production of a good tone requires a subtle coordination between the main bowing parameters bow velocity, bow force, and bow-bridge distance. From a motor control perspective, bowing requires a precise control and coordination of the different parts of the bowing arm, which is only achieved after extensive training. All these aspects are subordinate to the music to be performed, which can therefore be considered as a third type of constraint.

The conditions for the production of an acceptable tone have already a long history of research, beginning with the mechanical bowing experiments by Raman, leading to a theoretical description of the constraints for steady-state Helmholtz motion by Schelleng. These constraints are known as the Schelleng equations for maximum and minimum bow force. In the so-called Schelleng diagram, a log-log representation of bow force versus bow-bridge distance at constant bow velocity, the bow-force limits form straight lines, demarcating a triangular-shaped playable region.

Similarly, the conditions for the creation of Helmholtz motion during attacks were formalized by Guettler in terms of bow force, bow acceleration, and bow-bridge distance. The conditions for a perfect attack, characterized by Helmholtz motion right from the start, are confined to a triangular-shaped region in the Guettler diagram of bow force versus bow acceleration at constant bow-bridge distance. Both the Schelleng and the Guettler diagram provide a macroscopic description of the dependence of string vibrations on a limited set of control parameters, and have therefore become a hallmark of playability evaluations of physical bowed-string models, allowing for comparison with realistic measurements and study of the effect of different friction models and other synthesis parameters.

Measurements of bowing parameters in real violin performance were undertaken by Askenfelt in two pioneering studies. These studies provided a first insight in the use and coordination of the main bowing parameters under a variety of conditions, including dynamic level, note duration, and typical bowing techniques such as spiccato and martellato. The measured bowing parameters proved also useful for analysis of timing and phrasing in musical fragments as well as the expression of mood, clearly revealing the means used by the players to transform their intentions into sounding results. The bowing parameters showed a clear relation with the vibration level of the violin, measured by a contact microphone on the top plate. Moreover, it was shown that the violinists kept a safe margin with respect to the Schelleng bow-force limits in the production of sustained notes, mov-
ing more or less in parallel with the bow-force limits through the parameter space when playing at different dynamic levels.

More recently, the interest in the measurement of bowing gestures has been revived in connection with augmented interactive performance and the control of virtual violins.\textsuperscript{7,10-13} Physical models are bound to similar constraints of the coordination of bowing parameters as real violins. For example, the quality of the attack is strongly dependent on detailed features of the acceleration and force envelopes.\textsuperscript{13} Moreover, a realistic synthesis of violin sound, including the reproduction of a variety of bowing techniques, can only be achieved when the control parameters are varied in a way resembling a real violin performance. Real-time controllers, as well as score-based synthesis methods, must be able to produce realistic control envelopes, and their design therefore requires an in-depth understanding of the mechanics and acoustics of bowing, along with a detailed insight in typical bowing strategies in a variety of musical contexts.

B. Aims of the study

The major goal of the current study is to provide a detailed description of the interaction between the player and the instrument and to show how the player adapts to the constraints imposed by the bow-string interaction. In earlier studies it has been shown that players generally respect the Schelleng limits of bow force.\textsuperscript{9,10} However, no detailed studies are available of how players adapt the bowing parameters to the physical properties of, e.g., different strings. The degree in which the player adapts to the varying physical constraints will therefore provide interesting new insights in the sensitivity of the player with respect to the playability of bowed-string instruments.

A method developed for measuring bowing parameters developed by Schoonderwaldt and Demoucron\textsuperscript{14} was used for an extensive study of violin and viola performances. The experiment covered a wide variety of bowing techniques, both isolated in basic performance tasks and applied in a proper musical context. The measurements represent an inventory of typical aspects of bowing technique and should provide a detailed insight in the coordination and control of bowing parameters by players under realistic performance conditions.

The analyses in this paper will be restricted to the “steady” part of bowing, focusing on aspects of coordination and control in sustained notes. The study consists of three parts. In the first part the coordination of bowing parameters will be analyzed in relation with the bow-force limits in the Schelleng diagram. The chosen combinations of bowing parameters by the participating players can be directly compared to the bow-force limits found in an earlier empirical study using a bowing machine.\textsuperscript{15} Bowing strategies for producing different dynamic levels will be analyzed dependent on note duration, shedding further light on the trade-off between bow velocity and bow-bridge distance in setting (or changing) the dynamic level, as suggested by Askenfelt.\textsuperscript{9}

Further, individual patterns in the ranges and variation of bowing parameters will be identified and compared to the common strategies.

In the second part, the dependence of sound level and spectral centroid on the main bowing parameters will be analyzed and compared to findings of an earlier study using a bowing machine.\textsuperscript{16} The results provide further insight in the control of these sound features in terms of the main bowing parameters, and give an indication of the typical ranges found in violin performance.

In the third part subtle aspects of control exerted by players will be studied by analyzing how the participants adapted their combinations of bowing parameters to the physical properties of the string and the instrument. Within the same instrument the physical properties of the strings, in particular the characteristic impedance and internal damping, differ significantly from the lowest to the highest strings, giving rise to, among others, different bow-force limits. Such differences are even more pronounced between different members of the violin family, for example, the violin and the viola. In the latter case the difference in size of the instruments also requires an adaptation of posture, which presents an additional difficulty for a player when switching from one instrument to another.

II. METHOD

A. Setup

In the experiment, motion data, sensor signals, and sound were synchronously recorded. The position and orientation of the bow and the violin/viola were tracked using a Vicon 460 optical motion capture system. The bow was equipped with a bow-force sensor and an accelerometer, mounted on the frog. A detailed specification of the setup, the calibration steps involved, and the definitions and calculations of the bowing parameters is provided in a companion paper.\textsuperscript{14}

B. Participants

A total of six violin and/or viola players participated in the study. All players were advanced master and postgraduate students from two schools of music in Montreal [Schulich School of Music at McGill University and the faculty of music at the Université de Montréal (UdeM)]. Two of the players performed on both the violin and the viola, making a total of eight recording sessions. All players were paid 20 Canadian dollars in compensation.

For the current analyses two sessions were discarded, due to problems with the calibration of the bow force in the upper half of the bow. A total of six sessions remained, three with violin and three with viola. An overview of the sessions and participants included in the analyses is shown in Table I.

C. Experimental procedure and tasks

During the experiment the players were seated on a piano stool. The wires from the sensors on the bow were taped to the right lower and upper arm to avoid interference in playing, taking care that there was ample freedom for the...
performance of full bow strokes. After these preparations the players were given time to familiarize with the experimental situation, making sure that they could play comfortably.

The same instrument and bow combinations were used in all recording sessions, with exception of player P3, who played on a smaller viola. The provided bows and instruments were of master quality.

Before the start of the experiment, the participants received general oral instructions regarding the details of the experimental procedure and the calibration of the bow-force sensor. Force calibration was performed at regular intervals during a session (six to seven times) by the players themselves.14

A variety of bowing techniques were recorded, including détaché (slow and fast), spiccato, martelé, tremolo, and different types of attacks. The tasks consisted of basic tasks (repeated notes on different strings and at different dynamic levels), scales, and musical excerpts, all presented in normal musical notation. The different types of tasks were intertwined so that the basic tasks were followed by the application of the same bowing technique in a musical context, which helped to keep the participants motivated. All participants performed the tasks in the same order. The sessions lasted typically 2 h, including two 5–10 min breaks.

Explicit reference to bowing parameters was avoided in the instructions, leaving the decisions up to the players. The players did not receive feedback about their performance; only in exceptional cases feedback was given, for example, when the dynamic contrast was judged inappropriate.

The basic tasks were performed in strict tempo indicated by a metronome signal, presented to the players’ right ear via light earplug headphones. The metronome signal was recorded on a separate audio track. For the musical excerpts the target tempo was indicated by a two-bar cue from the metronome via a loudspeaker.

The basic tasks were performed on all strings, stopping the string with the third finger in first position, a musical fourth above the open string (C4, G4, D5, and A5 on the violin G, D, A, and E strings, stopped string length 244 mm). The players were allowed to change fingering to prevent fatigue. For viola the tasks were similar, transposed a fifth down (stopped string length 282–285 mm).

The tasks selected for analysis consisted of sustained notes (whole notes and half notes) and détaché 16th notes, played at 3 dynamic levels (forte, mezzoforte, and pianissimo), as well as 4 half-note conditions with various crescendo-diminuendo patterns (between notes and within notes). The nominal durations of the whole and half notes were 4 and 2 s (metronome at 60 BPM). For the 16th notes the nominal duration was 0.2 s (76 BPM). The long-note conditions consisted of 4 notes per string and dynamic level, and the 16th-note conditions of 24 notes.

D. Analysis

In the following analyses, only the steady parts of the notes were considered. Samples were collected from the separate trials by an automatic procedure, respecting a certain margin before and after bow changes (200 and 50 ms in long notes and 16th notes, respectively). Another requirement was that the bow force should exceed a minimum threshold of 0.01 N, to make sure that the bow was in contact with the string. The collected points were more sparsely sampled than the original data, with effective sample rates of 12.5 and 50 Hz for long notes and 16th notes, respectively. The effective playing time accounted for in the analyses per player, string, and dynamic level amounted to 15, 7, and 2.5 s for the whole-note, half-note, and 16th-note conditions, respectively, corresponding to about 188, 88, and 120 samples.

The accumulated data of all participants per instrument (violin/viola) included 144 notes for each of the long-note conditions (48 per dynamic level), and 864 notes (288 per dynamic level) for the 16th-note conditions. The effective playing times considering only the steady part of the notes were about 540, 250, and 86 s for the whole-note, half-note, and 16th-note conditions, respectively.

The bowing parameters included were bow position, bow velocity, bow-bridge distance (absolute and normalized with respect to effective string length), and bow force. The sound features included sound level (rms converted to decibel) and spectral centroid (compensated for background noise). All data points were labeled with respect to instrument (violin/viola), player, string played, note duration, dynamic level, and bowing direction.

The recording level of the sound was not calibrated. The sound level of the violin performances was therefore posthoc compensated for the level difference between recording sessions. The differences were estimated by comparing the sound levels in specific central regions of the bowing-parameter spaces in the sustained part of long notes. This method of compensation was judged appropriate for the purpose of the current study by the apparent predictability of sound level by the $v_B/\beta$ ratio (see Sec. IV A).

III. COORDINATION OF BOWING PARAMETERS IN THE SCHELLENG DIAGRAM

Typical examples of bowing-parameter signals as a function of time are shown in Fig. 1. The example shows performances of the basic tasks with three different note lengths (whole notes, half notes, and 16th notes) played $mf$. The bowing parameters shown from the top down are bow position ($x_B$), bow velocity ($v_B$), bow force ($F_B$), and relative bow-bridge distance ($\beta$). The following analyses will mainly involve distributions of the latter three main bowing parameters.

<table>
<thead>
<tr>
<th>Player ID</th>
<th>Sex</th>
<th>Violin</th>
<th>Viola</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>P3</td>
<td>f</td>
<td>Violin</td>
<td>Master student viola (final year)</td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>m</td>
<td></td>
<td>Master student viola (final year)</td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>m</td>
<td></td>
<td>Post-graduate student viola</td>
<td></td>
</tr>
<tr>
<td>P6</td>
<td>f</td>
<td></td>
<td>Master student (final year)</td>
<td></td>
</tr>
</tbody>
</table>
A. Sustained notes

Figure 2 shows the density distributions of bow force and relative bow-bridge distance when the players performed on the violin D string, represented in Schelleng diagrams at six adjacent ranges of bow velocity. The shown data provide a global representation of all dynamic levels in the whole-note and half-note conditions by the three violin players (P4, P5, and P6). The densities of the respective note-length conditions were normalized, ensuring an equal representation in the graphs. The limits of the playable region were estimated for the same violin and same type of string (Pirastro Obligato, D string stopped at pitch G4), with the use of a bowing machine. Additional information regarding the upper and lower bow-force limits is based on the findings by Schoonderwaldt et al. (see figure caption and footnote for more details).

The combined data from the three players formed clear coherent regions in the Schelleng diagrams, mostly following the contours of the indicated bow-force limits. The highest densities of points were found at bow velocities around 15 and 30 cm/s, corresponding to the whole- and half-note conditions, respectively. As the individual players used somewhat different ranges in bowing parameters (see Sec. III D), the distinction between dynamic levels has become somewhat blurred. However, in panel (c) ($v_B = 15$ cm/s), the three dynamic levels can still be distinguished as more or less separate clusters.

In general, the upper bow-force limit was respected with a reasonable margin, especially at higher bow velocities, and no occurrences of raucous motion could be heard in the sound recordings. Furthermore, it could be observed that higher bow forces were used at higher bow velocities, in accordance with the increase in the upper bow-force limit. In some cases at forte levels the pitch-flattening limit indicated...
by the short solid lines in panels (b) and (c) was exceeded, which was indeed audible in the recorded sound as a “pressed” tone with instable pitch.

As pointed out by Schoonderwaldt et al.,\textsuperscript{15} there was a higher degree of uncertainty associated with the determination of the lower bow-force limit, which was shown to be strongly influenced by finger damping. As this was an uncertain factor in the current experiment, the location of the lower bow-force limit could not be exactly estimated. The lower bow-force limit is therefore indicated by a band, representing the range of lower limits found under different conditions reported in that study.\textsuperscript{15} Another assumption based on the earlier findings is that the lower limit is independent of bow velocity, in contrast to Schelleng’s predictions. This behavior is extrapolated to bow velocities of 25 and 30 cm/s, not measured in the earlier study.

At the three highest bow velocities in Fig. 2 the data points fell mostly above the indicated lower bow-force limits. In contrast, at lower bow velocities, most notably 5 and 10 cm/s, combinations of bow force and \( \beta \) below the measured lower bow-force limit were observed, and there was a considerable overlap with the observed playing regions and the hatched area representing the possible transition to multiple slipping. In particular, for the three lowest bow velocities the data points for bow forces lower than 0.1 N and \( \beta \) values between 1/11 and 1/7 fell almost entirely in the indicated gray zone, and could be mainly attributed to the \( pp \) whole-note condition by one of the players. However, no prolonged episodes of multiple slipping were audible in the recorded sound, with some exceptions in the vicinity of some bow changes. More generally, it could be concluded that the displayed points in Fig. 2 were mostly associated with regular Helmholtz motion.

B. Extension to higher bow velocities

In normal violin performance, bow velocities of 1 m/s are not exceptional and bow velocities beyond 2 m/s can occasionally be observed. Usually, studies of the use of bowing parameters and playability take a limited range of bow velocity into account, covering only a small portion of the parameter range used in real performance. The 16th-note condition in this study allowed extending the analysis of the used bowing-parameter space to higher bow velocities up to 2 m/s.

In order to provide an overview of the various conditions with a wide range of bow velocity, an alternative graphical representation was chosen, as the Schelleng diagram is in principle only valid at a single value of bow velocity. In the alternative representation, \( F_B \) is plotted versus the \( v_B/\beta \) ratio with both axes logarithmically scaled. The maximum Schelleng limit can then be represented by a straight line with slope 1. The lower bow-force limit cannot be simply shown in this diagram for two reasons. Firstly, Schelleng’s lower bow-force limit is not uniquely determined by the \( v_B/\beta \) ratio; at a fixed value of bow velocity the minimum Schelleng limit could be represented by a straight line with slope 2, the offset being dependent on \( v_B/\beta \); on the other hand, at a fixed value of \( \beta \), the minimum bow force is then represented by a straight line with slope 1, parallel to the maximum bow-force limit.\textsuperscript{18} Secondly, the actual lower bow-force limit has not been reliably determined at high bow velocities, and it is unknown how the minimum bow force depends on bow velocity in a real bowed string. Measurements by Schoonderwaldt et al.\textsuperscript{15} showed that the minimum bow force did not significantly depend on bow velocity in the range \( v_B = 5–20 \) cm/s, and there were several indications that Schelleng’s equation for minimum bow force did not provide an adequate explanation of the observations.

Figure 3 shows the two-dimensional density distribution of the combined conditions (the long-note and 16th-note conditions at 3 dynamic levels, as well as crescendo-diminuendo conditions) in the alternative representation. There was a considerable overlap between the four included conditions (détaché whole notes, half notes, 16th notes, and crescendo-diminuendo). The used \( v_B/\beta \) range and average trend for each condition are indicated with fitted lines. There was a good agreement between the whole-note and half-note conditions. For the whole notes the fitted line was shifted to lower values of \( v_B/\beta \) and the slope was slightly steeper, indicating that the \( v_B/\beta \) ratio was generally smaller while the used bow forces were relatively high, especially at \( f \) level. In the crescendo-diminuendo condition the slope was similar to the half-note condition, but the range of \( v_B/\beta \) was extended to higher values, mainly due to the extended range in bow velocity. The slopes of the three long-note conditions (1.24–1.33) were all slightly steeper than the slope (1) of the upper bow-force limit.

In the 16th-note condition the behavior was different. The used range of \( v_B/\beta \) was considerably larger, but the bow forces observed at high values of \( v_B/\beta \) were generally not larger than in the other conditions and seemed to be limited to about 2.5 N. At \( f \) level, the maximum bow force of about 5–10 N was far from reached. As a result, the fitted slope (0.86) was much less steep, even smaller than that of the upper bow-force limit.

One particular area, indicated by a circle in Fig. 3,
clearly deviated from the diagonal. This area could be attributed to the performance of the 16th-note pp condition by one of the players, who used a relatively high bow velocity at small values bow-bridge distance. By listening to the recorded sound it could be concluded that even in this case Helmholtz motion was maintained most of the time. Many instances of multiple slipping could be detected at the bow changes, but that part of the tones was not considered in the above analysis. In contrast, on the G string, the same strategy led to multiple slipping throughout the entire condition, which could be clearly distinguished in the sound level (see Sec. IV A).

Figures 2 and 3 show that the three main bowing parameters as used by the players were clearly interrelated due to the influence of the constraints of the playable region in the Schelleng diagram. Generally, $F_B$ was negatively correlated with $β$, and positively correlated with $v_B$; the correlation between $v_B$ and $β$ was somewhat weaker. The correlations per condition could be clearly visualized in the $F_B$ versus $v_B/β$ diagram. The $R^2$ values of the fitted slopes were rather high in all conditions, indicating a strong correlation between $F_B$ and $v_B/β$ (Pearson correlation $r$=0.9).

C. Bowing strategies in setting dynamic level

In Helmholtz motion the amplitude of the string vibration is in principle determined by the $v_B/β$ ratio, making bow velocity and bow-bridge distance the main control parameters of dynamic level. However, it is known that the perceived loudness is not only dependent on the peak-to-peak amplitude, but also on the distribution of energy in the spectrum, or rather, the distribution of energy across critical bands in the ear. Since the amount of corner rounding—and thus the energy of the higher partials—is mainly influenced by bow force, $F_B$ also has a significant influence on the perceived dynamic level.

As $F_B$ was above shown to be highly correlated with $v_B/β$, the following analysis will focus on the role of bow velocity and bow-bridge distance. Figure 4 shows the used ranges of these parameters, along with the resulting $v_B/β$ ratio in the violin performances for the three note-length conditions and the combined crescendo-diminuendo conditions. In the whole-note and half-note conditions bow velocity was clearly constrained by the length of the bow, and the range in bow velocity across dynamic levels was small. The variation in $v_B/β$ was clearly dominated by the contribution of $β$, especially in the whole-note condition.

In the whole-note condition most players used the full length of the bow, corresponding to a bow velocity of 15 cm/s, and reduced bow velocity only slightly to about 11 cm/s when playing pp. In the half-note condition, the players reduced the average bow velocity from 30 down to 18 cm/s with a decreasing dynamic level. These observations are in agreement with Askenfelt’s findings that players preferred bow velocities in the range 20–40 cm/s, and only occasionally brought down the bow velocity to 10 cm/s.

For the 16th-note conditions, the opposite was found. The used range of bow velocity was extensive, showing large differences between dynamic levels, and bow velocity was clearly dominant in setting the string amplitude. The used range of $β$ was rather small and did not show large differences between dynamic levels.

The minimum bow-bridge distance in the 16th-note condition was significantly larger compared to the long-note conditions. There might be several reasons for this. Firstly, a high bow velocity in combination with a small bow-bridge distance entail the danger of partial slip phases due to the incompatibility of the string displacement with the finite width of the bow, which would result in audible noise. A second possible explanation might be related to the bow change. In the long-note conditions at forté level the peak accelerations during the bow change were mainly in the range 10–20 m/s², whereas in the 16th-note condition bow acceleration ranged from 20 to 50 m/s². In the latter case, a larger bow-bridge distance provides more benign conditions for the creation of Helmholtz motion, which might partly explain the strategy chosen by the players.

In the crescendo-diminuendo conditions the range of bow velocity used was extended to higher bow velocities compared to the half notes at fixed dynamic levels, indicating that bow velocity played a more prominent role in the variation of dynamic level.

D. Individual differences between players

As the number of players participating in this study was limited, it is important to consider the individual strategies. Figure 5 shows the average values of the main bowing parameters in the different conditions for the three violinists. Generally, the individual participants showed similar trends between conditions as shown above. Bow velocity was appreciably higher in the 16th-note condition for all players.
and the range of $\beta$ was extended to smaller values in the long-note conditions. Bow force was only slightly higher in the 16th-note condition.

In the long-note conditions there were only minor differences in the use of bow velocity between the players; only in the half-note pp condition players P4 and P6 clearly reduced the bow velocity. However, in the 16th-note condition, the differences between players were substantial. Player P6 used relatively high bow velocities at all dynamic levels: 80 and 138 cm/s in the pp and f conditions, respectively, corresponding to a factor 1.7. In comparison, player P5 used 19 and 80 cm/s (a factor of 4.2). Player P4 showed the largest range in bow velocity, 26–123 cm/s (a factor of 4.7).

Player P6 used by far the highest bow forces in all conditions but one, especially at forte level (from around 1 N in whole notes to 2 N in 16th notes). She also used the largest range in force across dynamic levels (e.g. 0.25–2 N in 16th notes). Player P5 used relatively low bow forces, typically less than half the force values for P6, except in pp. Player P4 could be placed in between.

Regarding bow-bridge distance, there was a high degree of agreement between players in the $f$ conditions. Interestingly, $\beta$ was approximately doubled by all three players from about 1/14 (17 mm from the bridge) in the whole-note $f$ condition to 1/7 (34 mm) in the 16th-note $f$ condition. However, at mf and pp levels, there were substantial individual differences. Player P6 used only a limited range of $\beta$ around 1/10 (25 mm). Player P5 used the largest range of $\beta$ between conditions (1/15–1/4, corresponding to 16–63 mm) utilizing more the fingerboard area. Player P4 could again be placed in between.

IV. RELATION BETWEEN MAIN BOWING PARAMETERS AND SOUND FEATURES

A. Sound level

Figure 6 shows the relation between sound level and $v_B/\beta$ for the combined data of all violinists (P4, P5, and P6) and the combined conditions (whole notes, half notes, 16th notes, and crescendo-diminuendo) on the violin G string. There was a good overlap between the conditions, and the combined clusters showed a strong linear relation ($R^2=0.90$, outliers discarded). The fitted slope (21.7 dB/decade) was close to the theoretical value of 20 dB/decade.

There were some notable outliers, as indicated by a circle. These could be attributed to the performance of the 16th-note pp condition by one of the players, which was dominated by multiple slipping. The proportionality of string amplitude to $v_B/\beta$ is lost in that case, and the vibrating string does no longer reach its full amplitude, leading to a significant decrease in sound level.

Similar results were found for the higher strings. The $R^2$ values (0.82–0.90) were generally high, indicating that the sound level was highly correlated with $v_B/\beta$ (in logarithmic scale). The fitted slopes (20.2–21.7) were slightly but significantly larger than the theoretical 20 dB/decade for all four strings. This might possibly be explained by the influence of bow force; the coordinated increase in $F_B$ with $v_B/\beta$ causes an increase in the energy of the higher partials, and the influence on the details of the wave shape might boost the rms value used for the calculation of sound level.

Table II shows the average sound levels per condition relative to the sound level of the whole-note pp condition. The value predicted by the $v_B/\beta$ ratio is indicated between parentheses. All levels were calculated relative to the whole-note pp condition.

<table>
<thead>
<tr>
<th></th>
<th>Whole</th>
<th>Half</th>
<th>16th</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>11.0 (10.1)</td>
<td>13.9 (13.4)</td>
<td>20.3 (18.7)</td>
</tr>
<tr>
<td>$mf$</td>
<td>7.2 (7.1)</td>
<td>11.1 (11.0)</td>
<td>15.0 (14.2)</td>
</tr>
<tr>
<td>$pp$</td>
<td>0 (0)</td>
<td>1.9 (2.1)</td>
<td>4.2 (4.2)</td>
</tr>
</tbody>
</table>

FIG. 5. Average bowing parameters per condition for the three violinists (P4, P5, and P6) across all strings. The error bars indicate 95% confidence intervals of the means.

FIG. 6. (Color online) Sound level (dB) versus $v_B/\beta$ (logarithmically scaled) for all conditions (whole notes (□), half notes (○), and 16th notes (+) at pp, mf, and $f$ levels, and crescendo-diminuendo half notes (+)), in the online version also indicated by different colors) performed by the three violinists on the G string. The black line indicates the theoretical dependence with a slope of 20 dB/decade. The points within the circle clearly deviated from the diagonal, and could be attributed to prolonged episodes of multiple slipping in the performance of the 16th-note pp condition by one of the players. The white line indicates the fitted linear relation (slope: 21.7 dB/decade, $R^2=0.90$; the outliers indicated by the circle were discarded).
The table shows that sound level was highest in the 16th-note $f$ condition. The sound level for a given dynamic level depended on note duration; for example, the average sound level in the 16th-note $mf$ condition exceeded that in the whole-note $f$ condition. The maximum difference of dynamic levels across note-duration conditions was about 20 dB. Within note-duration conditions the average ranges of sound level were between 11 and 16 dB. Earlier reported differences between dynamic levels are 9–10 dB between $p$ and $f$ and 14 dB between $pp$ and $ff$.\(^9\,22\)

The sound level range per string spanning from 2.5% to 97.5% of the distribution (i.e., containing 95% of the measured values) was about 30 dB (from the G to the E string: 28, 26, 28, and 34 dB). Taking all four strings into account, the 95% sound level range was about 31 dB, corresponding to the maximum dynamic range of sound with musically acceptable tone quality reported by Askenfelt.\(^9\) The 99% range was 37 dB, which is in close agreement with earlier reported values\(^23\,24\) as well as the total dynamic span found by Askenfelt\(^9\) when not paying full respect to the tone quality.

In Table III the separate contributions (in decibel) of bow velocity and bow-bridge distance to the $pp$–$f$ range of predicted sound level are shown for each note-duration condition. The $\beta$-weights indicating the relative weight of the bowing parameters in determining sound level are shown between parentheses. The values clearly reflect the varying role of bow velocity and bow-bridge distance in setting the dynamic level for the different note-duration conditions, as shown in Sec. III C. In the whole-note condition the contribution of bow-bridge distance was dominant with 73%, whereas in the 16th-note condition bow velocity dominated with 76%. In the half-note condition bow velocity and bow-bridge distance contributed rather equally to the total sound level.

For the crescendo-diminuendo conditions the $pp$–$f$ sound level range could not be specified, as these conditions did not comprise discrete dynamic levels. However, the $\beta$-weights obtained by multiple regression indicate that bow velocity was the dominant parameter in varying sound level.
sign of the coefficient of $\beta$ indicates that spectral centroid increased with increasing bow-bridge distance.

The partial contributions of the independent variables $F_B^a$, $v_B$, and $\beta$ to the spectral centroid are shown in Fig. 8. The range of variation of spectral centroid accounted for by bow force was about 2 kHz, followed by bow velocity (about 1 kHz) and bow-bridge distance (less than 0.5 kHz). The panels show clear linear relationships (i.e., after curvilinear transformation of $F_B$), indicating that spectral centroid could be well explained by the combination of bowing parameters. It should be noted that the contribution of bow force is in practice opposed by the contribution of bow velocity and bow-bridge distance, given the strong correlation between those parameters in playing. The resulting width of the range of spectral centroid per string was typically 1.0–1.5 kHz (see Table IV).

Table IV shows the outcome of the regression model for all four strings. The $\beta$-weights showed a similar trend across the strings. The $R^2$ value was highest for the G string; for the D and the A string the model explained 60% and 30% of the variance, respectively, and for the E string the regression model accounted for merely 3%. A possible reason might be that the players used vibrato, which especially on the higher strings resulted in sympathetic resonances (the notes played were one octave above the adjacent lower strings), causing large fluctuations of the spectral centroid. Also other factors might have played a role. The damping caused by the finger stopping the string is likely to have an influence on the spectrum as it plays an important role in the process of corner rounding. Variations in finger pressure due to vibrato might therefore cause additional fluctuations in spectral centroid without a direct relation with the bowing parameters. On the E string, which has a low characteristic impedance and a low internal damping, fluctuations in damping might have a larger influence on the spectrum compared to the lower strings.

V. ADAPTATION OF THE BOWING PARAMETERS TO STRING AND INSTRUMENT

A. Influence of string properties

Figure 9 shows the averages of the main bowing parameters per string on the violin. The bow force (middle panel) used on the G string was generally higher compared to the higher strings. This is in accordance with expectations, as the higher characteristic impedance, along with the higher inter-

![Figure 8](image-url)

**FIG. 8.** (Color online) Partial contributions of $F_B^a$ (with $\alpha=0.2$), $v_B$, and $\beta$ to the spectral centroid, including four conditions [whole notes ($\square$), half notes ($\bigcirc$), and 16th notes (+) at pp, mf, and f levels, and crescendo-diminuendo half notes (+), in the online version also indicated by different colors]. The solid lines indicate the linear relations in the regression model. Violin, G string, including all conditions and players [multiple slipping (16th-note pp performance by one of the players) discarded].

![Figure 9](image-url)

**FIG. 9.** Average bowing parameters per condition and string for the violin across all players.

TABLE IV. $\beta$-weights and $R^2$ values of regression model with spectral centroid as a dependent variable and bowing parameters as independent variables. The last column indicates the measured range of spectral centroid in kilohertz (from 2.5% to 97.5% of the distribution).

<table>
<thead>
<tr>
<th>String</th>
<th>$F_B^a$</th>
<th>$v_B$</th>
<th>$\beta$</th>
<th>$R^2$</th>
<th>95% range (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>1.16</td>
<td>−0.51</td>
<td>0.18</td>
<td>0.71</td>
<td>0.9–2.2</td>
</tr>
<tr>
<td>D</td>
<td>0.84</td>
<td>−0.39</td>
<td>−0.05</td>
<td>0.60</td>
<td>1.9–2.9</td>
</tr>
<tr>
<td>A</td>
<td>0.69</td>
<td>−0.24</td>
<td>0.11</td>
<td>0.30</td>
<td>2.0–3.3</td>
</tr>
<tr>
<td>E</td>
<td>0.28</td>
<td>−0.11</td>
<td>0.11</td>
<td>0.03</td>
<td>2.0–3.5</td>
</tr>
</tbody>
</table>
nal damping of the G string, leads to higher upper and lower bow-force limits compared to the other strings. In the \( f \) conditions the force used on the E string was also consistently higher compared to the two middle strings (D and A). A possible explanation for this somewhat surprising behavior might be related to the geometry of the instrument. On the middle strings the bow force must be applied more carefully to prevent the bow from touching the neighboring strings. On the outer strings (G and E) the bow inclination is less constrained, giving the player more freedom to apply higher bow forces.

Furthermore, the average bow-bridge distance (lower panel) was slightly, but consistently, larger on the G string compared to the other strings, especially in the two long-note conditions. This might be another adaptation of the players to the higher bow-force limits for the G string. By increasing the bow-bridge distance, the increase in bow force required on the G string compared to the three higher strings will be smaller. A larger bow-bridge distance also facilitates bow changes in heavier strings (a higher \( Z_0 \)).

Regarding bow velocity (upper panel), there were no noteworthy differences between different strings in the long-note conditions. However, the 16th-note \( f \) and \( mf \) conditions showed a clear increase in bow velocity from lower to higher strings. The amplitude of string vibration, which is proportional to the fundamental period, might offer a possible explanation. A large string amplitude gives rise to an increase in pitch due to an increase in effective string tension, an effect which players might want to avoid. This effect is especially noticeable on low strings, in particular, when they have a steel core.

The average values of \( v_B/\beta \) and the resulting amplitude (maximum displacement amplitude at the middle of the string, given by \( \eta_{\text{max}} = v_B T_1 / 8 \beta \)) and pitch rise for each string are shown in Table V. The pitch rise under influence of the bowing parameters was estimated for the used strings via the procedure described by Schoonderwaldt (see Appendix in Ref. 16). It can be seen that the maximum string amplitude increased from the higher to the lower strings (E: 1.5 mm; G: 2.5 mm), despite the fact that the players halved the \( v_B/\beta \) ratio. If the same bow velocity as for the E string would have been used on the G string, the maximum amplitude would have been 5 mm, which would have resulted in a pitch rise of 33 cent instead of 8 cent. Interestingly, the calculated pitch rise is small and differs only a factor 2 across all 4 strings (4–8 cent).

It is further interesting to note that the peak transverse bridge force \(^\text{27}\) was 1.8–1.9 N for all strings, indicating that the driving force on the violin bridge was almost equal for all four strings, despite the fact the \( Z_0 \) was a twice as high for the G string as for the E string. An equal driving force might have been another reason for players to adapt bow velocity to the specific properties of the string (characteristic impedance in this case). The motivation for an equal driving force is, however, not clear-cut. Obtaining a similar sound level and timbre on all strings is not warranted by an equal bridge force, as the interfacing bridge has rather different impedance transformation ratios for the G and E strings. In any case, the observation is an interesting aspect on the choice of strings (\( Z_0 \), Young’s modulus), which matches playing style as well as instrument properties.

An alternative explanation for the reduced bow velocity on lower strings can be found in the attack. It is more difficult to obtain short Helmholtz transients during attacks or bow changes in strings with a higher characteristic impedance.\(^\text{25}\) The players might therefore have used lower accelerations, resulting in lower peak velocities in fast notes characterized by sine-like velocity patterns (see Fig. 1).

### VI. DISCUSSION AND CONCLUSIONS

The analyses presented in this study provided a detailed description of the use of bowing parameters in the steady part of détaché notes, giving a deepened insight in the interaction between the player and the instrument. The distributions and averages represented many notes with a total effective playing time of about 15 min, allowing for a detailed

<table>
<thead>
<tr>
<th>String</th>
<th>( v_B/\beta ) (m/s)</th>
<th>( \eta_{\text{max}} ) (mm)</th>
<th>Pitch rise (cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>5.2</td>
<td>2.5</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>7.5</td>
<td>2.4</td>
<td>6</td>
</tr>
<tr>
<td>A</td>
<td>9.5</td>
<td>2.0</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>10.5</td>
<td>1.5</td>
<td>5</td>
</tr>
</tbody>
</table>

**TABLE V.** Average values of \( v_B/\beta \) per string of the 16th-note \( f \) performances and the resulting peak amplitude \( \eta_{\text{max}} \) in the middle of the string and the pitch rise due to the increase in effective string tension. The latter values are based on empirically determined Young’s moduli of the used strings.
analysis of the different factors influencing the choice of bowing parameters, as well as individual differences between players.

The results of this study were generally in agreement with earlier findings by Askenfelt.\textsuperscript{8,9} However, in the current study, the display of distributions of bowing parameters, rather than selected points or averages, provides a more comprehensive overview of the use of the available bowing-parameter space. Similar typical ranges of bowing parameters were found in violin performance: $v_B$ ranged from about 5 cm/s to 2 m/s, $F_B$ from slightly less than 0.1 N to about 2–2.5 N (in extreme cases, not included in the current analyses, peak bow forces up to 4 N were observed), and $\beta$ from about 1/22 to 1/4, corresponding to 11–63 mm on the stopped string (cf. 15–84 mm on an open string). In the current experiment, it seems that the bow forces observed at soft dynamics were somewhat lower than reported by Askenfelt,\textsuperscript{9} who found that bow forces below 0.5 N were rarely used, and that the lowest bow force easily accessible (at the tip) was about 0.15 N. In the current study bow forces around 0.1 N were normally observed at $pp$ level, and even at $mf$ level bow force was on average found to be slightly below 0.5 N.

It was shown that the strategy for the production of dynamic differences involved a trade-off between bow velocity and bow-bridge distance, confirming earlier findings by Askenfelt.\textsuperscript{9} In the current study the range of note durations was further expanded, leading to higher contrasts in the observed control strategies, as well as larger differences between the extreme values of the bowing parameters. For the longest notes (4 s) dynamic level was almost entirely determined by bow-bridge distance, whereas for the 16th notes (0.2 s) bow velocity was the dominating parameter. Bow-bridge distance showed much less variation between dynamic levels, mainly because small values of bow-bridge distance were avoided at high bow velocities.

There were several indications of that players adapted the bowing parameters to the physical properties of the strings and the instrument. Bow force was clearly highest on the lowest strings, especially on the C string on the viola. This indicates that players are sensitive to the differences in bow-force limits across strings, constantly optimizing their performance. On the viola higher bow forces were used in general, in accordance with the higher characteristic impedances of the strings. Also bow velocity in détaché 16th notes performed at $f$ level was adapted to the string played: Bow velocity was decreased from higher to lower strings, possibly to limit the amplitude of vibration and to facilitate the bow changes.

The distributions of bow force and bow-bridge distance in the Schelleng diagram (Fig. 2) showed that the (empirically determined) bow-force limits were generally well respected. When changing the dynamic level the players moved diagonally through the Schelleng diagram, following the contours of the bow-force limits. Bow force was also observed to increase with bow velocity. However, at extreme bow velocities in loud 16th notes, the full range of bow force was not utilized. The bowing parameters were highly interrelated, as indicated by the high correlations observed for $F_B$ versus $v_B/\beta$ across dynamic level.

Sound level and spectral centroid showed a clear dependence on the main bowing parameters. As expected, the $v_B/\beta$ ratio was found to play a major role in setting dynamic level. However, the coordinated increase in bow force with $v_B/\beta$ also led to an increase in spectral centroid with increasing dynamic level. As the perceived loudness is not only dependent on amplitude, but also on the energy of the higher partials, this will reinforce the perceived contrast between dynamic levels. The role of spectral centroid in perceived dynamic level might be more important in long notes, in which the contrast of spectral centroid was found to be larger than in fast détaché notes (see Fig. 7).

Concerning spectral centroid, the analyses confirmed earlier observations by Guettler et al.\textsuperscript{29} and Schoonderwaldt.\textsuperscript{16} Similar results were also obtained by Demoucron\textsuperscript{13} in an extensive study of the influence of bowing parameters on the sound produced by a virtual violin (physical model). It was found that bow force was by far the most dominant control parameter. The spectral centroid increased with increasing bow force and decreased with increasing bow velocity. It was further confirmed that spectral centroid increased slightly with increasing bow-bridge distance. The latter result is rather counterintuitive, but can be explained by the high correlation between bow-bridge distance and bow force: Bow force is mostly decreased when bow-bridge distance is increased, leading to a net decrease in spectral centroid.

It can be concluded that our method for measuring bowing gestures allows for a detailed analysis of bowing strategies and subtle aspects of control, taking most relevant parameters into account. It is hoped that the current study will contribute to a deepened understanding of tone production in string instrument performance, as well as of the interaction between the player and the instrument. Only a small portion of all the subtleties, which can be readily observed in the data, could be accounted for in the current study. Preliminary analyses of some of these aspects have been presented in Ref. 30 including the use and possible control functions of the bow angles $tilt$ and $skewness$. Follow-up studies will be needed to focus on more advanced aspects of bow control, such as attacks, bow changes, and complex note patterns involving string crossings.

**ACKNOWLEDGMENTS**

This work was partly supported by the Swedish Science Foundation (Contract No. 621-2001-2537) and the Natural Sciences and Research Council of Canada (NSERC-SRO). The experiment was performed in collaboration with Lambert Chen, who selected the musical fragments, and greatly contributed to the design of the tasks and the recruitment of the participants for the experiment. Part of the results were incorporated in his D.Mus. thesis. Many thanks go to Marcelo M. Wanderley and Anders Askenfelt for their inspiring supervision, and to the players for their enthusiastic participation in the experiment.
The player and the bowed string


The limits of the “gray zones” indicated by the hatched areas are based on the fitted Schelleng limits on an open D string on a monochord and a stopped D string on a violin (the same violin as used in the current study, but a different string, stopped at pitch E♭4). The lower bow-force limit was considered to be independent of bow velocity, as found by Schoonderwaldt et al. (Ref. 15).

A comparable two-dimensional representation of the bowing-parameter space was proposed by Askenfelt (Ref. 9) with vB/β on the abscissa, instead of vF/β. This representation is more favorable for visualization of the minimum bow force limit, under the assumption that Schelleng's equation for minimum bow force holds.


In the bowing machine study (Ref. 16) spectral centroid was calculated from the string velocity signal under the bow, which required a modification of the spectrum. In both the current study and the bowing machine study a limiting frequency of 10 kHz was used in the calculation of the spectral centroid.

Some interesting observations on finger pressure have been made by H. Kinoshita, S. Obata, S. Furuya, and T. Aoki ["Fingering force during violin playing: Tempo, loudness and finger effects in single sound production," poster presented at Neuroscience and Music III, Montreal, QC, Canada (2008)].

The peak transverse bridge force was calculated using Eq. (3.23) in Ref. 20.


K. Guettler, E. Schoonderwaldt, and A. Askenfelt, "Bow speed or position—Which one influences spectrum the most?", in Proceedings of the Stockholm Music Acoustics Conference (SMAC03), Stockholm, Sweden (2003), pp. 67–70.