Oversubscription Planning: Complexity and Compilability

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Abstract

Oversubscription Planning: all goals are not simultaneously achievable and the planner needs to find a feasible subset.

We present complexity results for *partial satisfaction* and *net benefit* problems under various restrictions.

Our results reveal strong connections between these problems and with classical planning.

We also present a method for efficiently compiling oversubscription problems into the ordinary plan existence problem.

Problem Definition

Instance: $\Pi = (V, A, I, G, c, U, K)$

- Θ: a set of SAS$^+$ instances
- A SAS$^+$ instance: $(V, A, I, G) \in \Theta$
- The cost function $c : A \to \mathbb{N}_0$
- $U \in \mathbb{N}_0^n$, $K \in \mathbb{N}$

Questions:

Pe(Θ): (Plan Existence) Does Π have a solution, i.e. a plan from I to G?

Bcpe(Θ): (Bounded Cost Plan Existence) Does Π have a solution $(a_1, \ldots, a_n)$ such that $\sum_{i=1}^{n} c(a_i) \leq K$?

Psp(Θ): (Partial Satisfaction Problem) Is there a state $G' \sqsubseteq G$ such that $|G'| \geq K$ and (V, A, I, G') has a solution?

Nbp(Θ): (Net Benefit Problem) Is there a plan $p = (a_1, \ldots, a_t)$ starting from I and leading to a state S such that $m_{G,U}(S) - \sum_{i=1}^{t} c(a_i) \geq K$?

Restrictions:

P: (Post-unique) for every $v \in V$ and every $d \in \mathcal{D}$, there is at most one $a \in A$ such that $\mathbf{post}(a)[v] = d$.

U: (Unary) for every $a \in A$, $\langle \mathbf{post}(a) \rangle = 1$.

B: (Binary) $|\mathcal{D}| = 2$.

S: (Single-valued) for every $v \in V$ and every $a, b \in A$, if $\mathbf{pre}(a)[v] \neq \mathbf{u}$, $\mathbf{pre}(b)[v] \neq \mathbf{u}$ and $\mathbf{post}(a)[v] = \mathbf{post}(b)[v] = \mathbf{u}$, then $\mathbf{pre}(a)[v] = \mathbf{pre}(b)[v]$.

Summary of Results for PSP and NBP under Bylander Restrictions

		post: 1	post: ≥ 2	post: $*$
pre	0	P	NP-c. †	NP-c. †
pre	1	NP-h.	NP-h.	PSPACE-c.
pre	≥ 2	NP-h.	PSPACE-c.	PSPACE-c.
pre	$*$	PSPACE-c.	PSPACE-c.	PSPACE-c.

		post: 1	post: ≥ 2	post: $*$
+ pre	0	P	NP-c. †	NP-c. †
+ pre	1	NP-c. †	NP-h.	PSPACE-c.
+ pre	≥ 2	NP-c. †	PSPACE-c.	PSPACE-c.
+ pre	$*$	NP-c. †	PSPACE-c.	PSPACE-c.

		+ post: 1	+ post: ≥ 2	+ post: $*$
pre	0	P	NP-c. †	NP-c. †
pre	1	NP-c.	NP-c.	NP-c.
pre	≥ 2	NP-c.	NP-c.	NP-c.
pre	$*$	NP-c.	NP-c.	NP-c.

		+ post: 1	+ post: ≥ 2	+ post: $*$
+ pre	0	P	NP-c. †	NP-c. †
+ pre	1	NP-c. †	NP-c.	NP-c.
+ pre	≥ 2	NP-c. †	NP-c.	NP-c.
+ pre	$*$	NP-c. †	NP-c.	NP-c.

†: Complexity of Pe differs from Psp, Nbp $\quad$ $*$: no restriction

Notation

Example:

$$\text{Psp-B}_{2+}^{0}$$

is the class of **P**artial **S**atisfaction **P**roblems with **B**inary domain having no(**0**) preconditions and at most two(**2**) positive(**+**) postconditions.

PSP - Hardness Results

For Θ closed under goal substitution:

- **Important Lemma:**
  Pe(Θ) $\in$ NP $\Longrightarrow$ Psp(Θ) $\in$ NP
  - Why? Read the paper!
- Psp-B$_{2+}^{0}$ and Psp-B$_{1+}^{1+}$ are NP-hard
  - Reduction from Vertex Cover.
- Psp-PUBS$_+$ is NP-hard
  - Reduction from Independent Set.

NBP - Membership Results

For Θ closed under goal substitution:

- **Important Lemma:**
  Bcpe(Θ) $\in$ NP $\Longrightarrow$ Nbp(Θ) $\in$ NP
  - Why? Read the paper!
- Nbp$_1^0$ is in P
- Nbp^0 is in NP
- Nbp-B$_+$ is in NP
  - Optimal solution is shorter than $|V|$.
- Nbp-US and Nbp-B$_1^+$ are in NP
  - Optimal solution is shorter than $2|A|$.

Compiling NBP into PE

- **Introducing a new counter:**
  - A sequence of binary variables $X = (x_{k-1}, \ldots, x_0)$ and the triggers $C = \{c^i\}, D = \{d^i\}$. Define $(X \doteq m) = \{x_i \mid m_i = 1\} \cup \{\bar{x}_i \mid m_i = 0\}$ in which $m = (m_{k-1} \ldots m_1 m_0)_2$.
  - Add $+\ 2^n$ achievable by exactly one of the following:

$$\begin{aligned} a_1^n &: \bar{x}_n \to x_n \\ a_2^n &: \bar{x}_{n+1}, x_n \to x_{n+1}, \bar{x}_n \\ &\vdots \\ a_{k-n}^n &: \bar{x}_{k-1}, x_{k-2}, \ldots, x_n \to x_{k-1}, \bar{x}_{k-2}, \ldots \bar{x}_n \end{aligned}$$

  - For $0 \leq i < k$ and $1 \leq l \leq k - i$, the counter actions

$$\begin{aligned} inc_l^i &: c^i, \mathbf{pre}(a_l^i) \to \overline{c^i}, \mathbf{post}(a_l^i) \\ dec_l^i &: d^i, \mathbf{post}(a_l^i) \to \overline{d^i}, \mathbf{pre}(a_l^i) \end{aligned}$$

  - Finally, for arbitrary $0 \leq s < 2^k$, $C \doteq s$ and $D \doteq s$ are used for $+s$ and $-s$ operations, respectively.
- **Construction:**
  - Build a SAS$^+$ instance $\Pi' = (V', A', I', G')$ from the Nbp instance $\Pi = (V, A, I, G, c, U, K)$. Let $M = \sum_{i=1}^{|V|} U[i]$ and $m = [\log M] + 1$, define:
  - $I'[V] = I[V]$, $I'[X] = M$ and $I'(v) = 0$ otherwise.
  - $G'[X] = M + K$ and $G'(v) = \mathbf{u}$ otherwise.
    Extend A' with:
  - for every $a_i \in A$:

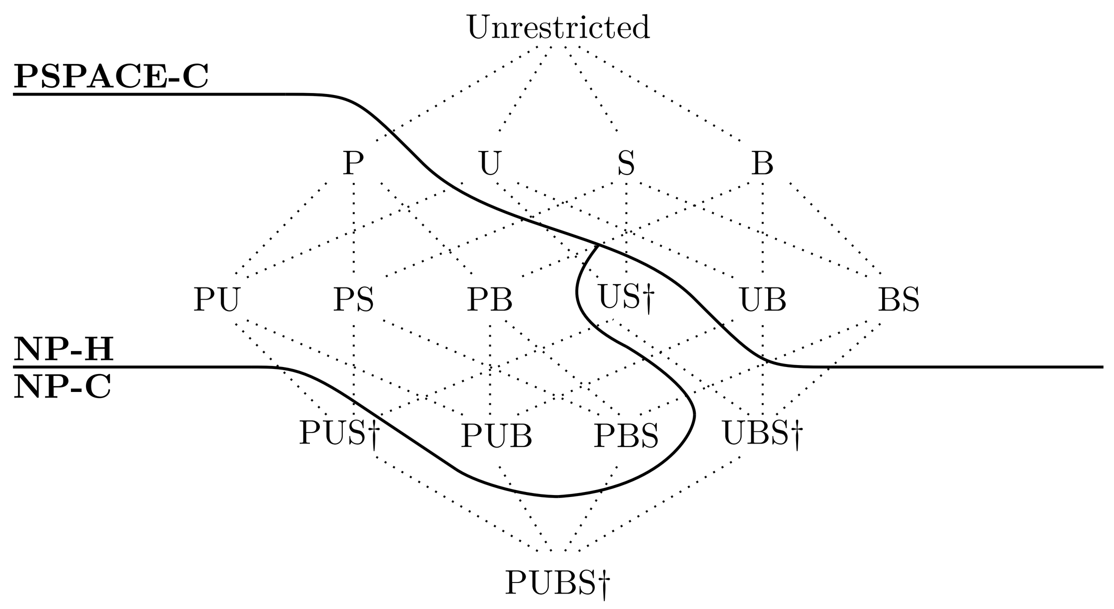
$$\begin{aligned} a_i' &: \mathbf{pre}(a_i), \overline{B}, \overline{E} \to (B \doteq i), (D \doteq c(a_i)) \\ a_i'' &: (B \doteq i), \overline{D} \to \overline{B}, \mathbf{post}(a_i). \end{aligned}$$

  - for every $v_i \in V$ such that $G[v_i] \neq \mathbf{u}$:

$$g_i' : \overline{B}, \overline{end}_{v_i}, \overline{C}, (v_i = G[v_i]) \to end_{v_i}, (C \doteq U[v_i])$$

  - *freesubtract$_l$*: $\mathbf{post}(a_l^0) \to \mathbf{pre}(a_l^0)$, $1 \leq l \leq m$
- **A polynomial-time reduction from Nbp to Pe with a very slow growth in size.**
- **From now on, we can use Pe planners to solve Nbp.**

Results for P,U,B,S Restrictions



References

Important references:

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Acknowledgements

Meysam Aghighi is partially supported by the *National Graduate School in Computer Science* (CUGS), Sweden.