Cost-optimal and Net-benefit Planning - A Parameterised Complexity View

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Introduction

- The usage of *zero-cost* or *rational-cost* actions **does** change the parameterised complexity of planning
- Analysis of a large number of subclasses, using both PUBS restrictions and restricting the number of preconditions and effects

Problem Definition

INSTANCE: $\mathbb{P} = \langle V, A, I, G, c, U\rangle$

- $\langle V, A, I, G\rangle \in C \subseteq \text{SAS}^+$

  V, A, I and G are the set of variables, the set of actions, initial state and goal state, respectively.
- $c : A \to \mathbb{D}$ (numeric) is a cost function
- $U : \text{vars}(G) \to \mathbb{D}$ is a utility function

PARAMETER: A non-negative integer k.

QUESTIONS:

COST-OPTIMAL PLANNING $\big($COP$(C, \mathbb{D})\big)$: Does $\mathbb{P}$ have a plan ω of cost $c(\omega) \leq k$?

NET-BENEFIT PLANNING $\big($NBP$(C, \mathbb{D})\big)$: Is there a state $s \in S(V)$ and a plan ω from I to s such that $U(s) - c(\omega) \geq k$?

RESTRICTIONS ON C:

P (post-unique): No two actions change the same variable to the same value.

U (unary): Each action has only one effect.

B (binary): Each variable takes only two values.

S (single-valued): When two actions have v as their precondition but not as effect, then they require the same value from v.

Parameterised Complexity Theory

Standard Complexity measures complexity as a function of the input size (n).

Tractable: solvable in time $O(n^c)$ for some constant c.

Parameterised Complexity measures complexity as a function of both input size (n) and a parameter (k) which is independent of n.

Fixed-parameter tractable: solvable in time $O(f(k) \cdot n^c)$.

Parameterised Complexity Classes:

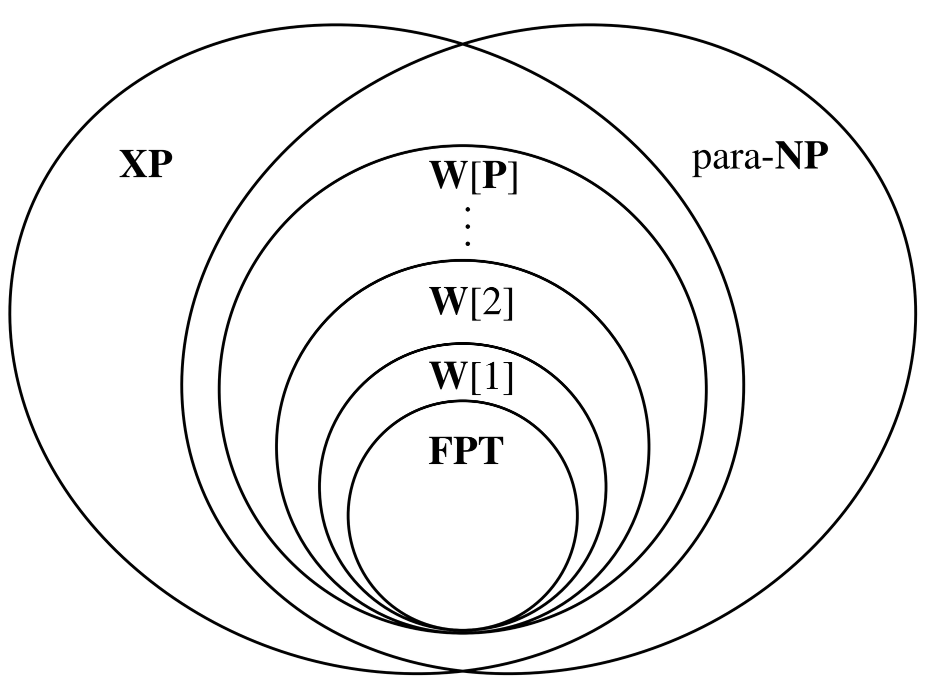
FPT: Fixed-parameter tractable problems

W[i]: Defined by WEIGHTED SATISFIABILITY PROBLEM (weight $\leq k$, literal alterations $\leq i$)

W[P]: Same as **W**[i] but with with unbounded alternations.

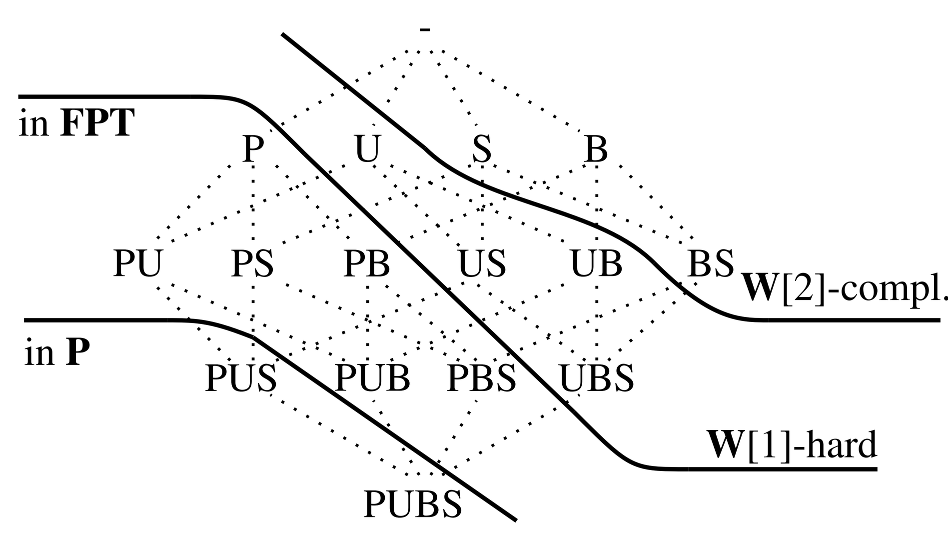
para-NP: Problems solvable in non-deterministic time $f(k) \cdot n^c$.

XP: Problems solvable in time $n^{f(k)}$.

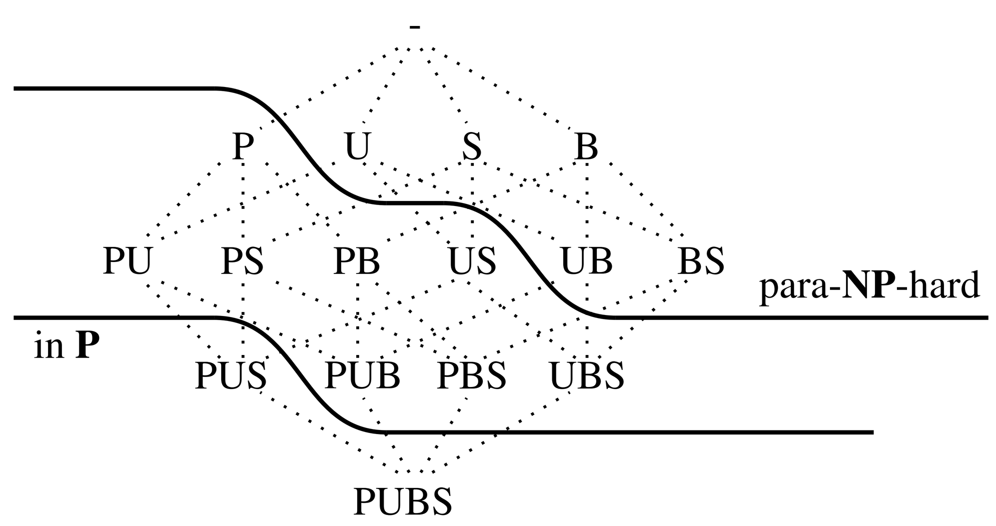


Results - PUBS Restrictions

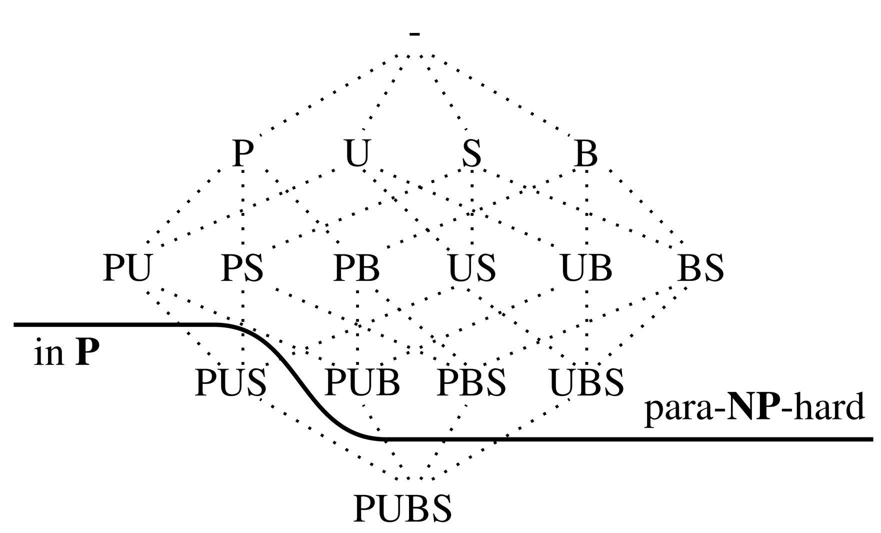
Positive Integers: COP$(C, \mathbb{Z}_+)$



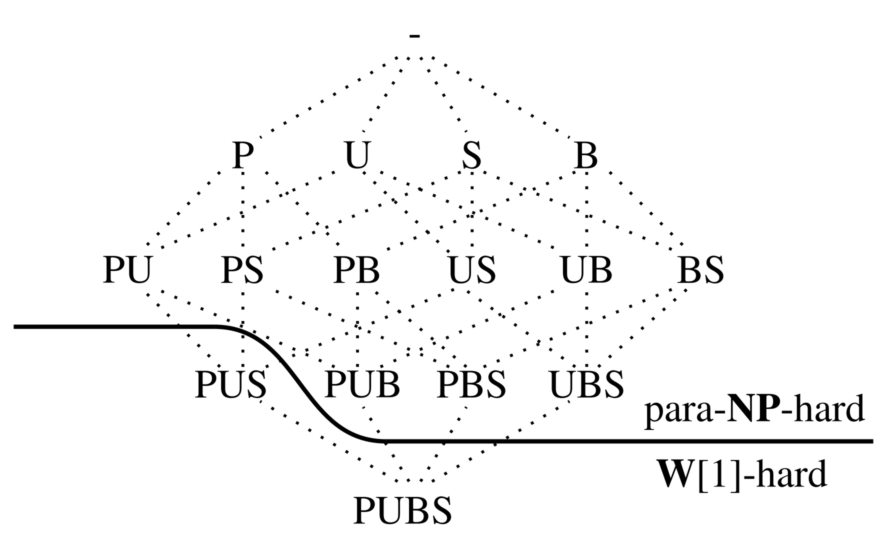
Non-negative Integers: COP$(C, \mathbb{Z}_0)$



Positive Rationals: COP$(C, \mathbb{Q}_+)$



NBP$(C, \mathbb{Z}_+)$, NBP$(C, \mathbb{Z}_0)$, NBP$(C, \mathbb{Q}_+)$



Results - Pre/Eff Restrictions

In this section, results based on restricting the number of preconditions and effects are brought.

COP$(C, \mathbb{Z}_+)$

pre \ eff	1	≥ 3	$*$
0	P	**W**[1]-hard	**W**[2]-hard
≥ 1	**W**[1]-hard	**W**[1]-hard	**W**[2]-hard
$*$	**W**[1]-hard	**W**[1]-hard	**W**[2]-hard

COP$(C, \mathbb{Z}_0)$

pre \ eff	1	≥ 3	$*$
0	P	**W**[1]-hard	**W**[2]-hard
≥ 1	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

COP$(C, \mathbb{Q}_+)$

pre \ eff	1	≥ 2	$*$
0	P	para-**NP**-hard	para-**NP**-hard
≥ 1	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

NBP$(C, \mathbb{Z}_+)$, NBP$(C, \mathbb{Z}_0)$, NBP$(C, \mathbb{Q}_+)$

pre \ eff	1	≥ 2	$*$
0	P	?	?
≥ 2	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

?: Open cases

Observation

Instead of ordinary polynomial-time reductions, an *fpt reduction* is used in parameterised complexity. An *fpt reduction* from a parameterised language $L \subseteq \Sigma^* \times \mathbb{Z}_0$ to another parameterised language $L' \subseteq \Pi^* \times \mathbb{Z}_0$ is a mapping $R : \Sigma^* \times \mathbb{Z}_0 \to \Pi^* \times \mathbb{Z}_0$ such that:

(1) $\langle \mathbb{I}, k\rangle \in L \Leftrightarrow \langle \mathbb{I}', k'\rangle = R(\mathbb{I}, k) \in L'$

(2) There is a computable function f and a constant c such that R can be computed in time $f(k) \cdot |\mathbb{I}|^c$

(3) There is a computable function g such that $k' \leq g(k)$

A COP$(\text{SAS}^+, \mathbb{Q}_+)$ instance can be polynomially reduced to a COP$(\text{SAS}^+, \mathbb{Z}_+)$ instance by multiplying all costs and the parameter with a suitable value α. This is, however, **not an fpt reduction** since α will typically not depend on the parameter (only), which contradicts condition (3) for fpt reductions. Hence, the membership results for COP$(C, \mathbb{Z}_+)$ do not transfer to COP$(C, \mathbb{Q}_+)$.

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