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Numerical Linear Algebra

Prestudy No. 2 (Autumn 2014)

Problem 2.1: Write a program in MATLAB to estimate the machine precision ε. It is by definition the smallest floating-point number with the property that

$$1 + \varepsilon > 1.$$

Problem 2.2: In a textbook from 1988, I found the following interesting exercise. Let us evaluate the expression

$$f(x) = \frac{(x+1)^2 - 1}{x} \tag{1}$$

at $x = .1, .01, .001, \ldots$ using a computer program on an IBM 370. The results are reported in table 1. Expanding the expression in eq. (1) we get

$$f(x) = \frac{(x+1)^2 - 1}{x} = \frac{x^2 + 2x + 1 - 1}{x} = x + 2.$$

x	Computed $f(x)$	Correct $f(x)$
10^{-1}	2.1000	2.1000
10^{-2}	2.0098	2.0100
10^{-3}	1.9999	2.0010
10^{-4}	1.9836	2.0001
10^{-5}	1.9073	2.0000
10^{-6}	1.9073	2.0000
10^{-7}	0.0000	2.0000

Table 1: Value of $f(x)$ computed.

Write a program in MATLAB to compute the values in eq. (1) and compare your results to the reported results. What do you abserve? How can you explain your observations?

Problem 2.3: Archimedes approximated π by calculating the perimeters of polygons inscribing and circumscribing a circle, starting with hexagons, and successively doubling the number of sides. Two forms of the recurrence formula for the circumscribed polygon are:

$$
\begin{aligned}
t_0 &= \frac{1}{\sqrt{3}}, \\
t_{i+1} &= \frac{\sqrt{t_i^2+1}-1}{t_i} \quad \text{first form,} \\
t_{i+1} &= \frac{t_i}{\sqrt{t_i^2+1}+1} \quad \text{second form.}
\end{aligned}
$$

The value of π is approximated by

$$\pi \approx 6 \cdot 2^i \cdot t_i.$$

Write a MATLAB program for the approximation of π to compare both forms. Hint: Use the instruction `format long` in MATLAB to show more digits.

Send your solutions to `barbel@kth.se` until October 11th, 2014.