Computational Blackbody Radiation

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1 Black-Body Radiation

> All these fifty years of conscious brooding have brought me no nearer to the answer to the question, ``What are light quanta?''. Nowadays every Tom, Dick and Harry thinks he knows it, but he is mistaken. (Einstein 1954)

1.1 Wave-Particle Duality and Modern Physics

Maxwell's equations represent a culmination of classical mathematical physics by offering a compact mathematical formulation of all of electromagnetics including the propagation of light and radiation, as electromagnetic waves. But as in a Greek tragedy, the success of Maxwell's equations prepared the way for the collapse of classical mathematical physics and the rise of modern physics based on a concept of *wave-particle duality* with a resurrection of Newton's old idea of light as a stream of light particles or photons, in its modern version combined with statistics.

But elevating wave-particle duality to a physical principle is a cover-up of a contradiction [3, 4, 12]: As a reasonable human being you may sometimes act like a fool, but duality is here called schizophrenia, and schizophrenic science is crazy science, in our time represented by CO_2 climate alarmism ultimately based on radiation as streams of particles. The purpose of this note is to show that particle statistics can be replaced by deterministic finite precision computational wave mechanics. We thus seek to open a door to restoring rational physics including climate physics, without any contradictory wave-particle duality.

1.2 Climate Alarmism, Greenhouse Effect and Backradiation

In particular, the objective is to show that the ``greenhouse effect'' of climate alarmism claimed to arise from ``backradiation'' of particle streams as depicted by NASA in Figure 1, is pure fiction without real physical meaning. This removes a main source of energy from climate alarmism, in the sense that various feedbacks will have to start from zero rather than an alarming warming from radiation alone. We first give a popular science description in words and then a mathematical one using formulas.

To express physics in precise terms it is necessary to use the language of mathematics, but main ideas can be captured also in ordinary language helping understanding, and so the two forms of expression complement each other. In particular we shall find that the term ``backradiation'' which can be contemplated without mathematics, when expressed mathematically reveals its true unstable nature, which makes it into a fictitious unphysical phenomenon without reality. We shall find that it represents the same form of fiction as a bubble-economy in real economic terms: fictitious values without real substance from a circulating self-propelling flow of paper money. Or with another analogy: As nonexistent as audio feedback between a microphone and loadspeaker with the amplifier turned off.

1.3 Blackbody Radiation in Words

A blackbody acts like a transformer of radiation which absorbs high-frequency radiation and emits low-frequency radiation. The temperature of the blackbody determines a *cut-off frequency* for the emission, which increases linearly with the temperature: The warmer the blackbody is, the higher frequencies it can and will emit. Thus only frequencies below cut-off are emitted, while all frequencies are being absorbed.

A blackbody thus can be seen as a system of resonators with different eigen-frequencies which are excited by incoming radiation and then emit radiation. An ideal blackbody absorbs all incoming radiation and re-emits all absorbed radiation below cut-off.

Conservation of energy requires absorbed frequencies above cut-off to

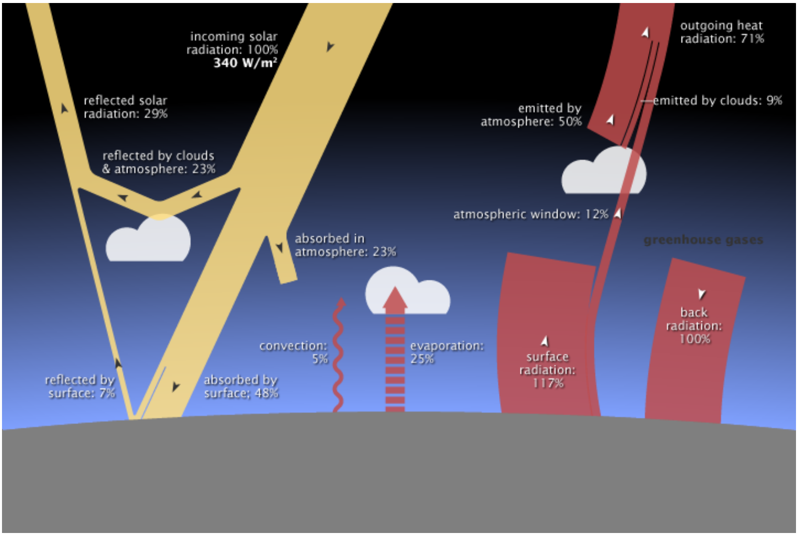

Figure 1: The Earth energy budget according to NASA [10] with incorrect unphysical 100% backradiation and $117\% = 390\, W/m^2$ outgoing radiation from the Earth surface, but with correct physical 30% out of absorbed 48% transported by convection/evaporation from the Earth surface to the atmosphere.

be stored in some form, more precisely as heat energy thus increasing the temperature of the blackbody.

As a transformer of radiation a blackbody thus acts in a very simple way: it absorbs all radiation, emits absorbed frequencies below cut-off, and uses absorbed frequencies above cut-off to increase its temperature. A blackbody thus acts as a semi-conductor transmitting only frequencies below cut-off, and grinding coherent frequencies above cut-off into heat in the form of incoherent high-frequency noise.

We here distinguish between coherent organized electromagnetic waves of different frequencies in the form of radiation or light, and incoherent high-frequency vibrations or noise, perceived as heat.

A blackbody thus absorbs and emits frequencies below cut-off without getting warmer, while absorbed frequencies above cut-off are not emitted but are instead stored as heat energy increasing the temperature.

A blackbody is thus like a high-pass filter, which re-emits frequencies below a cut-off frequency while capturing frequencies above cut-off as heat.

A blackbody acts like a censor which filters out coherent high-frequency (dangerous) information by transforming it into incoherent (harmless) noise. The IPCC acts like a blackbody by filtering coherent critical information, transforming it into incoherent nonsense perceived as global warming.

The increase of the cut-off frequency with temperature can be understood as an increasing ability to emit coherent waves with increasing temperature/excitation or wave amplitude. At low temperature waves of small amplitude cannot carry a sharp signal. It is like speaking at $-40\,C$ with very stiff lips.

We can also compare with a common teacher-class situation with an excited/high temperature teacher emitting information over a range of frequencies from low (simple stuff) to high (difficult stuff), which by the class is absorbed and re-emitted/repeated below a certain cut-off frequency, while the class is unable to emit/repeat frequencies above cut-off, which are instead used to increase the temperature or frustration/interest of the class. The temperature of the class can then never exceed the temperature of the teacher, because all coherent information originates from the teacher. The teacher and student connect in two-way communication with a one-way flow of coherent information.

The net result is that a warm blackbody can heat a cold blackbody, but not the other way around. A teacher can teach a student but not the other way around. The hot Sun heats the colder Earth, but the Earth does not heat the Sun. A warm Earth surface can heat a cold atmospheric layer, but a cold atmosphere cannot heat a warm Earth surface. A blackbody is heated only by frequencies which it cannot emit, but has to store as heat energy.

There is no ``backradiation'' from the atmosphere to the Earth. There is no ``greenhouse effect'' from ``backradiation''. fig. 5 propagated by NASA thus displays fictional non-physical recirculating radiation with an Earth surface emitting 117% while absorbing 48% from the Sun.

We shall see that the reason recirculation of energy is non-physical is that it is unstable. The instability is of the same nature as that of an economy with income tax approaching 100%, or interest rate 0%, or benefits without limits from taxes without limits. An economy with fictitious money circulating with increasing velocity creates financial bubbles which burst sooner or later from inherent instability, as we have been witnessing in recent times.

Backradiation would correspond to audio feedback "blowup" between a microphone and loudspeaker without amplifier energy input, which cannot happen because inevitable losses prevents blowup without energy input.

There is no ``backradiation'' for the same reason that there is no ``backconduction'' or ``backdiffusion'', namely instability. ``Backdiffusion'' would correspond to restoring a blurred diffuse image using Photoshop, which you can easily convince yourself is impossible: Take a sharp picture and blurr it, and then try to restore it by sharpening and discover that this does not work, because of instability. Blurring or diffusion destroys fine details which cannot be recovered. Diffusion or blurring is like taking meanvalues of individual values, and the individual values cannot be recovered from mean values. Mixing milk into your coffee by stirring/blurring is possible but unmixing is impossible by unstirring/unblurring.

Radiative heat can be transmitted by electromagnetic waves from a warm blackbody to a colder blackbody, but not from a cold to a warmer, thus with a one-way direction of heat energy, while the electromagnetic waves propagate in both directions. We thus distinguish between two-way propagation of waves and one-way propagation of heat energy by waves.

A cold body can heat up by eating/absorbing high-frequency, high temperature, coherent waves in a catabolic process of destruction of coherent waves into incoherent heat energy. A warm body cannot heat up by eating/absorbing low-frequency low-temperature waves, because catabolism involves destruction of structure. Anabolism builds structure, but a blackbody is only capable of destructive catabolism (the metabolism of a living cell consists of destructive catabolism and constructive anabolism).

Figure 2: A blackbody acts like a censor or high-pass filter which transforms coherent high-frequency high-interest information into incoherent noise, while it lets low-frequency low-interest information pass through.

2 Planck's Law

The particle nature of light of frequency ν as a stream of *photons* of energy $h\nu$ with h Planck's constant, is supposed to be motivated by Einstein's model of the photoelectric effect [2] viewed to be impossible [1, 8] to explain, assuming light is an electromagnetic wave phenomenon satisfying Maxwell's equations. The idea of light in the form of energy quanta of size $h\nu$ was introduced by Planck [11] in ``an act of despair'' to explain the *radiation energy* $R_\nu(T)$ emitted by a *blackbody* as a function of frequency ν and temperature T, per unit frequency, surface area, viewing solid angle and time:

$$R_\nu(T) = \gamma T \nu^2 \theta(\nu, T), \quad \gamma = \frac{2k}{c^2}, \tag{1}$$

with the *high-frequency cut-off* factor

$$\theta(\nu, T) = \frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1}, \tag{2}$$

where c is the speed of light in vacuum, k is Boltzmann's constant, with $\theta(\nu, T) \approx 0$ for $\frac{h\nu}{kT} > 10$ say and $\theta(\nu, T) \approx 1$ for $\frac{h\nu}{kT} < 1$. Since $h/k \approx 10^{-10}$, this effectively means that only frequencies $\nu \leq T10^{11}$ will be emitted, which fits with the common experience that a black surface heated by the high-frequency light from the Sun will not itself shine like the Sun, but radiate only lower frequencies. We refer to $\frac{kT}{h}$ as the *cut-off* frequency, in the sense that frequencies $\nu > \frac{kT}{h}$ will be radiated subject to strong damping. We see that the cut-off frequency scales with T, which is *Wien's Displacement Law*.

The term *blackbody* is conventionally used to describe an idealized object which absorbs all electromagnetic radiation falling on it, hence appearing to be black. The analysis to follow will reveal some of the real truth of a real blackbody such as the Earth radiating infrared light while absorbing light mainly in the visible spectrum from the Sun.

It is important to note that the constant $\gamma = \frac{2k}{c^2}$ is very small: With $k \approx 10^{-23}\, J/K$ and $c \approx 3 \times 10^8\, m/s$, we have $\gamma \approx 10^{-40}$. In particular, $\gamma\nu^2 << 1$ if $\nu \leq 10^{18}$ including the ultraviolet spectrum, a condition we will meet below.

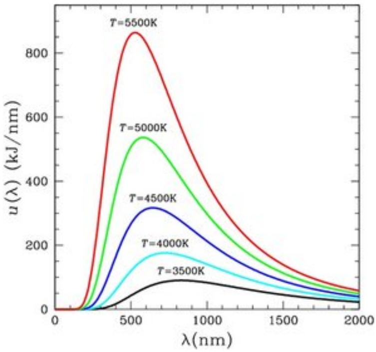

Figure 3: Radiation energy vs wave length/frequency at different temperatures of a radiating blackbody, per unit frequency. Observe that the cut-off shifts to higher frequency with higher temperature according to Wien's Displacement Law.

By integrating/summing over frequencies in Plancks radiation law (1), one obtains *Stefan-Boltmann's Law* stating the the total radiated energy $R(T)$ per unit surface area emitted by a black-body is proportional to T^4:

$$R(T) = \sigma T^4 \qquad (3)$$

where $\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8}\, W^{-1}m^{-2}K^{-4}$ is *Stefan-Boltzmann's constant*.

On the other hand, the classical *Rayleigh-Jeans Radiation Law* $R_\nu(T) = \gamma T\nu^2$ without the cut-off factor, results in an ``ultra-violet catastrophy'' with infinite total radiated energy, since $\gamma T \int_1^n \nu^2\, d\nu \approx \gamma T n^3 \to \infty$ as $n \to \infty$.

Stefan-Boltzmann's Law fits (reasonably well) to observation, while the Rayleigh-Jeans Law leads to an absurdity and so must somehow be incorrect. The Rayleigh-Jeans Law was derived viewing light as electromagnetic waves governed by Maxwell's equations, which forced Planck in his ``act of despair'' to give up the wave model and replace it by statistics of ``quanta'' viewing light as a stream of particles or photons. But the scientific cost of abandoning the wave model is very high, and we now present an alternative way of avoiding the catastrophy by modifying the wave model by *finite precision computation*, instead of resorting to particle statistics.

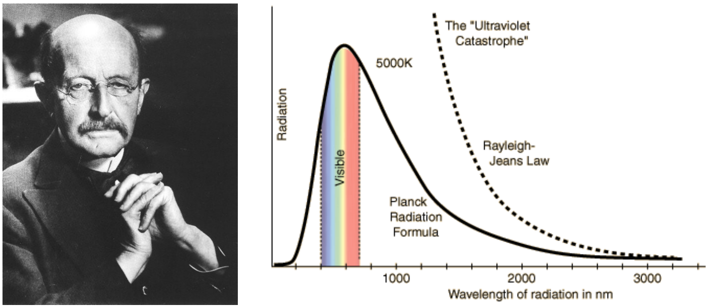

Figure 4: Planck on the ultraviolet catastrophe in 1900: *...the whole procedure was an* ***act of despair*** *because a theoretical interpretation had to be found at any price, no matter how high that might be...Either the quantum of action was a fictional quantity, then the whole deduction of the radiation law was essentially an illusion representing only an empty play on formulas of no significance, or the derivation of the radiation law was based on sound physical conception*. Planck in 1909: *Mechanically, the task seems impossible, and we will just have to get used to it (quanta)*.

We shall see that finite precision computation introduces a high-frequency cut-off in the spirit of the finite precision computational model for thermodynamics presented in [6].

The scientific price of resorting to statistical mechanics is high, as was clearly recognized by Planck and Einstein, because the basic assumption of statistical mechanics of microscopic games of roulette seem both scientifically illogical and impossible to verify experimentally. Thus statistical mechanics runs the risk of representing *pseudo-science* because of obvious difficulties of testability of basic assumptions.

The purpose of this note is to present an alternative to particle statistics for black-body radiation based on deterministic finite precision computation in the form of *General Galerkin G2* [5, 6].

To observe individual photons as ``particles'' without both mass and charge seems impossible, and so the physical reality of photons has remained hypothetical with the main purpose of explaining black-body radiation and the photoelectric effect. If explanations can be given by wave mechanics, both the contradiction of wave-particle duality and the mist of statistical mechanics can be avoided, thus fulfilling a dream of the late Einstein [3, 4].

2.1 The Enigma

The basic enigma of blackbody radiation can be given different formulations:

- Why is a blackbody black/invisible, by emitting infrared radiation when ``illuminated'' by light in the visible spectrum?
- Why is radiative heat transfer between two bodies always directed from the warmer body to the colder?
- Why can high frequency radiation transform to heat energy?
- Why can heat energy transform to radiation of a certain frequency only if the temperature is high enough?

We shall find that the answer is *resonance in a dissipative system of oscillators* (oscillating molecules/charges):

- outgoing radiation has a frequency spectrum $\sim \gamma T\nu^2$ for $\nu < \frac{T}{h}$, assuming all frequencies ν have the same temperature T, with a cut-off to zero for $\nu > \frac{T}{h}$, where h represents the finite precision (normalizing k to unity),
- incoming frequencies below cut-off are absorbed in resonance and re-emitted,

- incoming frequencies above cut-off are absorbed in dissipation and stored as internal heat energy.

2.2 Waves vs Particles in Climate Science

We shall find answers to these questions using a wave model where we can separate between propagation of waves and propagation of heat energy by waves, which combines two-way propagation of waves with one-way propagation of heat energy. In a particle model this separation is impossible since the heat energy is tied to the particles. Radiation as a stream of particles thus leads to an idea of ``backradiation'' with two-way propagation of heat energy carried by two-way propagation of particles. We argue that such two-way propagation is unstable because it requires cancellation, and cancellation in massive two-way flow of heat energy is unstable to small perturbations and thus is unphysical. We thus find that the supposed scientific basis of climate alarmism is unstable and therefore will collapse under perturbations, even small ones, with Climategate representing a perturbation which is big rather than small...

3 A Wave Equation with Radiation

> There are no quantum jumps, nor are there any particles. (H.D. Zeh [13])

3.1 A Basic Radiation Model

We consider the wave equation with radiation, for simplicity in one space dimension assuming periodicity: find $u = u(x,t)$ such that

$$\ddot{u} - u'' - \gamma\,\dddot{u} = f, \quad -\infty < x,\, t < \infty \tag{4}$$

where (x,t) are space-time coordinates, $\dot{v} = \frac{\partial v}{\partial t}$, $v' = \frac{\partial v}{\partial x}$, $f(x,t)$ models forcing in the form of incoming waves, and the term $-\gamma\dddot{u}$ models outgoing radiation with $\gamma > 0$ a small constant.

This models, in the spirit of Planck [11] before collapsing to statistics of quanta, a system of resonators in the form of a vibrating string absorbing energy from the forcing f of intensity f^2 and dissipating energy of intensity $\gamma\ddot{u}^2$ as radiation, while storing or releasing vibrational (heat) energy in energy balance.

The wave equation (4) expresses a force balance in a vibrating system of charged particles with u representing the displacement from a reference configuration with $\dot{u}$ velocity and $\ddot{u}$ accelleration, and $-\gamma\dddot{u}$ represents the *Abraham-Lorentz recoil force* from an accellerating charged particle [14]. Energy balance follows from the force balance by multiplication by $-\dot{u}$ followed by integration, which gives the dissipated radiated energy $\gamma\ddot{u}^2$ by integration by parts (from $-\gamma\dddot{u}$ multiplied by $\dot{u}$), referred to as *Lamours formula* [14].

In a mechanical analog the dissipative radiation term $-\gamma\dddot{u}$ is replaced by the dissipative viscous term $\mu\dot{u}$ with $\mu > 0$ a viscosity, with now dissipated energy $\mu\dot{u}^2$.

In both cases the model includes a dissipative mechanism describing energy loss (by radiation or viscosity) in the system, but the model does not describe where the lost energy ends up, since that would require a model for the receptor. The mechanical model has a direct physical representation as a forced vibrating string subject to a viscous damping force $\mu\dot{u}$. The radiative model is to be viewed as a conceptual model with radiative damping from an Abraham-Lorentz recoil force $-\gamma\dddot{u}$.

We shall see that the form of the damping term determines the energy spectrum, which thus is fundamentally different in the viscous and the radiative case.

3.2 Basic Energy Balance

Multiplying (4) by $\dot{u}$ and integrating by parts over a space period, we obtain

$$\int (\ddot{u}\dot{u} + \dot{u}'u')\,dx - \int \gamma\dddot{u}\dot{u}\,dx = \int f\dot{u}\,dx,$$

which we can write

$$\dot{E} = A - R \tag{5}$$

where

$$E(t) \equiv \frac{1}{2}\int (\dot{u}(x,t)^2 + u'(x,t)^2)\,dx \tag{6}$$

is the internal energy viewed as heat energy, and

$$A(t) = \int f(x,t)\dot{u}(x,t)\,dx, \quad R(t) = -\int \gamma \dddot{u}(x,t)\dot{u}(x,t)dx, \tag{7}$$

is the absorbed and radiated energy, respectively, with their difference $A - R$ driving changes of internal energy E.

Assuming time periodicity and integrating in time over a time period, we have integrating by parts in time,

$$\bar{R} \equiv \int R(t)\,dt = \int\int \gamma \ddot{u}(x,t)^2 dxdt \geq 0 \tag{8}$$

showing the dissipative nature of the radiation term.

If the incoming wave is an emitted wave $f = -\gamma \dddot{U}$ of amplitude U, then

$$\bar{A} - \bar{R} = \int\int (f\dot{u} - \gamma \ddot{u}^2)dxdt = \int\int \gamma(\ddot{U}\ddot{u} - \ddot{u}^2)\,dx \leq \frac{1}{2}(\bar{R}_{in} - \bar{R}), \tag{9}$$

with $\bar{R}_{in} = \int\int \gamma \ddot{U}^2\,dxdt$ the incoming radiation energy, and $\bar{R}$ the outgoing. We conclude that if $E(t)$ is increasing, then $\bar{R} \leq \bar{R}_{in}$, that is, in order for energy to be stored as internal/heat energy, it is required that the incoming radiation energy is bigger than the outgoing.

Of course, this is what is expected from conservation of energy. It can also be viewed as a 2nd Law of Radiation stating that radiative heat transfer is possible only from warmer to cooler. We shall see this basic law expressed differently more precisely below.

4 The Rayleigh-Jeans Radiation Law

> But the conception of localized light-quanta out of which Einstein got his equation must still be regarded as far from established. Whether the mechanism of interaction between ether waves and electrons has its seat in the unknown conditions and laws existing within the atom, or is to be looked for primarily in the essentially corpuscular Thomson-Planck-Einstein conception of radiant energy, is the all-absorbing uncertainty upon the frontiers of modern Physics. (Robert A. Millikan [9])

4.1 Spectral Analysis of Radiation

We shall show that the Rayleigh-Jeans radiation law $R_\nu(T) = \gamma T\nu^2$ is a direct consequence of the form of the radiation term $-\gamma\dddot{u}$, assuming that all frequencies have the same temperature T. This is elementary.

We shall also show that if the intensity of the forcing f in the model (4) has a Rayleigh-Jeans spectrum $\gamma T\nu^2$, then so has the corresponding radiation energy $R_\nu(T)$. More precisely, we show as a main result that

$$R_\nu(T) \sim \overline{f_\nu^2} \tag{10}$$

with the bar denoting integration in time and $\sim$ indicates proportionality with constant of unit size. This is less elementary and results from a (quite subtle) phenomenon of *near-resonance*, exhibited in more detail in [7] as a continuation to this article.

The prove (10) we first make a spectral decomposition in x, assuming periodicity with period 2π:

$$\ddot{u}_\nu + \nu^2 u_\nu - \gamma\dddot{u}_\nu = f_\nu, \quad -\infty < t < \infty, \quad \nu = 0, \pm 1, \pm 2, ..., \tag{11}$$

into a set of damped linear oscillators with

$$u(x,t) = \sum_{\nu=-\infty}^{\infty} u_\nu(t)e^{i\nu x}.$$

We then use Fourier transformation in t,

$$u_\nu(t) = \int_{-\infty}^{\infty} u_{\nu,\omega}e^{i\omega t}d\omega, \quad u_{\nu,\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} u_\nu(t)e^{-i\omega t}\,dt,$$

to get, assuming $u_\nu^{(3)}$ can be replaced by $-\nu^2\dot{u}_\nu$:

$$(-\omega^2 + \nu^2)u_{\nu,\omega} + i\omega\gamma\nu^2 u_{\nu,\omega} = f_{\nu,\omega}.$$

We have by Parseval's formula,

$$\overline{u_\nu^2} \equiv \int_{-\infty}^{\infty} |u_\nu(t)|^2\,dt = 2\pi\int_{-\infty}^{\infty} |u_{\nu,\omega}|^2\,d\omega$$

$$= 2\pi \int_{-\infty}^{\infty} \frac{|f_{\nu,\omega}|^2\, d\omega}{(\nu-\omega)^2(\nu+\omega)^2 + \gamma^2\nu^4\omega^2}$$

$$\approx \frac{2\pi}{\nu^2} \int_{-\infty}^{\infty} \frac{|f_{\nu,\omega}|^2\, d\omega}{4(\nu-\omega)^2 + \gamma^2\nu^4} = \frac{2\pi}{\gamma\nu^4} \int_{-\infty}^{\infty} \frac{|f_{\nu,\nu+\gamma\nu^2\bar{\omega}}|^2\, d\bar{\omega}}{4\bar{\omega}^2 + 1},$$

where we used the change of integration variable $\omega = \nu + \gamma\nu^2\bar{\omega}$.

We now assume as the definition of *near-resonance*, that

$$|f_{\nu,\omega}|^2 \sim \frac{1}{\pi^2}\overline{f_\nu^2} \quad \text{for} \quad |\nu - \omega| \le \frac{\pi}{4}, \tag{12}$$

and that $|f_{\nu,\omega}|^2$ is small else, where we use $\sim$ to denote proportionality with constant close to 1. With this assumption expressing that the forcing frequency ν is determined up to one Hz, we get

$$\overline{u_\nu^2} \sim \frac{1}{\gamma\nu^4}\overline{f_\nu^2},$$

that is,

$$R_\nu \equiv \gamma\overline{\ddot{u}_\nu^2} \sim \gamma\nu^4\overline{u_\nu^2} \sim \gamma\overline{\dot{u}_\nu}\nu^2 = \gamma T_\nu \nu^2 \sim \overline{f_\nu^2}, \tag{13}$$

where $R_\nu = R_\nu(T_\nu)$ is the intensity of the radiated wave of frequency ν, and we view $T_\nu = \frac{1}{2}(\overline{\dot{u}_\nu^2} + \nu^2\overline{u_\nu^2}) \approx \overline{\dot{u}_\nu^2}$ as the temperature of the corresponding frequency.

We read from (13) that

$$R_\nu(T_\nu) \sim \gamma T_\nu \nu^2, \tag{14}$$

which is the Rayleigh-Jeans Law. Further, if $\overline{f_\nu^2} \sim \gamma T\nu^2$, then also $R_\nu(T_\nu) \sim \gamma T\nu^2$ if $T_\nu \sim T$. The emitted radiation will thus mimic an incoming Rayleigh-Jeans spectrum, in temperature equilibrium with $T_\nu = T$ for all frequencies ν.

We note that the constant of proportionality in $R_\nu \sim \overline{f_\nu^2}$ is independent of γ and ν which reflects that the string has a certain absorbitivity (greater or equal to its emissivity).

Summing over frequencies we get

$$R \equiv \frac{1}{2\pi}\int_0^{2\pi} \gamma\overline{\ddot{u}^2}\, dx \sim \frac{1}{2\pi}\int_0^{2\pi} \overline{f^2}\, dx \equiv \|f\|^2, \tag{15}$$

that is, the intensity of the total outgoing radiation R is proportional to the intensity of the incoming radiation as measured by $\|f\|^2$, thus $R \sim \|f\|^2$. We summarize in

Theorem 1 *The radiation $R_\nu = \overline{\gamma \ddot{u}_\nu^2}$ of the damped oscillator (11) with forcing f_ν according to (12) satisfies $R_\nu \sim \overline{f_\nu^2}$, or after summation $R \sim \|f\|^2$. In particular, if $\overline{f_\nu^2} \sim \gamma T \nu^2$ then $R_\nu = R_\nu(T_\nu) \sim \gamma T \nu^2$ with $T_\nu = T$.*

In ([7]) we make a connection to near-resonance in acoustic appearing in the tuning of a piano with the three strings for each tone (except the single string bass tones) tuned with an offset of about 0.5 Hz, with the effect of a longer sustain and singing quality of the piano. In this perspective the radiation of blackbody is like the thick chord obtained by pressing all the keys of a piano.

5 Radiation from Near-Resonance with Dissipation

We have seen radiation resulting from forcing by a phenomeon of near-resonance in a damped oscillator of the form

$$\ddot{u}_\nu + \nu^2 u_\nu + \gamma \nu^2 \dot{u}_\nu = f_\nu, \tag{16}$$

where the forcing f_ν is balanced by the dynamics of the oscillator $\ddot{u}_\nu + \nu^2 u_\nu$ and the radiator $\gamma \nu^2 \dot{u}_\nu$ with an effect of dissipative damping (with $\gamma \nu^2 \leq 1$). In the case of large damping with $\gamma \nu^2 \approx 1$, then f_ν is mainly balanced by the radiator, that is, $\gamma \nu^2 \dot{u}_\nu \approx \dot{u}_\nu \approx f_\nu$ with the result that $R_\nu = \overline{f_\nu \dot{u}_\nu} \approx \overline{f_\nu^2}$. We see that in this case $\dot{u}_\nu$ is *in-phase* with the forcing f_ν, and there is little resonance with the oscillator.

We next consider the case $\gamma \nu^2 << 1$ with small damping and thus near-resonance. The relation $R_\nu = \overline{f_\nu \dot{u}_\nu} \sim \overline{f_\nu^2}$ tells us that in this case f_ν is balanced by the dynamics of both oscillator and radiator with u_ν in-phase and thus $\dot{u}_\nu$ *out-of-phase*. This is because if not, then $\gamma \nu^2 \dot{u}_\nu \approx f_\nu$ with $\dot{u}_\nu$ in-phase, which would give the contradicting $R_\nu = \overline{f_\nu \dot{u}_\nu} \sim \frac{\overline{f_\nu^2}}{\gamma \nu^2} >> \overline{f_\nu^2}$.

6 Absorption vs Emission

In the wave model (4) we have associated the term $-\gamma\dddot{u}$ with radiation, but if we just read the equation, we only see a dissipative term absorbing energy without information how this energy is dispensed with e.g. by being radiated away. The model thus describes *absorption by* the vibration string under forcing, and not really as written the process of *emission from* the string.

However, if we switch the roles of f and $-\gamma\dddot{u}$ and view $-\gamma\dddot{u}$ as input, then we can view f as an emitted wave, which can act as forcing on another system. For frequencies with $\gamma\nu^2 << 1$, we will then have

$$\overline{f_\nu^2} \sim \gamma\overline{\ddot{u}_\nu^2} >> \overline{(\gamma\dddot{u})^2} \approx \gamma\nu^2\gamma\overline{\ddot{u}_\nu^2}$$

with thus emission boosted by resonance, as in the resonant amplification of a musical instrument (e.g the body of a guitar).

In both cases, the relation $R_\nu \sim \overline{f_\nu^2}$ expresses that the energy of the absorbed radiation is equal to the outgoing emitted radiation.

7 Planck's Radiation Law

> Would it not be possible to replace the hypothesis of light quanta by another assumption that would also fit the known phenomena? If it is necessary to modify the elements of the theory, would it not be possible to retain at least the equations for the propagation of radiation and conceive only the elementary processes of emission and absorption differently than they have been until now? (Einstein)

7.1 The Alexander Cut-Off by Planck

The Rayleigh-Jeans Law leads to an ``ultraviolet catastrophe'' because without some form of high-frequency limitation, the total raditation will be unbounded. Classical wave mechanics thus appears to lead to an absurdity, which has to be resolved in one way or the other. In an ``act of despair'' Planck escaped the catastrophy by cutting the Gordian Knot simply replacing classical wave mechanics with a new statistical mechanics where high frequencies were assumed to be rare; ``a theoretical interpretation had to be found at any price, no matter how high that might be...". It is like kicking out a good old horse which has served fine for many purposes, just because it has a tendency to ``go to infinity" at a certain stimulus, and replacing it with a completely new wild horse which you don't understand and cannot control.

The price of throwing out classical wave mechanics is very high, and it is thus natural to ask if this is really necessary. Is there a form of classical mechanics without the ultraviolet catastrophe? Can a cut-off of high frequencies be performed without a Gordian Cut-off?

We believe this is possible, and it is certainly highly desirable, because statistical mechanics is difficult to both understand and apply. We shall thus present a resolution where Planck's statistical mechanics is replaced by deterministic mechanics viewing physics as a form of *analog computation with finite precision* with a certain dissipative diffusive effect, which we model by digital computational mechanics associated with a certain numerical dissipation.

It is natural to model finite precision computation as a dissipative/diffusive effect, since finite precision means that small details are lost as in smoothing by damping of high frequencies which is the effect of dissipation by diffusion.

We consider computational mechanics in the form of the *General Galerkin (G2)* method for the wave equation, where the dissipative mechanism arises from a weighted least squares residual stabilization [5]. We shall first consider a simplified form of G2 with least squares stabilization of one of the residual terms and corresponding simplified diffusion model. We then comment on full G2 residual stabilization.

7.2 Wave Equation with Radiation and Dissipation

We consider the wave equation (4) with radiation augmented by (simplified) G2 diffusion:

$$\begin{aligned} &\ddot{u} - u'' - \gamma\, \dddot{u} - \delta^2 \dot{u}'' = f, \quad -\infty < x, t < \infty, \\ &\dot{E} = \int f \dot{u}\, dx - \int \gamma \ddot{u}^2\, dx, \quad -\infty < t < \infty, \end{aligned} \tag{17}$$

where $-\delta^2\dot{u}''$ models dissipation/diffusion from velocity gradients, $\delta = h/T$ represents a *smallest coordination length* with h a *precision* or *smallest detectable change*, and T is temperature related to the internal energy E by $T = \sqrt{E}$.

The relation $\delta = \frac{h}{T}$ takes the form $|\dot{u}|\delta \sim h$ with $T \sim |\dot{u}|$. A signal with $|\dot{u}|\delta < h$ cannot be represented in coherent form and thus cannot be emitted. This is like the ``Mexican Wave'' around a stadium which cannot be sustained unless people raise their arms properly; the smaller the ``lift'' is (with lift as temperature), the longer is the required coordination length or wave length.

We see that the wave equation is here augmented by an equation for the internal energy E, which thus has a contribution from the dissipation $\int \delta^2(\dot{u}')^2\,dx$ (obtained as above by multiplication by $\dot{u}$).

We assume that incoming frequencies are bounded by a certain maximal frequency ν_{max}, we choose $\gamma = \nu_{max}^{-2}$ and assume $\nu_{max}^{-1} >> \delta^2 = \nu_{cut}^{-2} >> \gamma$, where $\nu_{cut} < \nu_{max}$ is a certain cut-off frequency.

We motivate this set up as follows: If u is a wave of frequency ν in x, then for $\nu > \nu_{cut} = \frac{T}{h} = \frac{1}{\delta}$, we have

$$\delta^2\dot{u}'' \sim \frac{h^2\nu^2}{T^2}\dot{u}$$

which signifies the presence of considerable damping in (17) from the dissipative term since $\frac{h^2\nu^2}{T^2} \geq 1$. Alternatively, we have by a spectral decomposition as above

$$\delta^2\nu^2\dot{u}_\nu^2 \sim f_\nu^2$$

and thus since $\gamma << \delta^2$

$$R_\nu = \frac{\gamma}{\delta^2}\delta^2\nu^2\dot{u}_\nu^2 << f_\nu^2.$$

Thus absorbed waves with $\nu > \nu_{cut}$ are damped and not fully radiated with the corresponding missing energy contributing to the internal/heat energy E and increasing temperature T.

We will also find cut-off for lower frequencies due to the design of the dissipative term $\delta^2\dot{u}''$ corresponding to a simplified form of G2 discretization. In real G2 computations the cut-off will have little effect on frequencies smaller than ν_{cut}. In the analysis we assume this to be the case, which corresponds to allowing δ to depend on ν so that effectively $\delta = 0$ for $\nu \leq \nu_{cut} = \frac{1}{\delta}$. We then obtain a Planck Law of the form

$$R_\nu(T) = \gamma T \nu^2 \theta_h(\nu, T) = \gamma T \min(\nu^2, \nu_{cut}^2) \tag{18}$$

with a computational high-frequency cut-off factor $\theta_h(\nu, T) = 1$ for $\nu \leq \nu_{cut}$ and $\theta_h(\nu, T) = \frac{\nu_{cut}^2}{\nu^2}$ for $\nu_{cut} < \nu < \nu_{max}$ with $\nu_{cut} = \frac{T}{h}$.

Clearly, it is possible to postulate different cut-off functions $\theta_h(\nu, T)$ for example exponential cut-off functions with the effect that $\theta_h(\nu, T) \approx 0$ for $\nu >> \nu_{cut}$. In the next section we study the cut-off in G2.

The net result is that absorbed frequencies above cut-off will heat the string, while absorbed frequencies below cut-off will be radiated without heating (in the ideal case with the dissipation only acting above cut-off).

If the incoming radiation has a Rayleigh-Jeans spectrum $\sim \gamma T \nu^2$, then so has the outgoing radiated spectrum $R_\nu(T_\nu) \sim \gamma T \nu^2$ with $T_\nu \sim T$ for $\nu \leq \nu_{cut}$. In particular, the outgoing radiated spectrum is equilibrated with all colors having the same temperature, if the incoming spectrum is equilibrated.

Another way of expressing this fundamental property of the vibrating string model is to say that frequencies below cut-off will be absorbed and radiated as *coherent* waves, while frequencies above cut-off will be absorbed transformed into internal energy in the form of incoherent waves which are not radiated. High frequencies thus may heat the body and thereby decrease the coordination length and thereby allow absorption and emission of higher frequencies.

Note that the internal energy E is the sum over the internal energies E_ν of frequencies $\nu \leq \nu_{cut} = \frac{T}{h}$ with $E_\nu \sim T$ assuming equilibration in temperature, and thus $E \sim T^2$ motivating the relation $T = \sqrt{E}$.

7.3 Cut-Off by Residual Stabilization

The discretization in G2 is accomplished by residual stabilization of a Galerkin variational method and may take the form: find $u \in V_h$ such that for all $v \in V_h$

$$\int (A(u) - f)v\,dxdt + \delta^2 \int (A(u) - f)A(V)\,dxdt = 0, \qquad (19)$$

where $A(u) = \ddot{u} - u'' - \gamma\dddot{u}$ and V is a primitive function to v (with $\dot{V} = v$), and V_h is a space-time finite element space continuous in space and discontinuous in time over a sequence of discrete time levels.

Here $A(u) - f$ is the residual and the residual stabilization requires $\delta^2(A(u) - f)^2$ to be bounded, which should be compared with the dissipation $\delta\ddot{u}^2$ in the analysis with $\ddot{u}^2$ being one of the terms in the expression $(A(u) - f)^2$. Full residual stabilization has little effect below cut-off, acts like simplified stabilization above cut-off, and effectively introduces cut-off to zero for $\nu \geq \nu_{max}$ since then $\gamma|\dddot{u}| \sim \gamma\nu^2|\dot{u}| = \frac{\nu^2}{\nu_{max}^2}|\dot{u}| \geq |\dot{u}|$, which signifies massive dissipation.

7.4 The Sun and the Earth

If an incoming spectrum of temperature T_{in} is attenuated by a factor $\kappa << 1$ (representing a solid viewing angle $<< 180°$), so that the incoming radiation $f_\nu^2 = \kappa\gamma T_{in}\nu^2$ with cut-off for $\nu > \frac{T_{in}}{h}$ (and not for $\nu > \frac{\kappa T_{in}}{h} << \frac{T_{in}}{h}$).

This may represent the incoming radiation from the Sun to the Earth with $\kappa \approx (\frac{R}{D})^2 \approx 0.005^2$ the viewing angle of the Sun seen from the Earth, R the radius of the Sun and D the distance from the Sun to the Earth. The amplitude of the incoming radiation is thus reduced by the factor κ, while the cut-off of the spectrum is still $\frac{T_{in}}{h}$.

The Earth at temperature T acting like the vibrating string will convert absorbed radiation into heat for frequencies $\nu > \frac{T}{h}$, that is as long a $T < T_{in}$, while radiating $\sim \gamma T^4$ and absorbing $\sim \kappa T_{in}^4$, thus reaching equilibrium with $\frac{T^4}{T_{in}^4} \approx \kappa$. With $T_{in} = 5778$ K and $\kappa = 0.005^2$, this gives $T \approx 273K$ (including a factor 4 from the fact that the the disc area of the Sun is πR^2 and the Earth surface area $4\pi r^2$ with r the Earth diameter).

The amplitude of the radiation/light emitted from the surface of the Sun at 5778 K when viewed from the Earth is scaled by the viewing solid angle (scaling with the square of distance from the Sun to the Earth), while the light spectrum covering the visible spectrum centered at a wavelength of $0.5\,\mu m$, remains the same. The Earth emits infrared radiation (outside the visible spectrum centered at $10\,\mu m$) at an effective blackbody temperature of 255 K (at a height of 5 km), thus with almost no overlap with the incoming Sunlight spectrum. The Earth thus absorbs high-frequency reduced-amplitude radiation and emits low-frequency radiation, and thereby acts as a transformer of radiation from high to low frequency: Coherent high-frequency radiation is absorbed and dissipated into incoherent heat energy, which is then emitted as coherent low-frequency radiation.

The transformation only acts from high-frequency to low-frequency, and is an irreversible process representing a 2nd Law.

7.5 The Temperature of Radiation

The temperature T_{in} of incoming radiation with an attenuated Planck spectrum $R_\nu = \kappa\gamma T_{in}\nu^2$ with cut-off for $\nu > \frac{T_{in}}{h}$, can be read from the cut-off (Wien's Law), while the amplitude does not carry this information unless the attenuation factor κ is known. For the outgoing spectrum $\gamma T\nu^2$, we noted that $T \leq T_{in}$ since heating requires dissipative cut-off after absorption, which requires that incoming radiation contains higher frequencies than outgoing and that is only possible if the temperature of the incoming radiation is higher than the present temperature of the absorbing body, as also expressed in the basic energy balance (5): Energy is transferred only from warmer to cooler.

7.6 Radiative Heating vs Resonance

We have seen that the vibrating string gets heated by incoming frequencies above cut-off through a dissipative mechanism acting in-phase through a forcing without resonance, while frequencies below cut-off trigger a radiative mechanism acting out-of-phase with resonance.

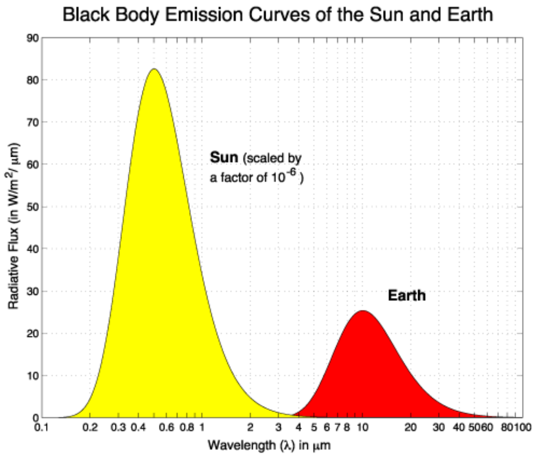

Figure 5: Blackbody spectrum of the Sun and the Earth.

7.7 A Fourier Law of Radiative Heat Transfer

Suppose an incoming radiation has a spectrum $\kappa\gamma T_{in}\nu$ of temperature T_{in} (with $\kappa \leq 1$) is absorbed and then emitted with spectrum $\gamma T\nu^2$. The heating effect from frequencies above cut-off at T, assuming $h = 1$, is then given by

$$\int_T^{T_{in}} \kappa\gamma T_{in}\nu^2\, d\nu \sim \kappa\gamma T_{in}(T_{in}^3 - T^3) \sim \kappa\gamma T_{in}^3(T_{in} - T) \qquad (20)$$

which can be viewed as a Fourier Law with heating proportional to temperature difference $T_{in} - T \geq 0$. Note that if $T_{in} < T$, then there is no heating since there is no cut-off: all of absorbed radiation is emitted.

7.8 The 2nd Law and Irreversibility

Radiative heating of a blackbody is an irreversible process, because the heating results from dissipation with coherent high frequency energy above cut-off being transformed into internal heat energy. We have shown that radiative heating requires that the temperature of the incoming radiation is higher than that of the absorbing body.

We assume that the dissipation is only active above cut-off, while the radiation is active over the whole spectrum. Below cut-off radiation is a reversible process since the same spectrum is emitted as absorbed. Formally, the radiation term is dissipative and thus would be expected to transform the spectrum, and the fact that it does not is a remarkable effect of the resonance.

7.9 Aspects of Radiative Heat Transfer

We can find aspects of radiative heating in many different settings, as heat conduction or communicating vessels with the flow always from higher level (temperature) to lower level. But radiative heat transfer is richer in the sense that it involves propagation of both waves and energy.

Let us try with a parallel in psychology: We know that trivial messages radiated from a parent may enter one ear of a child and go out through the other, while less trivial messages would not be listened to at all. However, the alertness of the child may be raised as a result of a ``high temperature'' outburst by the parent which could open the child's mind to absorbing/radiating less trivial messages. We would here distinguish between propagation of message and meaning.

7.10 Reflection vs Blackbody Absorption/Emission

A blackbody emits what it absorbs ($f^2 \rightarrow R$), and it is thus natural to ask what makes this process different from simple reflection (e.g. $f \rightarrow -f$ with $f^2 \rightarrow f^2$)? The answer is that the mathematics/physics of blackbody radiation $f \rightarrow \ddot{u} - u'' - \gamma\dddot{u}$, is fundamenatlly different from simple reflection $f \rightarrow -f$. The string representing a blackbody is brought to vibration in resonance with forcing and the vibrating string emits resonant radiation. Incoming waves thus are absorbed into the blackbody/string and then are emitted depending on the body temperature. In simple reflection there is no absorbing/emitting body, just a reflective surface without temperature.

7.11 Blackbody as Transformer of Radiation

The Earth absorbs incident radiation from the Sun with a Planck frequency distribution characteristic of the Sun surface temperature of about 5778 K and an amplitude depending on the ratio of the Sun's diameter to the distance of the Earth from the Sun. The Earth as a blackbody transforms the incoming radiation to an outgoing blackbody radiation of temperature about 288 K, so that total incoming and outgoing energy balances.

The Earth thus acts as a transformer of radiation and transforms incoming high-frequency low-amplitude radiation to outgoing low-frequency high-amplitude radiation under conservation of energy.

This means that high-frequency incoming radiation is transformed into heat which shows up as low-frequency outgoing infrared radiation, so that the Earth emits more infrared radiation than it absorbs from the Sun. This increase of outgoing infrared radiation is not an effect of backradiation, since it would be present also without an atmosphere.

The spectra of the incoming blackbody radiation from the Sun and the outgoing infrared blackbody radiation from the Earth have little overlap, which means that the Earth as a blackbody transformer distributes incoming high-frequency energy so that all frequencies below cut-off obtain the same temperature. This connects to the basic assumption of statistical mechanics of *equidistribution in energy* or thermal equilibrium with one common temperature.

In the above model the absorbing blackbody inherits the equi-distribution of the incoming radiation (below cut-off) and thereby also emits an equi-distributed spectrum. To ensure that an emitted spectrum is equi-distributed even if the forcing is not, requires a mechanism driving the system towards equi-distribution or thermal equilibrium.

7.12 Connection to Turbulence

The computational dissipation in our radiative model acts like turbulent dissipation in slightly viscous flow, in which high frequency coherent kinetic energy is transformed into heat energy in the form of small scale incoherent kinetic energy. The small coefficient γ in radiation corresponds to a small viscosity coefficient in fluid flow.

Since γ is small, the emitted wave is in one sense a small perturbation, but this is compensated by the third order derivate in the radiation term, with the effect that the radiated energy is not small. Or expressed differently: temperature involves first derivatives (squared) and radiated energy a second derivative multiplied by a small factor. Without the dissipative radiation term, the string cannot emit the energy absorbed and the temperature will then increase without limit. With radiation, the temperature will be limited by the temperature of the incoming wave.

7.13 Stefan-Boltzmann's Law for Two Blackbodies

The classical Stefan-Boltzmann's Law $R = \sigma T^4$ gives the energy radiated from a blackbody of temperature T into an exterior at absolute zero temperature ($0\,K$). For the case of an exterior temperature T_{ext} above zero, standard literature presents the following modification:

$$R = \sigma T^4 - \sigma T_{ext}^4, \tag{21}$$

where the term σT_{ext}^4 conventionally represents "backradiation" from the exterior to the blackbody. It is important to understand that this is a convention which by itself does not prove that there is a two-way flow of energy with σT^4 going out and σT_{ext}^4 coming in.

In our analysis, there is no such two-way flow of heat energy, only a flow of net energy as expressed writing (21) in the following differentiated form

$$R \approx 4\sigma T^3 (T - T_{ext}) \tag{22}$$

with just one term and not the difference of two terms. The mere naming of something does not bring it into physical existence.

7.14 IR Camera

An *IR camera* has sensors reacting to different frequencies of incoming infrared IR radiation outside the visible spectrum, and thus can produce a photographic image of an invisible object. By artificially coloring the image with red representing shorter IR wavelengths and blue longer wavelengths, and connecting frequency to temperature by Wien's displacement law, the image can be read as a thermometer acting at distance.

By connecting temperature to radiative heat energy by Stefan-Boltzmann's Law, an IR camera could mistakenly be viewed as an instrument measuring radiative flow of heat energy. Pointing this instrument to a (cloudy) sky, the reading of the instrument could be $324\,W/m^2$, which by alarmists may be present as experimental evidence of substantial backradiation: Since the instrument shows backradiation, backradiation must exist, right?

No, it is not so simple. What the instrument measures is frequency which can be connected to temperature reflecting some reality, but the connection to massive "downwelling" radiative energy, is just postulated through a certain formula and our analysis indicates that this is fiction rather than reality.

8 Climate Alarmism and Backradiation

> It is virtually certain that increasing atmospheric concentrations of carbon dioxide and other greenhouse gases will cause global surface climate to be warmer. (American Geophysical Union)
>
> We know the science, we see the threat, and we know the time for action is now (Arnold Schwarzenegger)
>
> There are many who still do not believe that global warming is a problem at all. And it's no wonder: because they are the targets of a massive and well-organized campaign of disinformation lavishly funded by polluters who are determined to prevent any action to reduce the greenhouse gas emissions that cause global warming out of a fear that their profits might be affected if they had to stop dumping so much pollution into the atmosphere. (Al Gore)

Global climate can be described as a thermodynamic system with gravitation subject to radiative forcing by blackbody radiation as described in Chapter 18. Understanding climate thus requires understanding blackbody radiation. A main lesson of this chapter is that ``backradiation'' is unphysical because it is unstable and serves no role, and thus should be removed from climate science, cf. fig. 4.

Since climate alarmism feeds on a ``greenhouse effect'' based on ``backradiation'', removing backradiation removes the main energy source of climate alarmism.

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