Newtonian Gravitation of Matter and Antimatter

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Abstract

We present a cosmological model based on Euler's equation for a self-gravitating medium consisting of matter and antimatter of density ρ generated by a gravitational potential φ as $\rho = |\Delta\varphi|$, with presence of matter where $\Delta\varphi > 0$ with visible matter at singularities and dark matter in regions of smoothness, and antimatter where $\Delta\varphi < 0$.

1 Lack of Unified Field Theory

Modern physics is troubled by the perceived incompatibility of the general theory of relativity for gravitation with Maxwell's equations for electromagnetics and quantum mechanics, which results from the fact that general relativity is a geometric theory of curved space-time while electro/quantum mechanics cannot be given such a geometric form. In particular, there is no unified field theory including both gravitation, fluid dynamics and electromagnetics, which can serve as a cosmological model.

Since Newtonian gravitation is compatible with electro/quantum mechanics, it is natural to ask if there is some form of Newtonian gravitation which can replace general relativity in a unified field theory. In this note we explore what a Newtonian fluid model can offer towards a unified field theory in the form of self-gravitating magnetohydrodynamics.

2 The Euler Equations with Gravitation

We consider a cosmological model in Euclidean space-time coordinates (x, t) of *matter* and *antimatter* subject to gravitation defined by a *gravitational potential* φ connected to matter/antimatter *density* ρ by the equation $\rho = |\Delta\varphi|$, where Δ is the Laplace operator and matter is present where $\Delta\varphi > 0$, antimatter present where $\Delta\varphi < 0$ and there is vacuum where $\Delta\varphi = 0$. We thus consider a Newtonian model in flat space-time with a generalized gravitational potential φ with $\Delta\varphi$ both positive and negative.

We adopt the Euler equations for a compressible perfect gas subject to gravitation expressing conservation of matter/antimatter density ρ, *momentum* $m = \rho u$ and *inter-*

nal energy e: Find (φ, m, e) as functions of (x, t) such that for $t > 0$ and all x

$$\begin{aligned} \dot{\rho} + \nabla \cdot m &= 0, \\ \dot{m} + \nabla \cdot (mu) + \nabla p + \rho \nabla \varphi &= 0, \\ \dot{e} + \nabla \cdot (eu) + p \nabla \cdot u &= 0, \\ \rho &= |\Delta \varphi|, \\ \varphi(x,0) = \varphi^0(x),\ m(x,0) = m^0(x),\ e(x,0) &= e^0(x), \end{aligned} \tag{1}$$

where $u = \frac{m}{\rho}$ is matter/antimatter *velocity*, $p = \gamma e$ is *pressure* with $\gamma \geq 0$ a *gas constant*, $\dot{v} = \frac{\partial v}{\partial t}$ and (φ^0, m^0, e^0) given initial values.

With $\Delta\varphi > 0$ this is the usual Euler equations for a prefect self-gravitating gas assuming $\Delta\varphi^0 > 0$ corresponding to the presence of only matter initially. The novelty here is that $\Delta\varphi^0$ is allowed to be also negative corresponding to the presence of both matter an antimatter initially.

We emphasize that we view the gravitational potential φ as a primary dependent variable with the density $\rho = |\Delta\varphi|$ as a derived quantity with matter present where $\Delta\varphi > 0$ and antimatter present where $\Delta\varphi < 0$. We may view the initial value φ^0 as a perturbation of zero generating both matter and antimatter as out of nothing, like writing 0 = +1 - 1.

We now turn to a basic conceptual analysis of the model, assuming first that $\gamma = 0$ so that $p = 0$.

3 Concentration-Thinning

The momentum equation can alternatively be expressed as Newton's 2nd law in the form

$$\rho \frac{Du}{Dt} = -\rho \nabla \varphi, \tag{2}$$

where $\frac{Dv}{Dt} \equiv \dot{v} + u \cdot \nabla v$ is the material time derivative. Matter/antimatter is thus subject to a gravitational force $-\rho\nabla\varphi$ in the direction of decreasing φ thus towards local minima of φ typically associated with $\Delta\varphi > 0$ and thus presence of matter, and away from local maxima associated with $\Delta\varphi < 0$ and presence of antimatter. The gravitational force is thus directed inward for matter regions and outward for antimatter regions.

The effect of the gravitational force is thus a tendency of concentration of matter with $\nabla \cdot u < 0$ and thinning of antimatter with $\nabla \cdot u > 0$. This connects to conservation of matter/antimatter expressed as

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot u \tag{3}$$

where we see ρ increasing where $\nabla \cdot u < 0$ and ρ decreasing where $\nabla \cdot u > 0$.

4 Visible and Dark Matter

We identify *visible matter* with point singularities at a point $\hat{x}$ of the gravitational potential φ of the basic form $\varphi(x) \sim |x - \hat{x}|^{-1}$ corresponding to matter density as a delta-function at $\hat{x}$. We refer to smooth $\Delta\varphi > 0$ as *dark matter* and thus decompose matter into visible matter and dark matter.

The formation of visible matter connects to concentration of matter regions in gravitational collapse.

5 Repulsion Matter-Antimatter

We have seen that a matter region tends to concentrate and thus two matter regions not separated by antimatter will attract each other. On the other hand, the attractive force between two matter regions is diminshed if there is an antimatter region in between because of its outward gravitational force. This effect can be interpreted as an effect of repulsion between matter and antimatter which comes out as repulsion between matter regions separated by antimatter.

This connects to the observed apparent repulsion between galaxies separated by large voids causing an apparent expansion of the Universe by *dark energy* [3, 4], if the large voids are identified as regions of antimatter.

6 Action at Distance vs Local Differentiation

We here view the gravitational potential φ as the primary variable, from which matter/antimatter density ρ is generated by differentiation as $\rho = |\Delta\varphi|$, and not as usual matter density as given from which the gravitational potential is generated by solving or integrating the differential equation $\Delta\varphi = \rho$ with $\rho \geq 0$. In the first case the generation is a local operation of differentiation while in the second case it is global with action at distance.

Viewing the gravitational potential as given thus relieves us from the seemingly impossible task of explaining action at distance.

7 2nd Law

Solutions of (1) satisfy a 2nd law of thermodynamics of the form ([1, 2]):

$$\dot{K} = W - D + P, \quad \dot{E} = -W + D, \quad D \geq 0, \tag{4}$$

where, assuming first the presence of only matter with $\Delta\varphi \geq 0$,

$$\begin{aligned}
K &= \int \frac{1}{2}\rho|u|^2\,dx, \quad E = \int e\,dx, \\
W &= -\int \nabla p\cdot u\,dx = \int p\nabla\cdot u\,dx, \\
P &= -\int \rho\nabla\varphi\cdot u\,dx = \int \varphi\nabla\cdot m\,dx = -\int \varphi\dot{\rho}\,dx = \dot{\Phi}, \\
\Phi &= -\frac{1}{2}\int \varphi\Delta\varphi\,dx = \frac{1}{2}\int |\nabla\varphi|^2\,dx,
\end{aligned}$$

and D is turbulent dissipation. By summation

$$\frac{d}{dt}(K + E - \Phi) = 0$$

stating that the *total energy*, the sum of *kinetic energy* K, *heat energy* E and *gravitational energy* $-\Phi$, is conserved. This formulation of the 2nd is fundamentally the same as a formulations in terms of entropy, with the advantage that turbulent dissipation has a clear physical significance while entropy remains mysterious, as explained in detail in [2].

In gravitational collapse the intensity $\frac{1}{2}|\nabla\varphi|^2$ of the gravitational energy Φ increases locally, with corresponding increase of kinetic energy K and transfer to internal heat energy E causing pressure increase until balance with the gravitational force.

With also antimatter present with $\Delta\varphi < 0$, we have (modulo boundary terms)

$$\Phi = \frac{1}{2}\int_{+} |\nabla\varphi|^2\, dx - \frac{1}{2}\int_{-} |\nabla\varphi|^2\, dx,$$

where $\int_+$ indicates integration over the domain with $\Delta\varphi \geq 0$, and $\int_-$ over $\Delta\varphi < 0$. We notice the the gravitational energy of antimatter is positive securing a smooth distribution in accordance with thinning.

8 Generalization to Finite Speed of Propagation

It is natural to consider a generalization of Poisson's equation $\rho = \Delta\varphi$ into a wave equation

$$\rho = \Delta\varphi - \frac{1}{c^2}\ddot{\varphi} \tag{5}$$

where c represents a finite speed of propagation of gravitation, with corresponding Euler equations

$$\begin{aligned} \dot{\rho} + \nabla\cdot m &= 0, \\ \dot{m} + \nabla\cdot(mu) + \nabla p + \rho\nabla\varphi &= 0, \\ \dot{e} + \nabla\cdot(eu) + p\nabla\cdot u &= 0, \\ \tfrac{1}{c^2}\ddot{\varphi} - |\Delta\varphi| &= -\rho, \end{aligned} \tag{6}$$

together with initial values for ρ, m, e, φ and $\dot{\varphi}$. In this case all variables (ρ, φ, m, e) appear with time-derivate and can be updated computationally by time-stepping.

We can view (6) as a relaxation of (1) corresponding to using a retarded potential. With proper choice of the speed of propagation c, this model can be made to conform with the observed anomalous precession of the perihelion of Mercury, which thus does not require general relativty.

Relaxation with instead a parabolic equation of the form

$$\frac{1}{c}\dot{\varphi} - |\Delta\varphi| = -\rho, \tag{7}$$

is also thinkable.

9 Summary

1. Matter/antimatter of (non-negative) density ρ is generated from a gravitational potential φ as $\rho = |\Delta\varphi|$ with presence of matter where $\Delta\varphi > 0$ and antimatter where $\Delta\varphi < 0$ and vacuum where $\Delta\varphi = 0$.

2. In matter regions visible matter may be associated with singularies of φ and dark matter with regions of smoothness of φ.

3. The gravitational force given by $-\nabla\varphi$ is directed inward for matter regions causing concentration and outward for antimatter regions causing thinnning with an effect of repulsion between matter and antimatter thus representing dark energy.

4. The initial value φ^0 with $\Delta\varphi$ of both signs may be viewed as a perturbation from zero with equal amounts of matter and antimatter.

5. With relaxation to finite speed of propagation of gravitation, the model conforms with the observed precession of Mercury.

6. The model appears to include both dark matter and dark energy as effects of a gravitational potential φ with $\Delta\varphi$ of both signs, and allows a start from a perurbation of zero rather than from a Big Bang.

References

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[3] L. Amendola and S. Tsujikawa, *Dark Energy, Theory and Observations*, Cambridge University Press, 2010.

[4] Yun Wang, *Dark Energy*, Wiley 2010.