

New Theory of Flight

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Abstract

We present a new mathematical theory explaining the miracle of flight which is fundamentally different from the existing theory by Kutta-Zhukovsky-Prandtl formed 100 year ago. The new theory is based on a new resolution of d'Alembert's paradox showing that slightly viscous bluff body flow can be viewed as zero-drag/lift potential flow modified by a specific separation instability into turbulent flow with nonzero drag/lift. For a wing the separation mechanism maintains the large lift of potential flow generated at the leading edge at the price of small drag, resulting in a lift to drag quotient in the range 10 – 70 which allows flight at affordable power. The new mathematical theory is supported by computed turbulent solutions of the Navier-Stokes equations with small friction boundary conditions in close accordance with observations.

1 Why is it Possible to Fly?

What keeps a bird or airplane in the air? How can the flow of air around a wing generate large *lift* L (balancing gravitation) at small *drag* D (requiring forward thrust) with a *lift to drag ratio* also referred to as *finesse* $\frac{L}{D}$ ranging from 10 for short wings to 70 for the long thin wings of extreme gliders, which allows flying at affordable power for both birds and airplanes?

An albatross with *finesse* $\frac{L}{D} = 50$ can glide 50 meters upon losing 1 meter in altitude. A 525 ton Airbus 380 with $\frac{L}{D} = 15$ is carried by a thrust of 35 tons, corresponding to $\frac{1}{4}$ of maximal thrust with $\frac{3}{4}$ required for acceleration at take-off. The dream of human-powered flight came true in 1977 on 60 m^2 wings of the *Gossamer Albatross* generating a lift of 100 kp at a thrust of 5 kp (thus with $\frac{L}{D} = 20$) at a speed of 5 m/s supplied by a 0.3 hp human powered pedal propeller.

The fundamental question of flight concerns *subsonic flight* with the flow of air being nearly incompressible. Experience shows that subsonic flight with $\frac{L}{D} > 10$ is possible if the *Reynolds number* is larger than about 5×10^5 [32], which includes larger birds, propeller airplanes and jetliners at takeoff and landing, but not small birds and insects because the Reynolds number is too small and not cruising jetliners in transonic flight or supersonic flight.

Experience shows that L increases quadratically with the speed and linearly with the *angle of attack*, that is the tilting of the wing from the direction of flight, until *stall* at about 15 degrees, when D abruptly increases and L/D drops below 5 making sustained flight impractical.

Is there a theory of subsonic flight explaining why $\frac{L}{D}$ of a standard wing can be as large as 20 until stall at 15 degrees? What is the dependence of lift and drag on wing form, wing area, angle of attack and speed? Can engineers compute the distribution of forces on an Airbus 380 during take-off and landing using mathematics and computers, or is model testing in wind tunnels the only way to figure out if a new design will work?

Subsonic flight is accurately modeled by the *incompressible Navier-Stokes equations*. The above questions can directly be translated into questions about solutions of the incompressible Navier-Stokes equations. But for the large Reynolds numbers of flight, solutions are turbulent and defy analytical expression. To understand flight thus requires understanding relevant aspects of turbulent solutions of Navier-Stokes equations. Let's see what books, media and authorities offer us.

2 Classical Text-Book Theory of Flight

The current mathematical theory of subsonic flight presented in standard text books [35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46] was developed by Kutta-Zhukovsky-Prandtl in the beginning of the last century after powered flight was shown to be possible by the Wright brothers in 1903. In short, Kutta-Zhukovsky formed a theory for lift without drag and Prandtl a theory for drag without lift, but a full theory has been missing through the development of aviation into our days, as illustrated by the 2003 New York Times headline *What Does Keep Them Up There?* [4]:

- *To those who fear flying, it is probably disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issue underlying this endeavor: what keeps planes in the air?*

NASA Glenn Research Center confirms on its web site by dismissing all popular science theories for lift, including your favorite one, as being incorrect, but refrains from presenting any theory claimed to be correct and ending with: *To truly understand the details of the generation of lift, one has to have a good working knowledge of the Euler Equations.*

Is it possible that NASA cannot explain what keeps planes in the air? Yes, it is possible: birds fly without explaining anything. The state-of-the-art theory

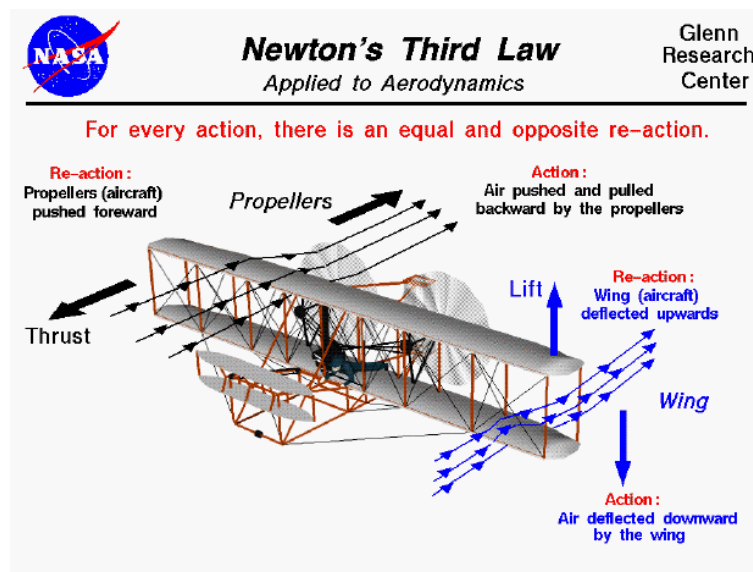


Figure 1: Tautological explanation of the flight of Wright brothers *The Flyer* by NASA:: There is upward lift on the wing from the air as a reaction to a downward push on the air from the wing.

of flight can be summarized as either (i) correct and trivial, or (ii) nontrivial and incorrect because essential turbulent effects are missing:

- Downwash generates lift: trivial without explanation of reason for downwash from suction on upper wing surface.
- Low pressure on upper surface: trivial without explanation why.
- Low pressure on curved upper surface because of higher velocity (by Bernouilli's law), because of longer distance: incorrect.
- Coanda effect: The flow sticks to the upper surface by viscosity: incorrect.

- Kutta-Zhukovsky: Lift comes from circulation: incorrect.
- Prandtl: Drag comes mainly from viscous boundary layer: incorrect.

3 New Flight Theory: Turbulent Navier-Stokes

In this article we present a new mathematical theory of both lift and drag in subsonic flight at large Reynolds number, which is fundamentally different from the classical theory of Kutta-Zhukovsky-Prandtl. The new theory is based on a new resolution[17] of d'Alembert's paradox showing that large Reynolds number incompressible bluff body flow can be viewed as zero-drag/lift potential flow modified by a specific separation instability referred to as *slip-separation* [19] into turbulent flow with nonzero drag/lift.

The resolution of d'Alembert's paradox opens to understanding the generation of both lift and drag of a wing from an analysis of potential flow and slip-separation as the determining factor of turbulent flow.

Our analysis shows that lift does not originate from circulation, in contradiction to Kutta-Zhukovsky, and that drag does not originate from a boundary layer, in contradiction to Prandtl. Our analysis shows that flight can be understood because the relevant aspect of turbulence of slip-separation can be described in analytical mathematical terms.

The new theory of flight is the result of a new capability of computing turbulent solutions of the incompressible Navier-Stokes equations at affordable cost for large Reynolds number using slip or small friction force boundary condition documented in detail in [18, 48, 48] allowing in particular accurate computation of lift and drag of arbitrary bodies. This is a new capability to be compared with state-of-the-art restricted by Prandtl's dictate to use no-slip velocity boundary conditions generating thin boundary layers requiring impossible quadrillions of mesh points [25] to resolve.

The slip boundary condition thus is crucial: It is good model of physics for large Reynolds number, makes computational simulation possible, and opens to a mathematical theory of flight because it builds on potential flow and slip-separation which are open to mathematical analysis.

The new theory, first presented in [15, 18] with preliminary computational results, is supported by computed turbulent solutions with lift and drag in close correspondence to experimental observation over the whole range of angles of attack including stall as shown in Fig. 3 with details in [22].

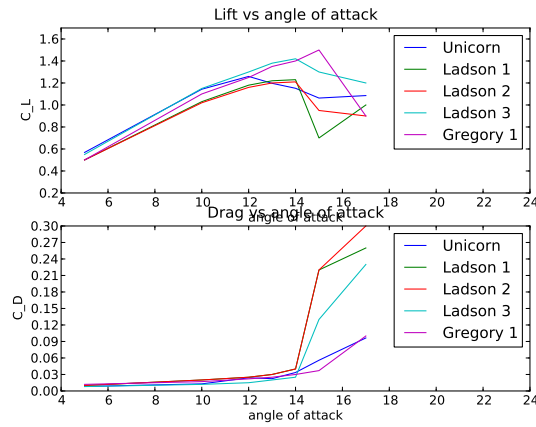


Figure 2: Lift and drag coefficients C_L and C_D of a long NACA012 wing for different angles of attack: The blue curve shows computed coefficients by solving the Navier-Stokes equations by Unicorn [?] compared to different wind tunnel experiments by Gregory/O’Reilly and Ladson.

In this article we focus on a description of the basic features of the aerodynamics of a wing revealed by computation, referring to our cited work for details of the computations. In short, we use a stabilized finite element method with automatic turbulence model and automatic duality-based error control guaranteeing correct lift and drag up to tolerances of a few percent.

4 Mathematical Theory of Flight

The miracle of flight can be explained qualitatively by the following mathematical properties of large Reynolds number incompressible flow around a wing as illustrated in Fig. 3:

- Potential flow can only separate at stagnation to zero flow velocity [19].
- Non-separation of potential flow before the trailing edge creates substantial lift from suction on the upper surface of wing.
- Slip-separation at the trailing edge creates downwash and maintains lift in contrast to potential flow separation without downwash destroying lift.

- Slip-separation switches the zones of high and low pressure at the trailing edge of potential flow and thus creates lift at the price of small drag.

Slip-separation, analyzed in detail in [19], results form a basic instability mechanism generating counter-rotating low-pressure rolls of streamwise vorticity initiated as surface vorticity resulting from meeting opposing flows as shown in Fig. 4 and 5.

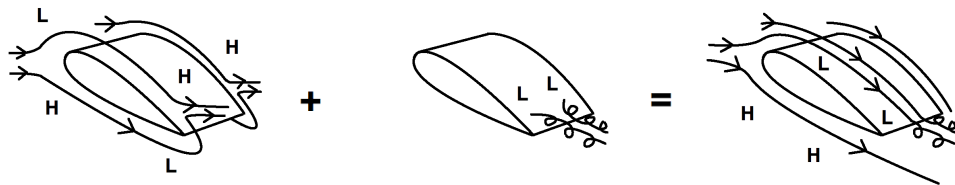


Figure 3: Correct explanation of lift by perturbation of potential flow (left) at separation from physical low-pressure turbulent counter-rotating rolls (middle) changing the pressure and velocity at the trailing edge into a flow with downwash and lift (right).

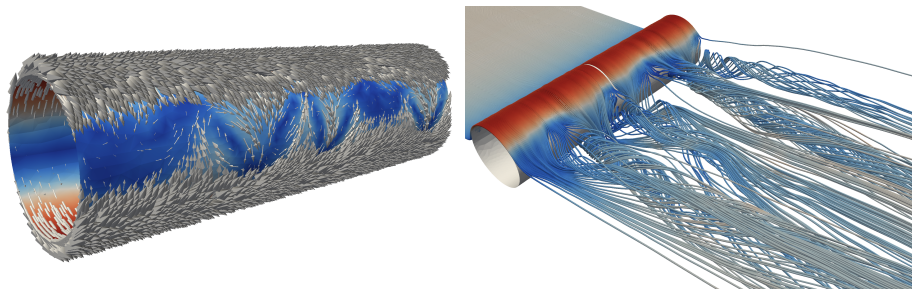


Figure 4: Turbulent separation by surface vorticity forming counter-rotating low-pressure rolls in flow around a circular cylinder, illustrating separation at the trailing edge of a wing [?].

5 Miracle of Flight in Computation

The qualitative mathematical explanation of the miracle of flight is confirmed by computational solution of the Navier-Stokes equations with slip boundary conditions for a NACA0012, which offers quantitative information on lift and drag

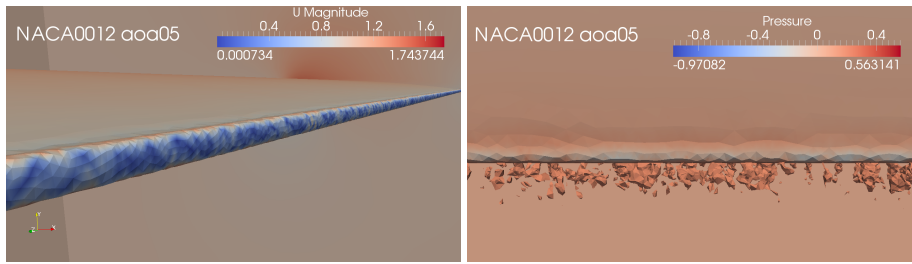


Figure 5: Trailing edge low-pressure slip-separation at $\alpha = 5$.

with total forces displayed in Fig. 3 and force distributions in Fig. 9, both in close agreement with measurement with $L/D \approx 30 - 50$ until beginning stall at $\alpha = 14$. We summarize the findings from the computational results displayed in Fig. 6-12 with details in [22], as follows :

Phase 1: $0 \leq \alpha \leq 8$

At zero angle of attack with zero lift there is high pressure at the leading edge and equal low pressures on the upper and lower crests of the wing because the flow is essentially potential and thus satisfies Bernoulli's law of high/low pressure where velocity is low/high. The drag is about 0.01 and results from slip-separation with low-pressure streamwise vorticity attaching to the trailing edge as shown above. As α increases the low pressure below gets depleted as the incoming flow becomes parallel to the lower surface at the trailing edge for $\alpha = 6$, while the low pressure above intensifies and moves towards the leading edge. The streamwise vortices of the slip-separation at the trailing edge essentially stay constant in strength but gradually shift attachment towards the upper surface. The high pressure at the leading edge moves somewhat down, but contributes little to lift. Drag increases only slowly because of negative drag at the leading edge (leading edge suction).

Phase 2: $8 \leq \alpha \leq 14$

The low pressure on top of the leading edge intensifies as the normal pressure gradient preventing separation increases, thus creating lift peaking on top of the leading edge. The high pressure at the leading edge moves further down and the pressure below increases slowly, contributing to the main lift coming from suction above. The net drag from the upper surface is close to zero because of the negative drag at the leading edge, known as *leading edge suction*, while the drag

from the lower surface increases (linearly) with the angle of the incoming flow, with somewhat increased but still small drag slope. This explains why the line to a flying kite can be almost vertical even in strong wind.

Phase 3: $14 \leq \alpha \leq 16$

Beginning stall with constant lift and quickly increasing drag.

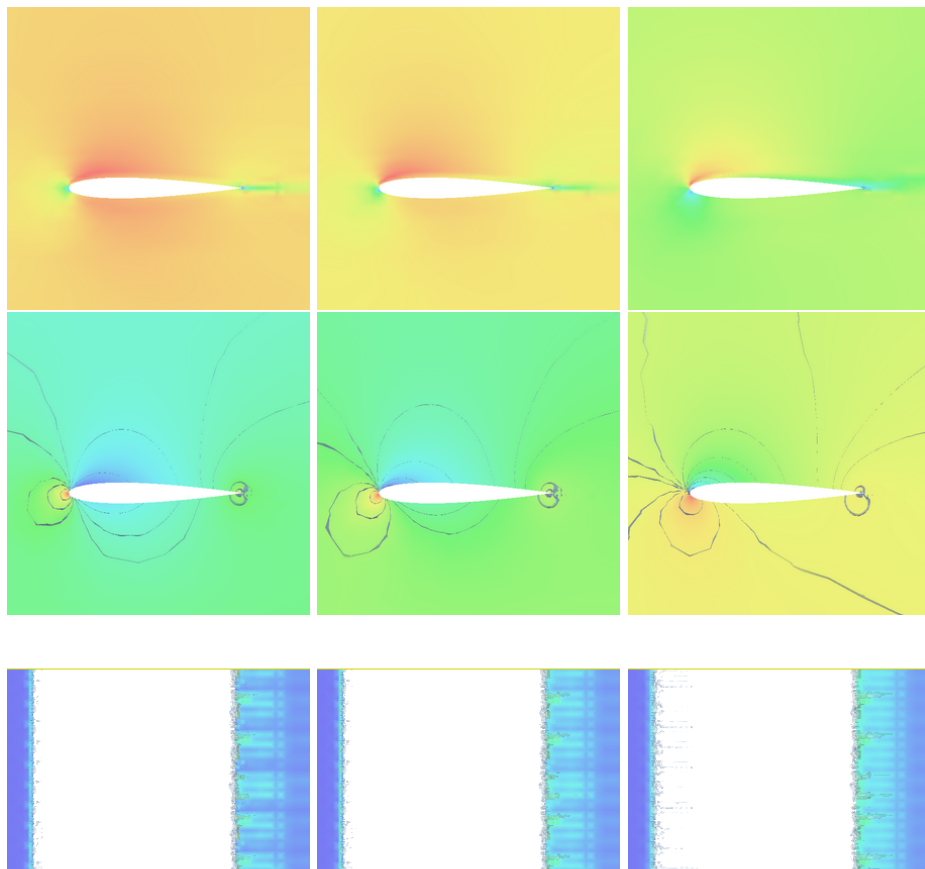


Figure 6: G2 computation of velocity magnitude (upper), pressure (middle), and non-transversal vorticity (lower), for angles of attack 2, 4, and 8°. Notice in particular the rolls of streamwise vorticity at separation.

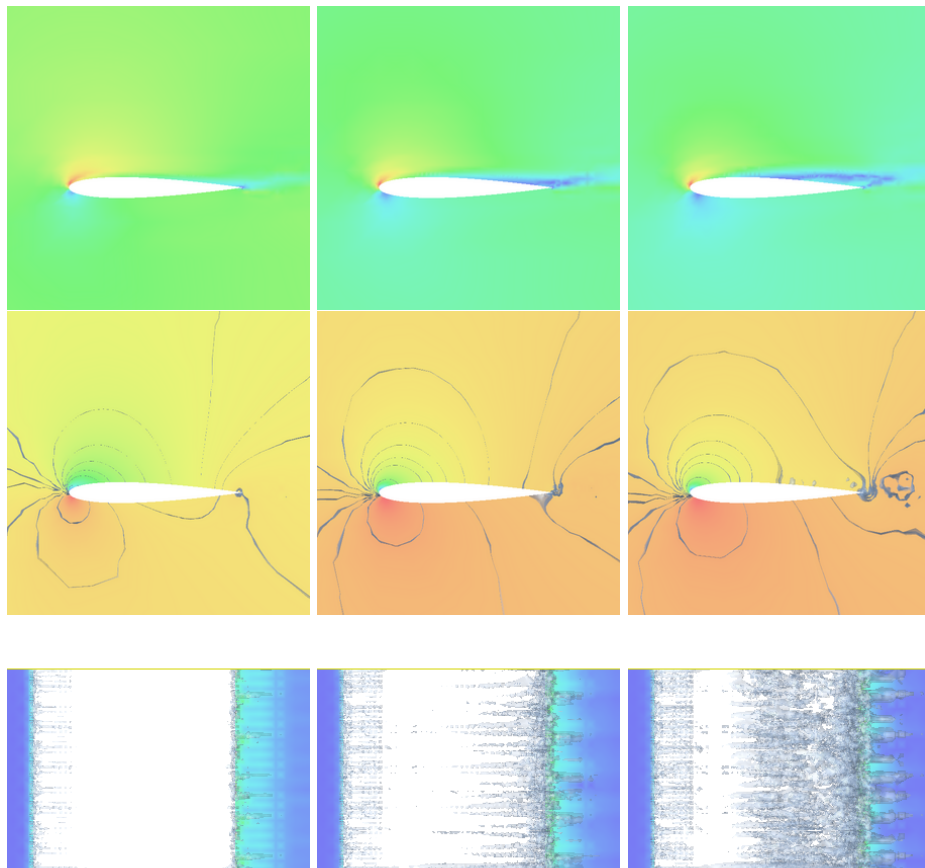


Figure 7: G2 computation of velocity magnitude (upper), pressure (middle), and topview of non-transversal vorticity (lower), for angles of attack 10, 14, and 18° (from left to right). Notice in particular the rolls of streamwise vorticity at separation.

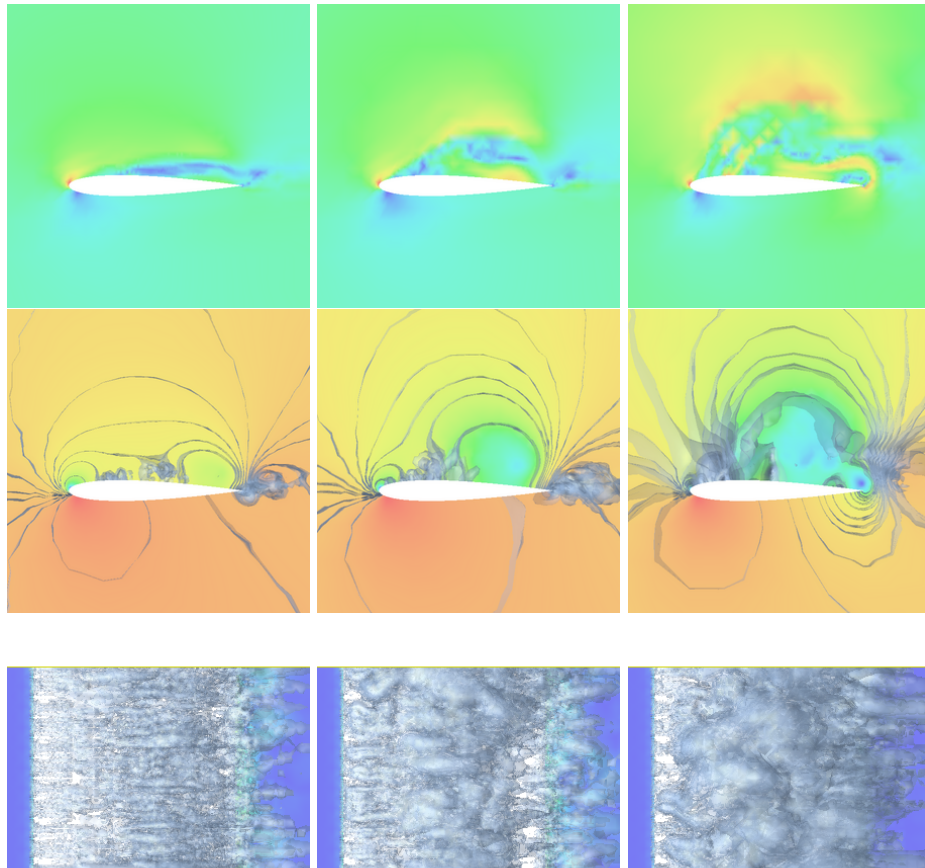


Figure 8: G2 computation of velocity magnitude (upper), pressure (middle), and non-transversal vorticity (lower), for angles of attack 20, 22, and 24° (from left to right).

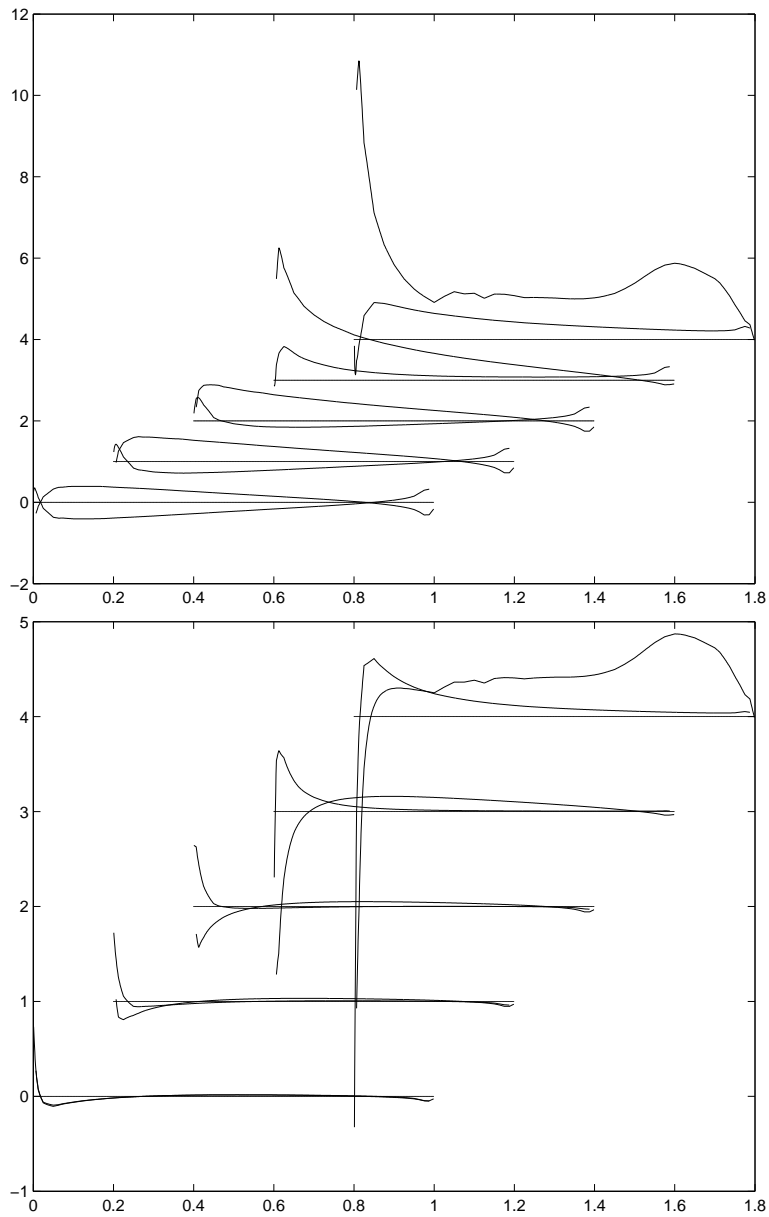


Figure 9: G2 computation of normalized lift (upper) and drag (lower) force distribution acting along the lower and upper parts of the wing, for angles of attack 0, 2, 4, 10 and 18°, each curve translated 0.2 to the right and 1.0 up, with the zero force level indicated for each curve.

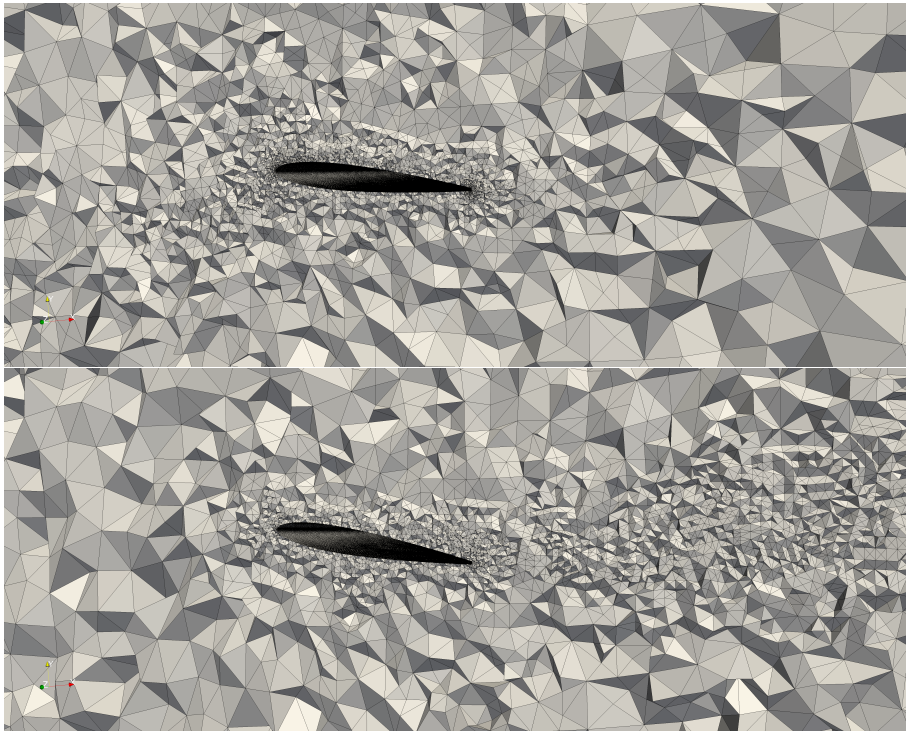


Figure 10: Automatically adapted meshes for $aoa = 10$ and $aoa = 14$.

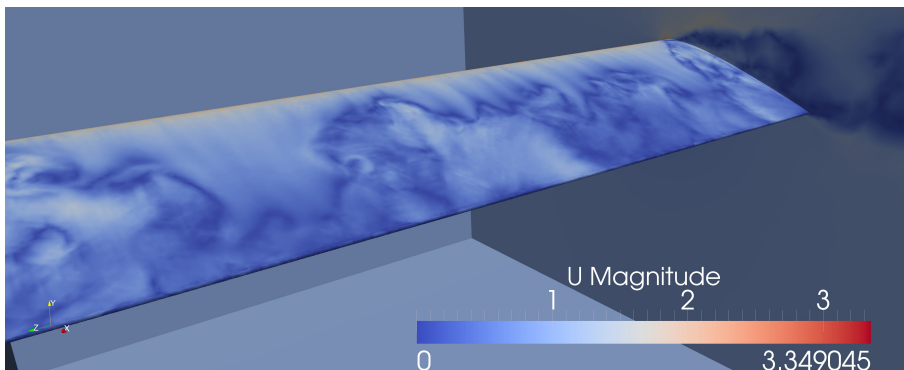
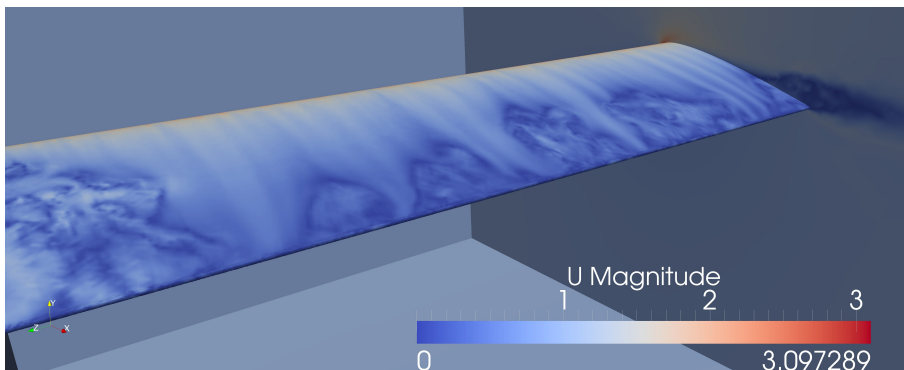
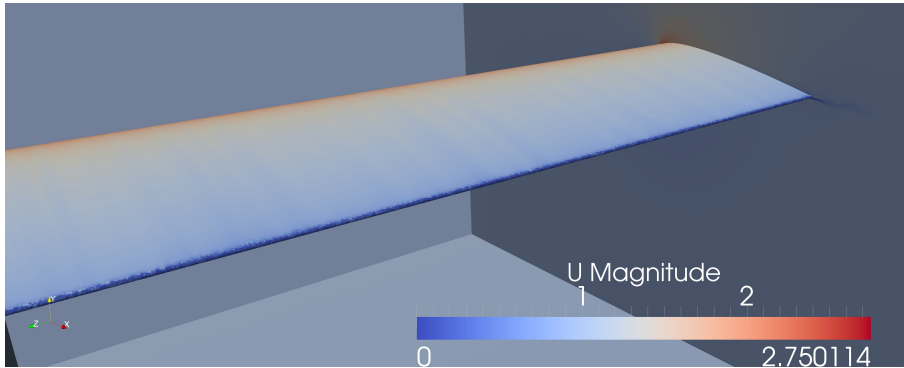


Figure 11: Velocity magnitude on the airfoil surface for $\alpha = 10$ (top), 14 (center) and 17 (bottom) showing that separation pattern moves up the airfoil with increasing α towards stall.

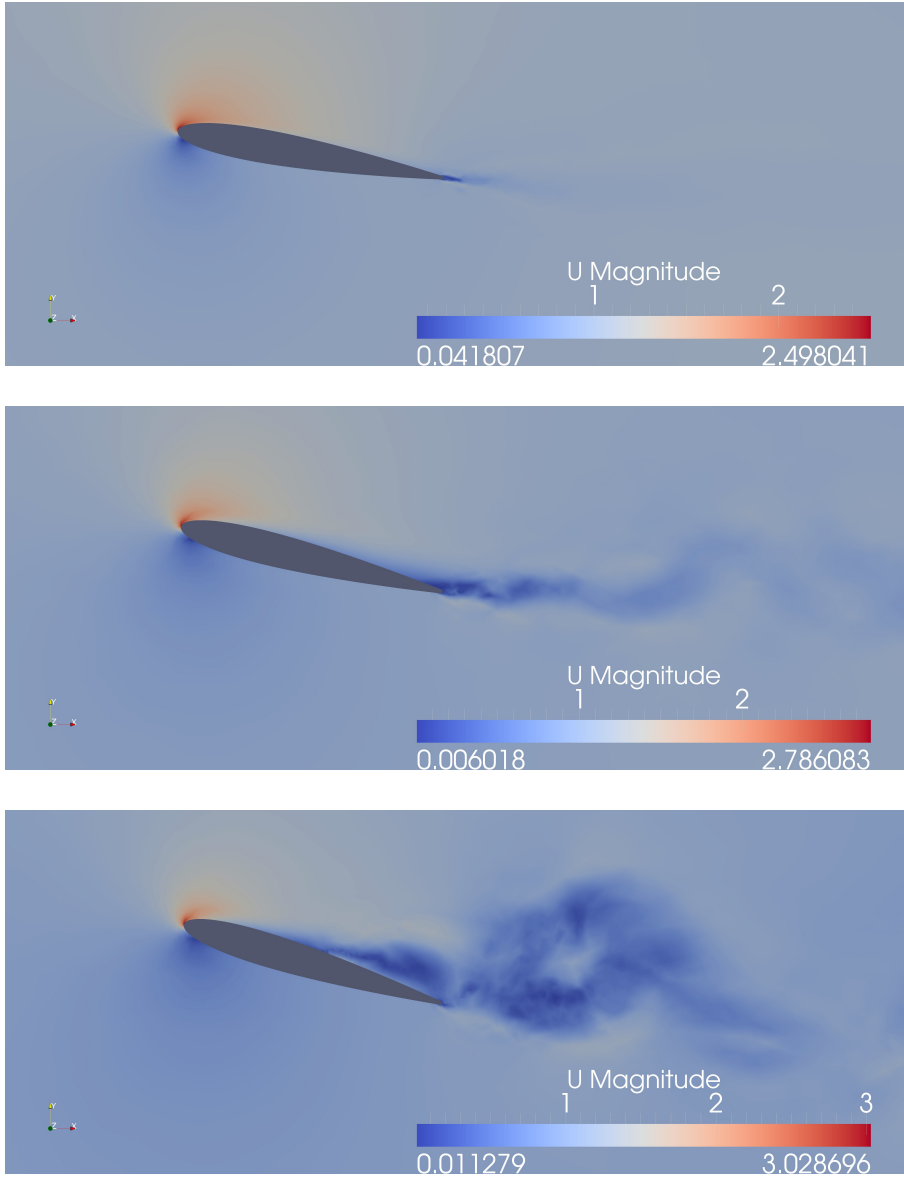


Figure 12: Velocity magnitude around the airfoil for $\alpha = 10$ (top), 14 (center) and 17 (bottom).

References

- [1] John D. Anderson, Ludwig Prandtl's Boundary Layer, <http://www.aps.org/units/dfd/resources/upload/prandtlvol58no12p4248.pdf>
- [2] Y. Bazilevs, C. Michler, V.M. Calo and T.J.R. Hughes, Turbulence without Tears: Residual-Based VMS, Weak Boundary Conditions, and Isogeometric Analysis of Wall-Bounded Flows, Preprint 2008.
- [3] G. Birkhoff, Hydrodynamics, Princeton University Press, 1950.
- [4] Kenneth Chang; Staying Aloft: What keeps them up there?, New York Times, Dec 9, 200
- [5] S. Cowley, Laminar boundary layer theory: A 20th century paradox, Proceedings of ICTAM 2000, eds. H. Aref and J.W. Phillips, 389-411, Kluwer (2001).
- [6] A. Crook, Skin friction estimation at high Reynolds numbers and Reynolds-number effects for transport aircraft, Center for Turbulence Research, 2002.
- [7] A. Ferrante, S. Elghobashi, P. Adams, M. Valenciano, D. Longmire, Evolution of Quasi-Streamwise Vortex Tubes and Wall Streaks in a Bubble-Laden Turbulent Boundary Layer over a Flat Plate, Physics of Fluids 16 (no.9), 2004.
- [8] A. Ferrante and S. E. Elghobashi, A robust method for generating inflow conditions for direct numerical simulations of spatially-developing turbulent boundary layers, J. Comp. Phys., 198, 372-387, 2004.
- [9] J.Hoffman, Simulation of turbulent flow past bluff bodies on coarse meshes using General Galerkin methods: drag crisis and turbulent Euler solutions, Comp. Mech. 38 pp.390-402, 2006.
- [10] J. Hoffman, Simulating Drag Crisis for a Sphere using Friction Boundary Conditions, Proc. ECCOMAS, 2006.
- [11] J. Hoffman, Lift and drag of a delta wing by EG2.
- [12] J. Hoffman, Drag and lift of a car by EG2.
- [13] J. Hoffman and C. Johnson, Blowup of Euler solutions, BIT Numerical Mathematics, Vol 48, No 2, 285-307.

- [14] J. Hoffman and C. Johnson, *Mathematical Theory of Flight*, 2009.
- [15] J. Hoffman and C. Johnson, *Mathematical Secret of Flight*, Normat, 2009.
- [16] J. Hoffman and C. Johnson, *Computational Turbulent Incompressible Flow*, Springer 2007, home page at www.bodysoulmath.org/books.
- [17] J. Hoffman and C. Johnson, Resolution of d'Alembert's paradox, *Journal of Mathematical Fluid Mechanics*, Online First Dec 10, 2008.
- [18] J. Hoffman and C. Johnson, *Computational Turbulent Incompressible Flow*, Springer 2008.
- [19] J. Hoffman and Claes Johnson, Large Reynolds Number Turbulent Flow Separation.
- [20] J. Hoffman and C. Johnson, Modeling Turbulent Boundary Layers by Small Friction.
- [21] J. Hoffman and Claes Johnson, Knol articles.
- [22] Johan and Niklas, Circular cylinder,
- [23] Direct Computation of Lift and Drag of a Wing, J. Hoffman, J. Jansson and C. Johnson.
- [24] J. M. Delery, R. Legendre and Henri Werle: Toward the elucidation of three-dimensional separation, *Annu. Rev. Fluid. Mech.* 2001 33:129-54.
- [25] K. Stewartson, D'Alembert's Paradox, *SIAM Review*, Vol. 23, No. 3, 308-343. Jul., 1981.
- [26] J. Kim and P. Moin, Tackling Turbulence with Supercomputer, *Scientific American*.
- [27] F. W. Lancaster, *Aerodynamics*, 1907.
- [28] Article in Ny Teknik.
- [29] L. Prandtl, On Motion of Fluids with Very Little Viscosity, Third International Congress of Mathematics, Heidelberg, 1904.
- [30] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, 1979.

- [31] James J. Stoker, *Bull. Amer. Math Soc.*
- [32] D. You and P. Moin, *Large eddy simulation of separation over an airfoil with synthetic jet control*, Center for Turbulence Research, 2006.
- [33] G. Schewe, *Reynold's-number effects in flow around more or less bluff bodies*, 4 Intern. Colloquium Bluff Body Aerodynamics and Applications, also in *Journ. Wind Eng. Ind. Aerodyn.* 89 (2001).
- [34] V. Theofilis, *Advance in global linear instability analysis of nonparallel and three-dimensional flows*, *Progress in Aerospace Sciences* 39 (2003), 249-315.
- [35] R. Legendre and H. Werle, *Toward the elucidation of three-dimensional separation*, *Annu. Rev. Fluid Mech.* 33 (2001), 129-54.
- [36] Prandtl, *Essentials of Fluid Mechanics*, Herbert Oertel (Ed.)
- [37] *Aerodynamics of Wingd and Bodies*, Holt Ashley and Marten Landahl,
- [38] *Introduction to the Aerodynamics of Flight*, Theodore A. Talay, Langley Reserach Center,
- [39] *Aerodynamics of the Airpplane*, Hermann Schlichting and Erich Truckenbrodt, Mac Graw Hill
- [40] *Airplane Aerodynamics and Performance*, Jan Roskam and C T Lan,
- [41] *Fundamentals of Aerodynamics*, John D Anderson,
- [42] *Fuhrer durch die Stromungslehre*, L Prandtl,...
- [43] *Aerodynamicss of Wind Turbins*, Martin Hansen,
- [44] *Aerodynamics, Aeronautics and Flight Mechanics*, McCormick,
- [45] *Aerodynamics*, Krasnov,
- [46] *Aerodynamics*, von Karmann,
- [47] *Theory of Flight*, Richard von Mises.

[48] How Stuff Works: *“It is important to realize that, unlike in the two popular explanations described earlier (longer path and skipping stone), lift depends on significant contributions from both the top and bottom wing surfaces. While neither of these explanations is perfect, they both hold some nuggets of validity. Other explanations hold that the unequal pressure distributions cause the flow deflection, and still others state that the exact opposite is true. In either case, it is clear that this is not a subject that can be explained easily using simplified theories. Likewise, predicting the amount of lift created by wings has been an equally challenging task for engineers and designers in the past. In fact, for years, we have relied heavily on experimental data collected 70 to 80 years ago to aid in our initial designs of wing.*

[49] *“Few physical principles have ever been explained as poorly as the mechanism of lift.*

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[51] *http : //www.youtube.com/watch?v = uUMlnIwo2Qo,
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