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Limit theorems for extremal processes generated by a point process with correlated time and space components

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ABSTRACT

The point process $\mathcal{N} = \{(T_k, X_k), k = 0, 1, 2, 3 \ldots\}$ defines the sequence of maxima $\mathcal{M}(t) = \bigvee_{\{k:T_k \le t\}} X_k$. Using time and space scaling it is possible to define different sequences of random time changed extremal processes. The convergence of such sequences to nondegenerate extremal processes is proved in case where the time and space components of the point process are correlated.

1. Introduction

Let us consider the point process $\mathcal{N} = \{(T_k, X_k), k = 0, 1, 2, 3 \ldots\}$ where the time points T_k form an ordinary renewal process determined by the sequence of iid positive random variables (r.v.) Y_k, $k = 1, 2, \ldots$, i.e.

$$T_0 = Y_0 = 0, \quad \text{a.s. and} \quad T_{k+1} = T_k + Y_{k+1}, \quad k = 0, 1, 2, \ldots.$$

Denote by $N(t) = \max\{n : T_n \le t\}$, $t \ge 0$ the corresponding counting process.

The space points X_k, $k = 1, 2, \ldots$ are assumed to be iid nonnegative random variables, and $X_0 = 0$ a.s. Thus, the random vectors $\{(X_k, Y_k), k = 1, 2, \ldots\}$ are independent and identically distributed but the random variables in each pair are correlated. Denote the joint cumulative distribution function (cdf) $F_{XY}(x, y) = \mathbf{P}(X_k \le x, Y_k \le y)$ and the marginal cdfs by $F_X(x) = F_{XY}(x, \infty)$ and $F_Y(y) = F_{XY}(\infty, y)$. Denote also $\bar{F}_X(x) = 1 - F_X(x)$, $\bar{F}_Y(y) = 1 - F_Y(y)$, and $\bar{F}_{XY}(x, y) = 1 - F_X(x) - F_Y(y) + F_{XY}(x, y)$.

Define the sequence of maxima of the space variables X_k

$$M_n = \bigvee_{k=0}^{n} X_k, \quad n = 0, 1, 2, \ldots \tag{1}$$

and the random time changed extremal process

$$\mathcal{M}(t) = M_{N(t)} = \bigvee_{k=0}^{N(t)} X_k, \quad t \ge 0.$$

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The weak convergence of sequences of extremal processes and extremal processes subordinated to random time has been studied extensively in the recent years. Different functional limits are obtained in rather general settings, like multivariate space components and nonlinear normalization, see e.g. Balkema and Pancheva (1996), Silvestrov and Teugels (1998), Pancheva (1998), Pancheva et al. (2006), Meerschaert and Stoev (submitted for publication) and the references therein. In most of the studies concerning the extremal processes *the main assumption is the independence of the time process and the space (or magnitude) variables*. The aim of this note is to prove limit theorems (Theorems 4 and 5) for suitably scaled sequences of random time changed extremal processes $\mathcal{M}_n(.)$, $n = 0, 1, 2, \ldots$ in case where the *time and space components are correlated.* In order to do this we use the duality between the process $\{\mathcal{M}(t), t \geq 0\}$ and the first hitting time process $\{\mathcal{T}(x), x \geq 0\}$ defined by $\mathcal{T}(x) = \inf\{t : \mathcal{M}(t) > x\}$, $x \geq 0$, or equivalently

$$\{\mathcal{T}(x) \leq t\} \iff \{\mathcal{M}(t) > x\}. \tag{2}$$

The pair of processes $\mathcal{M}(t)$ and $\mathcal{T}(x)$ arises naturally in the investigations of stochastic systems that are subject to random shocks at random times. The random variable X_n represents the magnitude of the nth shock which occurs at time T_n. Assuming that the system fails when the magnitude of the shock is greater than the level x, the time $\mathcal{T}(x)$ is then the failure time of the system. This model is well known as the general shock model. It is widely studied in the literature, see e.g. Shanthikumar and Sumita (1983), Anderson (1987), Gut and Huesler (1999), Gut (2001) and the references therein. In these investigations the main object of study is the process $\{\mathcal{T}(x), x \geq 0\}$ and its limiting distributions as $x \to x_F$. Here and later x_F denotes the right endpoint of the support of the r.v.'s X_k, i.e. $x_F = \sup\{x : F_X(x) < 1\} \leq \infty$. We continue the investigation of the process $\mathcal{T}(x)$ by proving a functional limit theorem for it (Theorem 3).

2. Conditions and preliminaries

We assume one of the following conditions for the interarrival times of the renewal sequence.

Condition 1. *The mean of the interarrival times Y_k is finite,*

$$\mu_Y = \mathbf{E}[Y_n] = \int_0^\infty \bar{F}_Y(y)\mathrm{d}y \in (0, \infty).$$

Condition 2. *The interarrival times Y_k have an infinite mean, and*

$$\mu_Y(t) = \int_0^t \bar{F}_Y(y)\mathrm{d}y \sim \frac{t^{1-\beta}L(t)}{\Gamma(2-\beta)}, \quad t \to \infty, \beta \in (0, 1],$$

where $L(.)$ is a function slowly varying at infinity (s.v.f.).

If Condition 1 holds then from the well-known results of renewal theory it follows that for every $t > 0$,

$$\frac{N(tn)}{n/\mu_Y} \xrightarrow{a.s.} t, \quad n \to \infty. \tag{3}$$

If Condition 2 holds let us denote the function

$$\tilde{r}(t) := \frac{t}{\mu_Y(t)\Gamma(2-\beta)} \sim \frac{t^\beta}{L(t)}, \quad \text{as } t \to \infty.$$

Its asymptotic inverse $r(t),\ t > 0$ is defined as follows (see Bingham et al. (1987), Theorem 1.5.12)

$$r(\tilde{r}(t)) \sim \tilde{r}(r(t)) \sim t \quad \text{as } t \to \infty. \tag{4}$$

The function $r(t)$ is regularly varying with exponent $1/\beta \geq 1$.

These two functions provide the proper normalizations for the following limits (see e.g. Meerschaert and Scheffler (2004))

$$\frac{T_{[nt]}}{r(n)} \Rightarrow D_\beta(t) \quad \text{and} \quad \frac{N(nt)}{\tilde{r}(n)} \Rightarrow W_\beta(t), \quad \text{as } n \to \infty, \text{for } t \geq 0$$

in the Skorohod topology. Here and later $\Rightarrow$ denotes the weak convergence in the space $D([0, \infty))$.

The process $D_\beta(t)$ is a one-sided β-stable Lévy motion, $D_\beta := D_\beta(1)$ is almost surely positive r.v. and $\mathbf{E}\left[\mathrm{e}^{-\lambda D_\beta}\right] = \mathrm{e}^{-\lambda^\beta}$, $\lambda > 0$. The process $W_\beta(t)$ is the first hitting time process of $D_\beta(t)$, i.e. $W_\beta(t) = \inf\{s : D_\beta(s) > t\}$. Its Laplace transform is

$$\mathbf{E}[\mathrm{e}^{-\lambda W_\beta(t)}] = \sum_{n=0}^\infty \frac{(-\lambda t^\beta)^n}{\Gamma(1+n\beta)}. \tag{5}$$

For more details about these processes see e.g. Meerschaert and Scheffler (2004).

In the case when the mean interarrival time is finite there are no restrictions on the correlation between X_k and Y_k, whereas in the case when the mean interarrival time is infinite the correlation between X_k and Y_k will be specified by the following condition (Anderson, 1987).

Condition 3. *There exists a function $m(x)$ such that $m(x) \to 1, x \to x_F$, and for every x sufficiently close to x_F, the following relation holds*

$$\bar{F}_Y(y) - \bar{F}_{XY}(x, y) \sim m(x)\bar{F}_Y(y), \quad \text{as } y \to \infty.$$

Proposition 1. (i) *(Shanthikumar and Sumita, 1983; Gut and Huesler, 1999) Assume Condition 1. Then*

$$\frac{\mathcal{T}(x)}{\mu_Y/\bar{F}_X(x)} \xrightarrow{d} \xi, \quad \text{as } x \to x_F,$$

where ξ is a standard exponential random variable.

(ii) *(Anderson, 1987) Assume Conditions 2 and 3. Then*

$$\frac{\mathcal{T}(x)}{r(1/\bar{F}_X(x))} \xrightarrow{d} \xi^{1/\beta} D_\beta, \quad \text{as } x \to x_F,$$

where ξ is a standard exponential random variable independent of D_β.

The last condition concerns the extremal limit laws for the sequence $\{M_n, n = 0, 1, 2, \dots\}$ defined by (1).

Condition 4. *The sequence of random variables $X_k, k = 0, 1, 2, \dots$ belongs to the domain of attraction of the max-stable law $G(x)$, i.e. there exist sequences $A(n) > 0$ and $B(n)$ such that*

$$F_X(A(n)x + B(n))^n \to G(x), \quad \text{as } n \to \infty,$$

for every $x > 0$.

It is well known that the limiting distribution $G(x)$ can take one of the three standard forms (Gumbel, Frechet or Weibull). Further, the following limit exists

$$\frac{M_{[nt]} - B(n)}{A(n)} \Rightarrow E(t), \quad \text{as } n \to \infty, \tag{6}$$

where $E(t)$ is a G-extremal process, whose one-dimensional distributions are $\mathbf{P}(E(t) \le x) = G(x)^t, t > 0$ (see e.g. Resnick (1987) or Lamperti (1964), Theorem 3.2).

Assume that independent copies of the G-extremal process $E(t), t > 0$, and the process $W_\beta(t), t > 0$ are given on a common probability space. Since $W_\beta(t)$ has nondecreasing sample paths then the subordinated process

$$\mathcal{E}(t) = E(W_\beta(t)), \quad t > 0 \tag{7}$$

is well defined.

Proposition 2. *The subordinated process $\mathcal{E}(t), t > 0$ has the following one-dimensional distributions*

$$\mathbf{P}(\mathcal{E}(t) \le x) = \mathbf{E}[G(x)^{W_\beta(t)}] = 1 + \sum_{n=1}^{\infty} \frac{(\log G(x))^n t^{n\beta}}{\Gamma(1 + n\beta)}. \tag{8}$$

Proof. The proof follows immediately from the independence and (5), applying the total probability formula. □

3. Limit theorems

Now we are ready to prove limit theorems for the processes $\mathcal{T}(x)$ and $\mathcal{M}(t)$ under the conditions stated in the previous section. The first theorem extends the result of Gut and Huesler (1999) (Section 4.3) to the case where the interarrival times have infinite mean.

Theorem 3. *Assume Conditions 2 and 3 and*

$$\bar{F}_X(x) \sim x^{-\alpha} L_X(x), \quad x \to \infty, \tag{9}$$

for $\alpha > 0$ and a s.v.f. $L_X(.)$. Then for $x > 0$,

$$\frac{\mathcal{T}(xz)}{r(1/\bar{F}_X(z))} \Rightarrow x^{\alpha/\beta} \xi^{1/\beta} D_\beta, \quad \text{as } z \to \infty,$$

where ξ is a standard exponential random variable independent of D_β.

Proof. Let $x > 0, y > 0$ be fixed. From (9) and the fact that $r(.)$ is regularly varying with exponent $1/\beta$ one gets that $r(1/\bar{F}_X(z))$ varies regularly with exponent α/β. Therefore

$$y\frac{r(1/\bar{F}_X(z))}{r(1/\bar{F}_X(xz))} \to yx^{-\alpha/\beta} \quad \text{as } z \to \infty.$$

Using this limit, the fact that the cdf of $\xi^{1/\beta}D_\beta$ is continuous, and Proposition 1(ii) one obtains that ((9) yields that $x_F = \infty$)

$$\begin{aligned}\lim_{z\to\infty} \mathbf{P}\left(\frac{\mathcal{T}(xz)}{r(1/\bar{F}_X(z))} \le y\right) &= \lim_{z\to\infty} \mathbf{P}\left(\frac{\mathcal{T}(xz)}{r(1/\bar{F}_X(xz))} \le y\frac{r(1/\bar{F}_X(z))}{r(1/\bar{F}_X(xz))}\right)\\ &= \mathbf{P}\left(\xi^{1/\beta}D_\beta \le yx^{-\alpha/\beta}\right) = \mathbf{P}\left(x^{\alpha/\beta}\xi^{1/\beta}D_\beta \le y\right).\end{aligned}$$

Thus, the convergence of the one-dimensional distributions is proved.

The convergence of the finite-dimensional distributions follows in the same way as in Gut and Huesler (1999) (Section 4.3) under Condition 1. Furthermore, we have also that for every fixed $z > 0$, the process $\left\{\frac{\mathcal{T}(xz)}{r(1/\bar{F}_X(z))}, x \ge 0\right\}$ has nondecreasing sample paths and the limiting process $\left\{\theta(x) := x^{\alpha/\beta}\xi^{1/\beta}D_\beta, x \ge 0\right\}$ is stochastically continuous. Applying Theorem 3 of Bingham (1971) we complete the proof. □

The next two theorems establish the convergence of the sequences of maxima of a random number of random variables to nondegenerate random time changed extremal processes.

First we consider the case when $\mu_Y < \infty$. Define the following sequence of maxima

$$\mathcal{M}_n(t) = \frac{\mathcal{M}(\mu_Y nt) - B(n)}{A(n)} = \bigvee_{k=1}^{N(\mu_Y nt)} \frac{X_k - B(n)}{A(n)}, \quad n = 0, 1, 2, \ldots.$$

Theorem 4. *Let Conditions 1 and 4 be satisfied. Then for every $t > 0$*

$$\mathcal{M}_n(t) \xrightarrow{d} E(t),$$

where $E(t)$ is the G-extremal process determined in (6) and $\xrightarrow{d}$ means the convergence of the one-dimensional distributions.

Proof. Let $t > 0$ and $x > 0$ be fixed. Then from the duality relation (2) one has

$$\begin{aligned}\mathbf{P}(\mathcal{M}_n(t) \le x) &= \mathbf{P}(\mathcal{M}(\mu_Y nt) \le A(n)x + B(n))\\ &= \mathbf{P}(\mathcal{T}(A(n)x + B(n)) > \mu_Y nt)\\ &= \mathbf{P}\left(\frac{\bar{F}(A(n)x + B(n))}{\mu_Y}\mathcal{T}(A(n)x + B(n)) > n\bar{F}(A(n)x + B(n))t\right).\end{aligned}$$

Condition 4 provides that

$$A(n)x + B(n) \to \infty \quad \text{and} \quad n\bar{F}(A(n)x + B(n)) \to -\log G(x), \quad n \to \infty.$$

These two relations and Proposition 1(i) give

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{\bar{F}(A(n)x + B(n))}{\mu_Y}\mathcal{T}(A(n)x + B(n)) > n\bar{F}(A(n)x + B(n))t\right) = \exp(-(-t\log G(x))) = G(x)^t. \quad □$$

Comment 1. *The theorem shows that the limiting process is the same as in the case without subordination (see (6)). This fact can be explained by the SLLN which provides the almost surely linear increase of the indexing process $N(t)$ (see (3)).*

Now we turn to the case when $\mu_Y = \infty$. In this case we define

$$\mathcal{M}_n(t) = \frac{\mathcal{M}(r(n)t) - B(n)}{A(n)} = \bigvee_{k=1}^{N(r(n)t)} \frac{X_k - B(n)}{A(n)}, \quad n = 0, 1, 2, \ldots,$$

where $r(.)$ is defined in (4).

Theorem 5. *Let Conditions 2–4 be satisfied. Then for every $t > 0$*

$$\mathcal{M}_n(t) \xrightarrow{d} \mathcal{E}(t)$$

where $\mathcal{E}(t)$ is the subordinated process defined in (7).

Proof. We first prove that

$$\lim_{n\to\infty} \mathbf{P}(\mathcal{M}_n(t) \le x) = \mathbf{P}(\xi^{1/\beta} D_\beta > t(-\log G(x))^{1/\beta}). \tag{10}$$

Let $t > 0$ and $x > 0$ be fixed. Then by the duality relation (2) it follows that

$$\begin{aligned} \mathbf{P}(\mathcal{M}_n(t) \le x) &= \mathbf{P}\left(\mathcal{M}(r(n)t) \le A(n)x + B(n)\right) = P\left(\mathcal{T}(A(n)x + B(n)) > r(n)t\right) \\ &= \mathbf{P}\left(\frac{\mathcal{T}(A(n)x + B(n))}{r\left(1/\bar{F}_X(A(n)x + B(n))\right)} > \frac{r(n)t}{r\left(1/\bar{F}_X(A(n)x + B(n))\right)}\right). \end{aligned} \tag{11}$$

Condition 4 provides that, as $n \to \infty$,

$$A(n)x + B(n) \to \infty \quad \text{and} \quad n\bar{F}_X(A(n)x + B(n)) \to -\log G(x). \tag{12}$$

Taking in view that $r(t)$ is regularly varying with exponent $1/\beta$, one gets

$$\frac{r(n)t}{r\left(1/\bar{F}_X(A(n)x + B(n))\right)} \to t(-\log G(x))^{1/\beta}$$

as $n \to \infty$ from the second relation in (12). Using this and the first relation there, an application of Proposition 1(ii) to the right-hand side of (11) yields (10).

We still have to prove that the right-hand sides of (10) and (8) are equivalent. It is known that (see Meerschaert and Scheffler (2004) or Feller (1971), Section 13.6) $H_\beta(x) := \mathbf{P}(W_\beta(1) \le x) = \mathbf{P}(D_\beta > x^{-1/\beta})$. Then

$$\begin{aligned} \mathbf{P}(\xi(D_\beta)^\beta > z) &= \int_0^\infty \mathbf{P}(D_\beta^\beta > z/u)\mathrm{d}\mathbf{P}(\xi \le u) \\ &= \int_0^\infty \mathbf{P}(D_\beta > (u/z)^{-1/\beta})\mathrm{d}\mathbf{P}(\xi \le u) = \int_0^\infty H_\beta(u/z)\mathrm{e}^{-u}\mathrm{d}u = \int_0^\infty H_\beta(v)z\mathrm{e}^{-vz}\mathrm{d}u. \end{aligned}$$

An integration by parts gives that (see also (5) with $t = 1$)

$$\mathbf{P}(\xi(D_\beta)^\beta > z) = \int_0^\infty \mathrm{e}^{-zv}\mathrm{d}H_\beta(v) = \sum_{n=0}^\infty \frac{(-z)^n}{\Gamma(1+n\beta)}.$$

This equation shows that

$$\begin{aligned} \mathbf{P}(\xi^{1/\beta} D_\beta > t(-\log G(x))^{1/\beta}) &= \mathbf{P}(\xi(D_\beta)^\beta > -\log(G(x))^{t^\beta}) \\ &= \int_0^\infty (G(x))^{t^\beta u}\mathrm{d}H_\beta(u) = \mathbf{E}\left[G(x)^{t^\beta W_\beta}\right] = 1 + \sum_{n=1}^\infty \frac{(\log G(x))^n t^{n\beta}}{\Gamma(1+n\beta)}, \end{aligned}$$

which together with (8) completes the proof of the theorem. □

Corollary 6. *For $t > 0$, the following relation holds $\mathcal{E}(t) \overset{d}{=} G^{\leftarrow}\left(\mathrm{e}^{-\xi.(D_\beta/t)^\beta}\right)$, where $G^{\leftarrow}(.)$ is the inverse function of $G(.)$.*

Proof. From (10) one has, for $t > 0$ and $x > 0$

$$\begin{aligned} \mathbf{P}(\mathcal{E}(t) \le x) &= \mathbf{P}\left(\xi^{1/\beta} D_\beta > t(-\log G(x))^{1/\beta}\right) = \mathbf{P}\left(\xi(D_\beta/t)^\beta > -\log G(x)\right) \\ &= \mathbf{P}\left(-\xi(D_\beta/t)^\beta < \log G(x)\right) = \mathbf{P}\left(\mathrm{e}^{-\xi(D_\beta/t)^\beta} < G(x)\right) = \mathbf{P}\left(G^{\leftarrow}\left(\mathrm{e}^{-\xi(D_\beta/t)^\beta}\right) < x\right). \quad \square \end{aligned}$$

4. Conclusion remarks

Finally, we give some properties of the limiting process $\mathcal{E}(t) \overset{d}{=} E(W_\beta(t))$ which follows immediately from the known properties of the G-extremal process $E(t)$ and the hitting time process $W_\beta(t)$.

1. Since both $E(t)$ and $W_\beta(t)$ (see Meerschaert and Scheffler (2004)) have nondecreasing sample paths, the limiting process $\mathcal{E}(t), t > 0$ also has nondecreasing sample paths.

2. Recall that the process $W_\beta(t)$ is β-selfsimilar (see Meerschaert and Scheffler (2004)). Then

- If $G(x) = \Phi_\alpha(x)$, the G-extremal process $E(t)$ is $1/\alpha$-selfsimilar and the compound process $\mathcal{E}(t), t > 0$ is selfsimilar with exponent β/α.

- If $G(x) = \Psi_\alpha(x)$, the G-extremal process $E(t)$ is $-1/\alpha$-selfsimilar and the compound process $\mathcal{E}(t), t > 0$ is selfsimilar with exponent $-\beta/\alpha$.

- If $G(x) = \Lambda(x)$ then it is not difficult to check directly that

$$\mathbf{P}\left(\mathcal{E}(tc) \leq x\right) = \mathbf{P}\left(\mathcal{E}(t) + \beta \log c \leq x\right).$$

3. Since the process $\mathcal{E}(t)$ has nondecreasing sample paths, it is possible to define its first passage time process by the relation $\{\tau(x) \leq t\} \Leftrightarrow \{\mathcal{E}(t) > x\}$. The one-dimensional distributions of $\tau(z)$ are

$$\mathbf{P}(\tau(x) \leq t) = -\sum_{n=1}^{\infty} \frac{(\log G(x))^n t^{n\beta}}{\Gamma(1+n\beta)}.$$

It is not hard to check that $\tau(x) \stackrel{d}{=} \theta(x) = x^{\alpha/\beta}\xi^{1/\beta}D_\beta$, $x > 0$, where $\theta(x)$ is the limiting process obtained in Theorem 3.

Comment 2. *Meerschaert and Stoev (submitted for publication) considered similar processes in the case where the random variables X_k and Y_k in each pair are independent. Under this condition they proved the weak convergence in Skorohod J_1 topology in $D(0,\infty) \times (-\infty,\infty)$ under similar normalization as above. (See also Pancheva and Jordanova (2004)).*

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