

Online Kinematics Estimation for Active Human-Robot Manipulation of Jointly Held Objects

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Abstract—This paper introduces a method for estimating the constraints imposed by a human agent on a jointly manipulated object. These estimates can be used to infer knowledge of where the human is grasping an object, enabling the robot to plan trajectories for manipulating the object while subject to the constraints. We describe the method in detail, motivate its validity theoretically, and demonstrate its use in co-manipulation tasks with a real robot.

I. INTRODUCTION

There is an increasing interest in letting humans and robots share the same workspace and perform object manipulation tasks together. Apart from safety issues, the main enabling technology necessary to realize this is the design of control systems that let the robot cooperate smoothly with the human, working towards the same goal, which may not be explicitly communicated to the robot before the task is initiated. The traditional approach is to let the robot be a passive agent in the interaction while the human agent controls the motion of the object.

However, when two humans perform an object manipulation task together, the role of leader and follower may typically alternate between the two agents, depending on task geometry, load distribution, limited observability, or other reasons. For human-robot interaction to become as efficient as human-human interaction, it is natural to assume that the robot must be able to perform both the active and passive parts of the interaction, just as a human would. For the robot to take the active part in the interaction, and to be able to plan and execute trajectories of the object, it must have knowledge about the passive agent and what constraints the human imposes on the object.

The main contribution of the present paper is a method for estimating hinge-like constraints imposed by a passive human agent on a jointly manipulated object. These estimates can be used to infer knowledge of where the human is grasping an object, enabling the robot to plan trajectories for manipulating the object while subject to these constraints. We describe the method in detail, motivate its validity theoretically, and demonstrate its use in some example co-manipulation tasks with a real robot, as illustrated in Figure 1.

The paper has the following structure: Section II reviews the state of the art in related work, Section III formalizes the constraint kinematics, Section IV describes the proposed



Fig. 1 : A robot and a human manipulating a jointly held object.

approach and motivates it theoretically, while Section V describes the experimental implementation of the method on real human-robot comanipulation tasks. Finally, conclusions are presented in Section VI.

II. RELATED WORK

An important problem for *physical human-robot interaction* (pHRI) is cooperative manipulation of an object jointly held by both a human and a robot. The earliest works in human-robot co-manipulation consider the robot mainly as a passive agent taking care of load compensation while the human acts as a leader for planning and guiding the cooperative task. One of the most common approaches has been to use different types of impedance control. Impedance control has been proposed with the goal of improving the safety for the human using variable impedance [1] or impedance based on human impedance characteristics [2]. Impedance controllers have also been proposed for co-manipulation with advanced robot setups consisting of robot manipulators mounted on mobile platforms [3], [4] or dual arm manipulators [5]. To enable robots to understand human intention particularly in case of rotation or translation, impedance control that imposes virtual non-holonomic constraints to the object or the combination of impedance control and voice instruction have been considered [6], [7]. In contrast to these impedance control based techniques, another approach is to use the interaction force only to generate the direction of motion, while the velocity profile is given as a minimum jerk trajectory [8].

The above approaches only consider robotic assistants that react to the actions of the human leader, hence their capabilities are limited. By shifting the role of robot in the co-manipulation task towards a more active one, it is possible to reduce human effort and exploit the advanced performance capabilities of robots in terms of precision. Recently, effort sharing policies through a dynamic role allocation scheme have been proposed to enable the manipulation of bulky objects [9]. Precise positioning of the object through human-robot collaboration has been treated in [10]. Estimating intended human motion has been used to enable a robot to give proactive assistance to an active human partner [11], and human motion estimation has been used in to set the stiffness matrix for impedance control to achieve proactive robot behavior for an a priori defined role of the robot. In this framework, confidence measures have been exploited to enable automatic switching of roles [12]. Other works combine motion learning to allow the robot to anticipate the human partner’s impedance with adaptive control to compensate for unmodelled uncertainties in the human part of the system [13] and understanding of human behavior when the robot is the leader [14].

The reviewed literature considers the manipulation of objects that are known to the robot in terms of task-related geometrical characteristics (kinematic parameters) such as center of mass, grasping points, pose of the object in hand (e.g. the human is connected to the object through a handle with an a priori known position). When the role of robot is changed and proactive robotic behavior is required, the human can be considered as a source of uncertainty. This is particularly true for cases when the collaboration takes place in domestic environments, where objects may not have handles and markers for determining the grasping points. Hence, it is important to design controllers that are able to handle these uncertainties in the kinematics, since they are important for mapping the intention of the human typically expressed through forces to the robot frame.

In the present paper, we propose an adaptive controller that estimates online the kinematic constraints imposed by the human. This enables the robot to actively perform some task on the jointly held object, while adhering to the human’s constraints. Furthermore, the proposed controller limits the interaction forces, allowing the human to impose the constraints without having to exert large forces on the manipulated object.

III. CONSTRAINTS’ FORMULATION

We consider a setting in which a human and a robot jointly hold an object. Specifically, we consider the scenario where the human is acting as a passive revolute joint — conceptually as part of a more complex interaction scenario — which means that the robot can move along a circular trajectory in the plane in order to rotate the object. This scenario enables the robot to actively affect the motion of the object while the human role is more passive. The human can impose the kinematic constraints on the object in several possible ways. Not only can the human grasp the object in

an arbitrary position, but for each possible grasp, the virtual axis of rotation can be adjusted by changing the stiffness of the wrists, elbows, or shoulders, by rotating the entire body, or by any combination of these or other actions, all of which are a priori unknown to the robot.

We assume that the robot has a fixed grasp of the object i.e. there is no relative rotation and translation between object and robot. In the following we consider that velocities and forces are expressed with respect to the frame of the end-effector $\{e\}$; the position and the orientation of $\{e\}$ with respect to the base frame $\{B\}$ (world frame) can easily be calculated through forward kinematics. We define the frame $\{h\}$ which consists of constrained motion axis \mathbf{y}_h and motion axis \mathbf{x}_h , see Fig 2. The main axes can be parameterized by the angle φ as follows:

$$\mathbf{x}_h = \mathbf{R} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_\varphi \\ -s_\varphi \end{bmatrix}, \mathbf{y}_h = \mathbf{R} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} s_\varphi \\ c_\varphi \end{bmatrix}, \quad (1)$$

where φ denotes the angle formed between each axis of $\{h\}$ with the corresponding axis of the end-effector frame $\{e\}$ while $\mathbf{R} \triangleq \begin{bmatrix} c_\varphi & s_\varphi \\ -s_\varphi & c_\varphi \end{bmatrix} \in SO(2)$ describes the orientation of $\{h\}$ with respect to $\{e\}$. Let $\mathbf{v} \in \mathbb{R}^2$ and $\omega \in \mathbb{R}$ denote the translational velocity of the robot expressed at the end-effector frame and the rotational velocity respectively. Furthermore, let ρ be the distance between the robot end-effector and the human virtual joint which corresponds to the radius of the robot motion.

The human imposes the following constraints to the motion of the robot:

$$\mathbf{y}_h^\top \mathbf{v} = 0, \quad (2)$$

$$v = \rho\omega, \text{ with } v = \mathbf{x}_h^\top \mathbf{v}. \quad (3)$$

Both φ and ρ are constant parameters since i) the human is passive (no translational velocity), and ii) the robot grasp is fixed, but they are unknown since they depend on human intention as well as object geometrical parameters (e.g. length). Note that the position of the human hinge can be calculated given the parameters φ and ρ as follows:

$$\mathbf{p}_h = \rho \begin{bmatrix} s_\varphi \\ c_\varphi \end{bmatrix} \quad (4)$$

Obtaining accurate estimates of φ and ρ is important during the manipulation as it allows the robot controller to include feed-forward terms that minimize interaction forces, which enables smoother and more comfortable interaction.

IV. METHODOLOGY

In this section we propose a control law and formalize the application problem of levelelling a jointly held board.

A. Controller

Let v_d be the desired velocity trajectory along the axis of allowed motion \mathbf{x}_h and f_d, τ_d the desired force along the constraint direction \mathbf{y}_h and torque around the rotation axis which is perpendicular to the plane of the motion. In order to define robot reference velocities which are consistent with

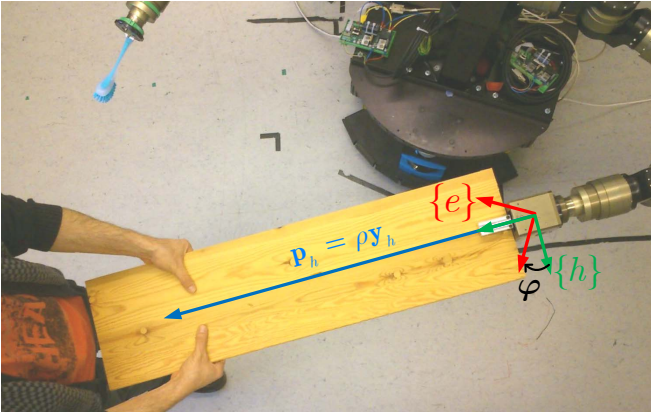


Fig. 2 : Kinematics of the human robot interaction task

the constraints imposed by the human the knowledge of φ and ρ is important. If the robot exerts high forces along a direction in which the human does not want to allow motion, extra effort will be needed from the human side and hence the manipulation task becomes more difficult for the human co-worker. To deal with this problem we propose the following velocity controllers for the translational and the rotational velocities:

$$\mathbf{v}_{\text{ref}} = \hat{\mathbf{x}}_h v_d + \hat{\mathbf{y}}_h w(\tilde{f}), \quad (5)$$

$$\omega_{\text{ref}} = \hat{d} v_d + w(\tilde{\tau}), \quad (6)$$

where:

- $w(x) = -\alpha_x x - \beta_x \int_0^t x(\zeta) d\zeta$ is a function of proportional and integral action on a control variable $x(t)$ e.g. force, torque errors with α_x, β_x being positive control gains. In (5) and (6) the argument of $w(\cdot)$ is the force error along the estimated constrained direction \tilde{f} and the torque error $\tilde{\tau}$ respectively:

$$\tilde{f} = \hat{f} - f_d, \quad \text{with } \hat{f} = \hat{\mathbf{y}}_h^\top \mathbf{f} \quad (7)$$

$$\tilde{\tau} = \tau - \tau_d, \quad (8)$$

where \mathbf{f}, τ are force and torque readings obtained from a force/torque sensor attached at the end-effector.

- $\hat{\mathbf{x}}_h = [\mathbf{c}_{\hat{\varphi}} \quad -\mathbf{s}_{\hat{\varphi}}]^\top$ and $\hat{\mathbf{y}}_h = [\mathbf{s}_{\hat{\varphi}} \quad \mathbf{c}_{\hat{\varphi}}]^\top$ are estimates of allowed motion and constrained direction respectively that are inherently unit vectors and they are updated following the adaptation of $\hat{\varphi}$ (online estimate of the parameter φ):

$$\dot{\hat{\varphi}} = -\gamma v_d w(\tilde{f}) \quad (9)$$

with γ being positive gain for tuning the estimation rate.

- \hat{d} is the estimate of the curvature of the cyclic trajectory i.e. $d \triangleq \frac{1}{\rho}$ that is updated according to the following law:

$$\dot{\hat{d}} = \gamma_d v_d w(\tilde{\tau}) \quad (10)$$

with γ_d being a positive gain for tuning the estimation rate.

The force/torque feedback part of the controller (5), (6) can be regarded as a type of a damping controller – for safe pHRI – similar to [15] but along the estimated constrained directions.

The proposed controller is formulated in the end-effector velocity frame. To be applied to the joint velocity level, the following first order differential kinematic equation can be used:

$$\dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q}) \begin{bmatrix} \mathbf{v}_{\text{ref}} \\ \omega_{\text{ref}} \end{bmatrix} \quad (11)$$

with $\mathbf{q}, \dot{\mathbf{q}} \in \mathbb{R}^n$ being the joint positions and velocities and $\mathbf{J}(\mathbf{q})^+ = \mathbf{J}(\mathbf{q})^\top [\mathbf{J}(\mathbf{q})\mathbf{J}(\mathbf{q})^\top]^{-1}$ being the pseudo-inverse of the geometric Jacobian $\mathbf{J}(\mathbf{q}) \in \mathbb{R}^{3 \times n}$ expressed at the end-effector frame which relates the joint velocities $\dot{\mathbf{q}}$ to the end-effector velocities $[\mathbf{v}^\top \omega]^\top$.

A dynamic controller based on the proposed reference velocities (5), (6) can be designed and applied at the joint torque level following the steps of our previous work [16]. However, if we assume a high frequency current control loop with external forces' compensators and weak inertial dynamics, the theoretical analysis of this work is valid. Experimental results of Section V-B support the validity of the use of control action at the velocity level.

Theorem 1: Consider the system of a velocity controlled robot manipulator (11) which is rigidly grasping an object connected to a human imposing the constraints (2), (3). The proposed controller (5), (6) combined with the parameter update laws (9), (10) and applied to the system ensures the identification of the position of the joint $\hat{\mathbf{p}}_h \rightarrow \mathbf{p}_h$ as well as convergence of force and torque errors to zero.

The adaptive controller given by (5), (6), (9), (10) is a variation of the controller proposed in [17] in case the axis of rotation for the robot end-effector is a priori known. A sketched proof of the Theorem can be found in the Appendix while details for the general case of uncertainties can be found in [17]. In this work we mainly focus on experimentally validating that the proposed control method can be used to identify the position of the “human joint” in human-robot object co-manipulation. The use of the online estimates of the constraints imposed by the human can be used to achieve a specific control objective. An example described in the following section is the automated leveling of an object jointly held by a human and a robot.

B. Leveling

Here, we define leveling as aligning the end-effector frame of the robot with the horizontal axis of the world frame, while jointly holding an object, see Fig 3.

In early work [4], leveling has been dealt with by controlling the orientation of the robot end-effector so that the pointing vector becomes parallel with the ground; the problem is solved by considering that the axis of motion is known with respect to the end-effector while the inexact knowledge of the distance between the human and the robot grasping points can only deteriorate the rate of convergence. Here we deal with the leveling problem [4] but we consider

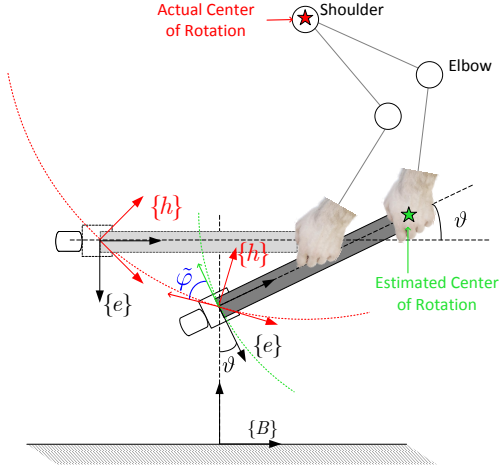


Fig. 3 : Automated leveling is achieved when the robot end-effector pointing vector is aligned with the horizontal axis of the inertial frame $\{B\}$

that the robot is not aware of the center of rotation, i.e. both axis of motion and distance are unknown.

In order to solve the leveling problem the velocity controller (5), (6) and the update laws (9), (10) are modified by considering the following specifications:

- 1) The gravity and any force exerted by the human along the vertical axis of the inertial frame will be filtered by projecting the measured force \mathbf{f}_m along \mathbf{x} that denotes the horizontal axis of the inertial frame (i.e. $[1 \ 0]^\top$) expressed at the end-effector frame, i.e. in (7) $\mathbf{f} = \mathbf{x}\mathbf{x}^\top \mathbf{f}_m$ is used.
- 2) The desired torque τ_d is set zero. If we consider the case of symmetric human and robot grasping points with respect to the center of mass of the object modeled as a beam, human and robot will share the load.
- 3) A projection operator that does not affect the convergence and stability properties of \hat{d} is used on (10) in order to ensure that $\hat{d}(t) \neq 0, \forall t$.
- 4) The desired velocity used in the feedforward terms of the controller is replaced by a feedback term based on the leveling angle $\vartheta \in (-\pi/2, \pi/2)$ shown in Fig. 3 which is the angle between a horizontal line and the pointing vector, defined as follows:

$$\vartheta = \arcsin(\mathbf{y}_h^\top \mathbf{y}) \quad (12)$$

where \mathbf{y} is the vertical axis of the inertial frame (i.e. $[0 \ 1]^\top$) expressed at the end-effector frame.

Instead of calculating the angle ϑ , $\sin \vartheta$ can be alternatively used in the controller, i.e.:

$$v_d = -\frac{\alpha}{\hat{d}} \sin \vartheta = -\frac{\alpha}{\hat{d}} \mathbf{y}_h^\top \mathbf{y} \quad (13)$$

where α is a positive control gain that is modulated by \hat{d} to enable the system a convergence rate independent of kinematic parameters of the task. Particularly, the convergence of the leveling angle to zero is described by the following

differential equation:

$$\dot{\vartheta} = -\frac{\alpha \hat{d}}{c_{\hat{\varphi}} \hat{d}} \sin \vartheta \quad (14)$$

which implies that $\vartheta \rightarrow 0$ since $c_{\hat{\varphi}} > 0, \hat{d} > 0$; thus the leveling objective can be achieved independently of the learning objective. However the rate of leveling angle convergence depends on the estimation error of the parameters including also uncertainty in the motion axis, which has not been considered in [4]. If the signal v_d ensures that after some time instant T the estimated parameters values are close to the actual ($\frac{\hat{d}}{d} \simeq 1, c_{\hat{\varphi}} \simeq 1$), then the convergence rate becomes approximately equal to α for $t \geq T$ in contrast to [4] that consider constant estimates. One way to achieve practical parameter convergence without affecting the stability properties of the closed loop system is to apply $v_d = \frac{v_d^*}{\hat{d}}$ for $|\vartheta| > \vartheta_0$ for some positive ϑ_0 and to use v_d given by (13) with $\alpha = \frac{v_d^*}{\sin \vartheta_0}$ for $|\vartheta| \leq \vartheta_0$.

V. EXPERIMENTS

The proposed method was verified by implementation on a dual arm robot. The robot has 7 degree of freedom (DoF) Schunk LWA arms that are velocity controllable at the joint level, and have a 6 DoF force/torque sensor at the wrist. See [18] for details.

A. Experimental Setup and Scenarios

The performance of the proposed method is demonstrated and verified in two scenarios. In scenario **A**, the human subject is completely passive, and the robot makes small perturbations of a jointly held object to estimate the kinematic constraints imposed on the object by the human agent. In scenario **B**, the human agent starts by moving the object, and the robot estimates the constraints on-line while following the action initiated by the human.

1) *Scenario A - Estimation of Kinematics of Passive Human*: In the first scenario, we demonstrate the capability of the proposed method to estimate the constraints imposed by a human on a jointly held object. The human is passively holding one end of a 95 cm long wooden board, with a mass of 2.75 kg, and the robot has a fixed grasp of the other end, see Figure 3. The task for the robot is to actively rotate the board around the virtual vertical axis imposed by the human at point \mathbf{p}_h .

We run two experiments in this scenario. In both of these experiments, the controller was run using $\gamma = 700, \gamma_d = 1000, \alpha_f = 0.001, \text{ and } \alpha_\tau = 0.002$.

In the experiment 1, the human agent keeps arms fixed and lets the board rotate around the grasping point (\mathbf{p}_{h1} in Figure 4). The point \mathbf{p}_{h1} was marked with a visible spot on the board, at $[0 \ 0.81]^\top$ in the end-effector frame, and a laser pointer was fixed to the ceiling and set to point to the spot. The human subject then tried to keep the spot and laser dot aligned during the robot's motion of the board, ensuring knowledge of the ground truth of the virtual rotational axis. The spot on the board was kept within 1 cm of the initial

point during the entire motion, and the estimate error e_1 was calculated as $e_1 = \|\mathbf{p}_{h1} - \hat{\mathbf{p}}_{h1}\|$.

In experiment 2, the human subject fixes the grasp, but keeps the arms stiff and rotates around the main vertical axis (\mathbf{p}_{h2} in Figure 4). For this experiment, the ground truth for the virtual axis of rotation is not as well defined as in the first scenario. Overhead video footage was used to approximate \mathbf{p}_{h2} at $[0 \ 1.25]^T$ in the end-effector frame, and the error e_2 was calculated as $e_2 = \|\mathbf{p}_{h2} - \hat{\mathbf{p}}_{h2}\|$.

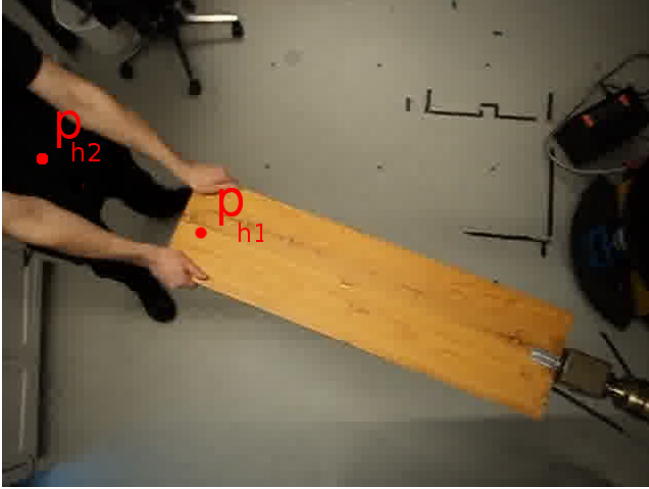


Fig. 4 : Scenario A. \mathbf{p}_{h1} and \mathbf{p}_{h2} mark the virtual rotational axis in the two experiments.

2) *Scenario B - Kinematic Estimation as a Secondary Task:* In the next scenario — experiment 3 — the human and robot are grasping the same board as in the previous scenario, but the robot is given the task of keeping the board horizontal. The human starts the action by raising the end of the board approximately 20 cm. As the robot raises the other end of the board to keep it horizontal, it estimates the position \mathbf{p}_{h3} of the virtual rotational axis imposed by the human. A laser point was projected onto a checkerboard pattern on the board to help the subject keep the virtual axis of rotation stable during the leveling motion of the robot, see Figure 5.

Controller settings were slightly different from scenario A, with $\gamma_d = 1500$ and $\alpha_\tau = 0.006$, the higher values making the controller more responsive to the initial human motion.

B. Experimental Results and Discussion

For experiments 1 and 3, the estimates of the virtual rotational axis converged to within a few centimeters after 5 cm perturbation by the robot (taking approx. 2 s), see Figure 8. For the experiment 2, the estimate initially converged to within approx 0.3 m in the same time, and to within 5 cm after approximately 20 cm perturbation, or 10 s. The estimates $\hat{\mathbf{p}}_{h1}$ and $\hat{\mathbf{p}}_{h2}$ are plotted in the end-effector reference frame in Figure 6, and $\hat{\mathbf{p}}_{h3}$ in Figure 7. Experiment 3 converges faster than the others, due to the higher gain settings.

The maximum force exerted by the robot on the human (via the board) was approximately 20 N for experiments 1

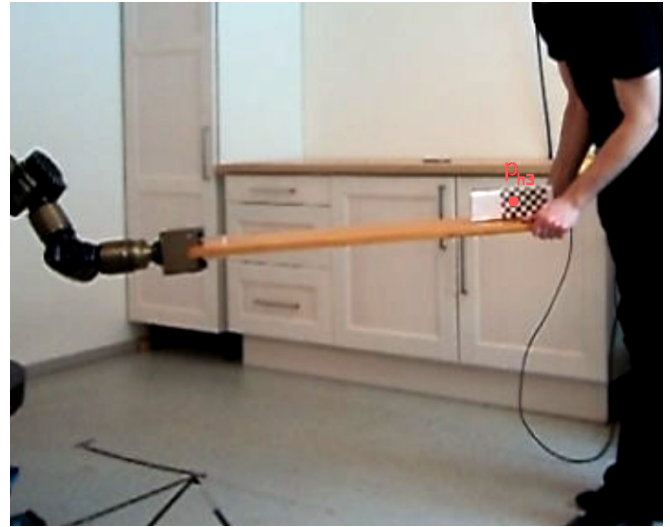


Fig. 5 : Scenario B. \mathbf{p}_{h3} marks the virtual rotational axis in the experiment.

and 3, and 13 N for the second trial, which is less than the gravitational load the board exerts on the human. After 4 s had passed and the initial error was mostly eliminated, forces stay below 6 N for all experiments.

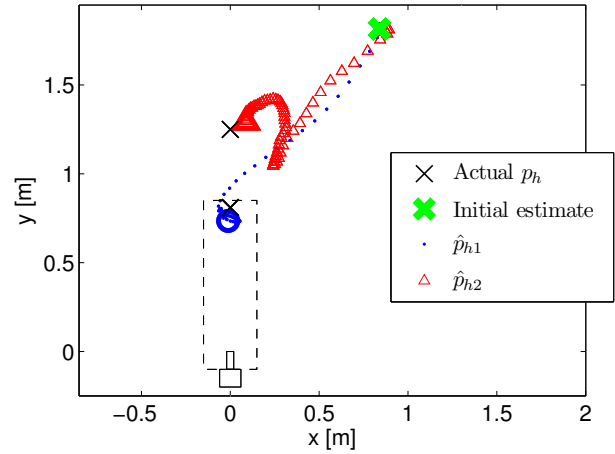


Fig. 6 : Scenario A. The plot shows the estimated positions $\hat{\mathbf{p}}_{h1}$ and $\hat{\mathbf{p}}_{h2}$, in the robot end-effector coordinate frame. Note that the actual value for \mathbf{p}_{h1} is $[0 \ 0.81]^T$, and $[0 \ 1.25]^T$ for \mathbf{p}_{h2} . The end-effector and the board are drawn in the image to aid orientation.

Experiments 1 and 3 had a very well defined virtual rotational axis \mathbf{p}_h , and the visual aid of the laser pointer made it easy for the human subject to fix \mathbf{p}_{h1} and \mathbf{p}_{h3} in a static position. This is probably why these estimates show good convergence. For experiment 2, \mathbf{p}_{h2} is much more difficult to fix for the human subject, so part of the error may be explained by a less well defined “true” virtual rotational axis.

The quick convergence from a large initial error (for all cases) indicates that even when \mathbf{p}_h is not well defined, the proposed method can make a rough estimate after only a

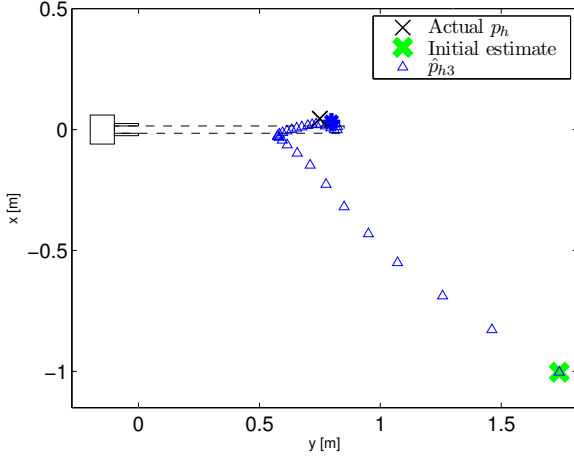


Fig. 7 : Scenario B. The plot shows the estimated position $\hat{\mathbf{p}}_{h3}$ in the robot end-effector coordinate frame. Note that the actual value for \mathbf{p}_{h3} is $[0.05 \ 0.75]^T$. The end-effector and the board are drawn in the image to aid orientation.

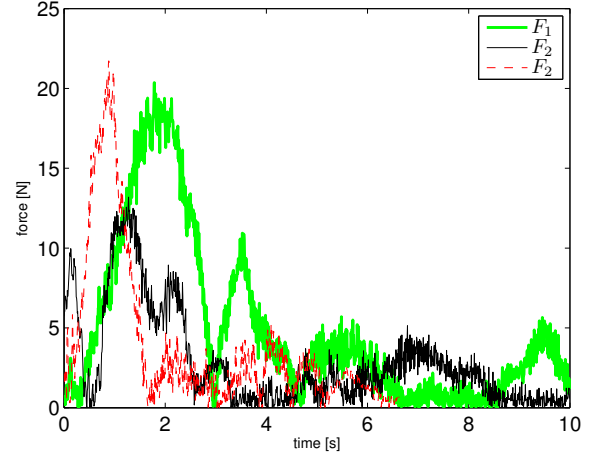


Fig. 9 : The magnitude of forces exerted in the horizontal plane in the experiments, as measured by the force/torque sensor in the robots wrist.

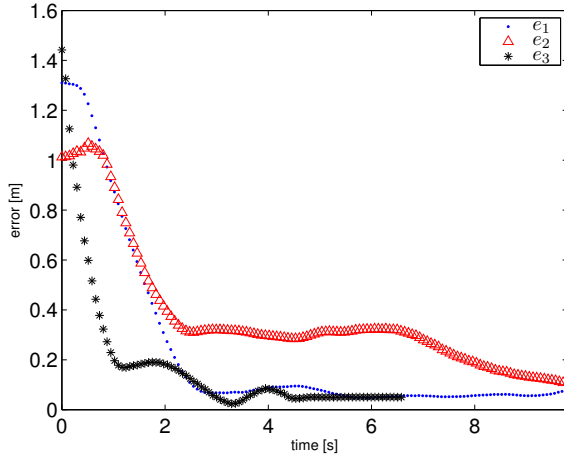


Fig. 8 : The estimation errors e_1 , e_2 , and e_3 as functions of time. The initial estimate is similar for all runs, while the true value of \mathbf{p}_h differs, hence the difference in initial error.

small perturbation of the jointly manipulated object has been performed. The small forces exerted by the robot on the human show that this rough estimate is enough to produce robot motion that follows the constraints imposed by the human.

VI. CONCLUSIONS

Motivated by our recent work for door/drawer opening [17] we propose a method for estimating the human joint position in a simple human-robot co-manipulation scenario in which the human is modeled as a rotational joint. The robot is controlled by a damping-type force controller for safe interaction while a feedforward or a feedback term – in case of a leveling task – drives the robot motion along the unconstrained directions as they are estimated online. Experimental results show that the method based on rigid modeling for robot grasp and external constraints can be

applied to cases where the human can be considered as a compliant joint. This work is a first step towards applying adaptive learning control to enhance safety and robot performance in complex human-robot object co-manipulation scenarios in which human and robot roles, effort sharing policies and human intention are dynamically changing. For future work, we plan to expand the treatment to a wider class of constraints, including those with more degrees of freedom.

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APPENDIX

Proof of Theorem 1: Substituting the control input (5) to (11), multiplying from left with the Jacobian matrix $\mathbf{J}(\mathbf{q})$, and using the constraint equations (2) and (3) we can derive relations connecting $w(\hat{t})$, and $w(\tilde{t})$ with the parameter errors $\tilde{\varphi}$ and \tilde{d} , that can be substituted into the parameter update laws (9), (10) in order to get the following differential equations describing the estimation error dynamics:

$$\dot{\tilde{\varphi}} = -\gamma v_d^2 \tan \tilde{\varphi} \quad (15)$$

$$\dot{\tilde{d}} = -\gamma_d v_d^2 \tilde{d} - \gamma_d v_d^2 d \sin \tilde{\varphi} \tan \tilde{\varphi} \quad (16)$$

Let $V(\tilde{\varphi}, \tilde{d}) : D \rightarrow \mathbb{R}^+$, with $D = \{\tilde{\varphi}, \tilde{d} \in \mathbb{R}, |\tilde{\varphi}| < \frac{\pi}{2}\}$ be a positive definite function given by:

$$V(\tilde{\varphi}, \tilde{d}) = -\frac{\xi^2}{\gamma} \log(\cos \tilde{\varphi}) + \frac{1}{2\gamma_d} \|\tilde{d}\|^2 \quad (17)$$

where ξ is a positive constant with $\xi > d/2$. By differentiating $V(\tilde{\varphi}, \tilde{d})$ with respect to time along the system trajectories (15), (16) we get:

$$\dot{V}(\tilde{\varphi}, \tilde{d}) \leq -(\xi^2 - \frac{d^2}{4})v_d^2 \tan^2 \tilde{\varphi} - v_d^2 \left(\frac{d}{2} \tan \tilde{\varphi} + \tilde{d}\right)^2 \quad (18)$$

Notice that $\dot{V} \leq 0$ which implies that $V(\tilde{\varphi}, \tilde{d}) \leq V(\tilde{\varphi}(0), \tilde{d}(0))$ and thus $\tilde{\varphi}, \tilde{d}$ are bounded and additionally $|\tilde{\varphi}(t)| < \pi/2, \forall t$, given that $|\tilde{\varphi}(0)| < \pi/2$. From system and constraint equations we can prove that $w(\tilde{f})$ and $w(\tilde{\tau})$ are bounded. Furthermore by integrating both sides (18) we get that $\int_0^{+\infty} v_d^2(\zeta) \tilde{\varphi}^2(\zeta) d\zeta, \int_0^{+\infty} v_d^2(\zeta) \left(\frac{d}{2} \tan \tilde{\varphi}(\zeta) + \tilde{d}(\zeta)\right)^2 d\zeta$ and are bounded and hence $\tilde{\varphi}, \tilde{d} \rightarrow 0$ when v_d satisfies persistent excitation condition. In the next step Barbalat's Lemma can be used in order to prove that $w(\tilde{f}), w(\tilde{\tau})$ converge to zero which implies (given that $\tilde{\varphi} \rightarrow \varphi$) that $f \rightarrow f_d$ and $\tau \rightarrow \tau_d$.

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