An Adaptive Control Approach for Opening Doors and Drawers under Uncertainties

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Abstract—**We study the problem of robot interaction with mechanisms that afford one degree of freedom motion, e.g. doors and drawers. We propose a methodology for simultaneous compliant interaction and estimation of constraints imposed by the joint. Our method requires no prior knowledge of the mechanisms' kinematics, including the type of joint — prismatic or revolute. The method consists of a velocity controller which relies on force/torque measurements and estimation of the motion direction, the distance and the orientation of the rotational axis. It is suitable for velocity controlled manipulators with force/torque sensor capabilities at the end-effector. Forces and torques are regulated within given constraints, while the velocity controller ensures that the end-effector of the robot moves with a task-related desired velocity. We give proof that the estimates converge to the true values under valid assumptions on the grasp, and error bounds for setups with inaccuracies in control, measurements, or modelling. The method is evaluated in different scenarios opening a representative set of door and drawer mechanisms found in household environments.**

I. Introduction

Robots operating in domestic environments need the ability to interact with doors, drawers, and cupboards, all of which exhibit various kinematic constraints due to the joints attaching them to the environment. The variation in size, orientation and type of joints makes it intractable to provide a robot with predefined kinematic models of all the mechanisms it may encounter. Prior knowledge of mechanisms could conceptually be combined with observations from cameras, laser-range finders or other distal sensors to infer a prior model of a mechanism. However, in domestic and other human-centric environments, occlusions, poor lighting, and the presence of previously un-encountered mechanism types make it very difficult to produce reliable systems based on these approaches. One could also imagine a situation where the constraints change dynamically during the manipulation, for example if the constraints are imposed on the mechanism by another agent — e.g. a human doing collaborative work with robot — whose intended actions cannot be inferred from prior observation alone [1]. Therefore, the performance, robustness, and generality of constrained manipulation tasks can be significantly improved if the need to have prior knowledge of the constraints is removed. In the general case, the uncertainties in the manipulation of such constrained kinematic mechanisms, e.g. doors and drawers, can be divided into two main categories:

Dynamic uncertainties which are related to the dynamic model of the door or the drawer: door's inertia, dynamics of the hinge mechanism etc.

Kinematic or Geometric uncertainties which are related to the kinematic model of the door or the drawer: type of the joint that models the kinematic mechanism, which may be prismatic or revolute, size of the door, location and orientation of the hinge, etc.

This categorization has been applied to several problems in robot control, like motion control [2] and force/motion control [3]. From a control perspective, the door opening problem can be regarded as a force/motion control problem in which the robot workspace can be divided into motion and force controlled subspaces according to the concept of hybrid force/motion control [4], [5]. In robot interaction tasks, the identification of geometric or kinematic uncertainties is crucial for defining a kinetostatically consistent Task Frame [6] to correspond to a real compliant motion. Several different methods have been proposed for directly calculating or estimating kinematic parameters, that can be twist-based or wrench-based, by exploiting the concept of reciprocity under ideal conditions [7]. For manipulation of kinematically constrained objects like doors and drawers, twist-based estimation has been used (Section II) since it is more robust when forces e.g. friction or rotational spring forces arise along the motion directions. A common characteristic shared by the majority of the works proposed in the literature is that the combined dynamics of estimation, tracking, and force control are not considered. This can been considered as a source of disturbance to the identification (which may result in inaccurate and unsafe task execution), as have been pointed out in e.g. [8].

In this work, we consider a general robotic setup with a manipulator equipped with a wrist force/torque sensor, and we propose an adaptive controller which can be easily implemented for dealing with the kinematic uncertainties of doors and drawers. The proposed control scheme which is inspired by the adaptive surface slope learning [9] does not require accurate identification of the constraints at each step of the door/drawer opening procedure as opposed to the majority of the solutions to this problem (Section II). It uses adaptive estimates of the motion and constraint parameters that converge to the actual dynamically changing radial direction during the procedure.

The paper is organised as follows: In Section II we make an overview of the related work to the door opening problem. Section III provides description of the kinematic model of the system and the problem formulation. The proposed solution and the corresponding stability analysis are given in Section IV followed by the simulation examples in Section V and the experimental results in Section VI. In Section VII the final

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TABLE I : Comparison of related works and this paper.

Publications	[10]	[11]	[12]	[13]	[14]	[15] [23]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	proposed approach
Force control	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓	✓	✓
Online, real-time	✓	✓	✓	✓	✓	✓	✓				✓	✓	✓	✓
Moderate H/W Spec.	✓	✓	✓	✓	✓	✓	✓	1	✓	✓	2	3	4	✓
Revolute Doors	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sliding Doors		✓	✓	✓		✓	✓		✓	✓	✓			✓
Estimate of Constraints		✓	✓	✓	✓	✓		✓	✓					✓
Estimate of Geometry					✓	✓		✓	✓					✓
Unknown Model		✓	✓	✓		✓			✓		✓		✓	✓
Unknown Parameters		✓	✓	✓	✓	✓		✓	✓				✓	✓
Proven Param. Identification														✓

[1] Multifingered hand with tactile sensors [2] Compliant joints (torque feedback at the joint level) – DLR lightweight robot II
[3] Joint compliance by using clutches to engage/disengage motors [4] Use of the humanoid robot HRP-2

outcome of this work is briefly discussed.

II. Related Work and our Contributions

Pioneering work on the door opening problem is presented in [10] and [11]. Experiments on door opening with an autonomous mobile manipulator assuming a known door model were performed in [10], using the combined motion of the manipulator and the mobile platform. In [11] a method estimating the constraints describing the door motion kinematics is proposed, based on the observation that ideally the motive force should be applied along the direction of the end-effector velocity. To overcome the problems of chattering due to measurement noise and ill-definedness of the normalization for slow end-effector motion, spatial filtering is proposed, but this may cause lag and affect the system stability. The use of velocity measurements to estimate the direction of motion has inspired the recent work of [12] using a moving average filter in the velocity domain. An estimator is used to provide a velocity reference for an admittance controller. Ill-defined normalizations and estimation lags are not treated. Estimating constraints with velocity measurements is also done in [13], applying velocity and impedance control along the tangent and the radial axis of the door opening trajectory respectively.

Several position-based estimation techniques have also been proposed to estimate geometric characteristics of the mechanism rather than the motion direction. Since estimation does not guarantee identification in each control step, those methods have been coupled with controllers providing the system with the proper compliance to absorb inaccuracies of the planned trajectories. In [14], the recorded motion of the end-effector is used in a least-squares approximation to estimate the center and the radius of the motion arc, and a compliant controller is used to cancel the effects of the high forces exerted due to inaccurate trajectory planning. A similar approach is presented in [24]. An optimization algorithm using the position of the end-effector was used in [15], [23]. The algorithm produces estimates of the radius and the center of the door and, subsequently of the control directions. The velocity reference is composed of a feedforward estimated tangential velocity and radial force feedback while an equilibrium point control law enables a viscoelastic behavior of the system around an equilibrium position. In [16], [25], an inverse Jacobian velocity control law with feedback of the force error following the Task Space Formalism [6] is considered. In order to obtain the natural decomposition of the task, which is essential within this framework, the authors propose to combine several sensor modalities so that robust estimation is established. In [25], the estimation is based on the end-effector trajectory, to align the task frame with the tangent of the hand trajectory.

On the other hand, probabilistic methods that are off-line and do not consider interaction force issues have been used for more advanced estimation tasks. In [18], a probabilistic framework for learning the kinematic model of articulated objects (object's parts connectivity, degrees of freedom, kinematic constraints) is proposed. The learning procedure requires a set of motion observations of the doors. The estimates are generated in an off-line manner and can feed force/position Cartesian controllers [26]. Probabilistic methods — particle filters and extended Kalman filters — for mobile manipulation have also been applied to simultaneously estimate the position of the robot and the angle of the door using, however, an a priori defined detailed model of the door [19].

Other work on door opening exploits advanced hardware capabilities. In [27], a combination of tactile-sensor and force-torque sensor is used to control the position and the orientation of the end-effector with respect to the handle. In [21], a specific hardware configuration with clutches that disengage selected robot motors from the corresponding actuating joints and hence enable passive rotation of these joints is used. Since no force sensing is present, a magnetic end-effector was used which cannot always provide the appropriate force for keeping the grasp of the handle fixed. The DLR lightweight robot controlled via Cartesian impedance control based on joint torque measurements is used for door opening in [20]. In [28], the authors present experiments using a force/torque sensor on a custom lightweight robot to define the desired trajectory for a door opening task. In [22], a method for door opening that uses an impulsive force exerted by the robot to a swinging door is proposed. A specific dynamic model for the door dynamics is used to calculate the initial angular velocity which is required for a specific change of the door angle, and implemented on the humanoid robot HRP-2. In [17], a multi-fingered hand with tactile sensors grasping the handle is used, and the geometry of the door is estimated by observing the positions of the fingertips while slightly and slowly pulling and pushing the door in position control. In a subsequent step, the desired trajectory is derived from the estimation procedure, and is used in a position controller.

Table I summarizes the literature on door opening and provides a comparison to our work. In the table, the term *force control* designates work that explicitly controls or limits the interaction forces, *online, real-time* implies that the method can be used to open a door directly, at human-like velocities, without any prior learning step, *moderate hardware requirements* means that the method can be used on a simple manipulator with velocity control and a force/torque sensor, and *revolute doors* and *sliding doors* describe what types of door kinematics that can be handled by the method. *Estimate of constraints* indicates methods that produce an estimate of the current kinematic constraints of a mechanism, while *estimate of geometry* indicates methods that produce an explicit estimate of the geometry of the door mechanics themselves. *Unknown model* indicates methods that will work properly even if the model (type of mechanism, i.e. revolute or prismatic joint) is not known a priori, and *unknown parameters* indicates methods that will work if the parameters of the mechanism (i.e. hinge position or motion axis of prismatic joint) are not known a priori. Finally, *proven parameter identification* states whether proofs are provided for the convergence of estimates.

In previous work, we presented a control algorithm for estimating the center of rotation for a revolute door, exploiting the torque or velocity inputs. We proved that we can identify the constraint direction as well as achieve velocity/force tracking for smooth door opening [29], [30]. The method assumes a revolute joint and free rotation of the hand, but the center of rotation is considered uncertain, thus limiting the approach to planar problems. The proposed update law uses a projection operator to guarantee well-defined updated estimates; the use of a projection set constrains the range of uncertainties that can be dealt with. In [31], we proposed a control scheme treating both sliding doors/drawers and revolute-joint doors with arbitrary hinge orientations, by assuming grasps to be fixed and will not rotate around the handle. Furthermore, the design of the update law does not require a projection operator since it produces inherently well-defined estimates that converge to the actual values.

In the work presented here, we propose a unified controller for both revolute and prismatic mechanisms, with formulations for both fixed and non-fixed grasps, and present experimental results on a real robot that demonstrate its performance on a range of different doors and drawers. The contribution of our work compared to the existing literature is a method that simultaneasly treats all of the following:
— Our method can be applied to open both rotational and sliding doors, without requiring ill-defined normalization.
— Our method is not based on unusual hardware capabilities and can be implemented in any velocity controlled manipulator with the capability to measure or estimate the forces and torques at the end effector.
— Our method is theoretically proven to achieve identification of the motion direction simultaneously with force/velocity convergence, by explicitly considering adaptive estimates in the controller design.

III. System and Problem Description

Generally, doors and drawers can be opened by grasping the handle and moving it along its intended trajectory of motion: along a circular path for hinged mechanisms, or along a linear path for sliding doors and drawers. We now formally define the problem of door/drawer opening under uncertainty, where the position of hinges, or direction of possible sliding motion is not known a priori.

A. Notation and Preliminaries

We introduce the following notation:

- Bold small letters denote vectors and bold capital letters denote matrices. Hat $\hat{\cdot}$ and tilde $\tilde{\cdot}$ denote estimates and errors between control variables and their corresponding desired values/vectors respectively. Notation $\cdot^\top$ denotes the transpose of a vector/matrix.
- The generalized position of a frame $\{i\}$ with respect to a frame $\{j\}$ is described by a position vector ${}^j\mathbf{p}_i \in \mathbb{R}^m$ and a rotation matrix ${}^j\mathbf{R}_i \in SO(m)$ where $m = 2$ or 3 for the planar and spatial case respectively. In case $\{j\} \equiv \{B\}$ where $\{B\}$ is the robot world inertial frame (typically located at the base of the robot) the left superscript is omitted. Each column of ${}^j\mathbf{R}_i$ is denoted by ${}^j\mathbf{x}_i \equiv \mathbf{R}_j^\top \mathbf{x}_i$, ${}^j\mathbf{y}_i \equiv \mathbf{R}_j^\top \mathbf{y}_i$, ${}^j\mathbf{z}_i \equiv \mathbf{R}_j^\top \mathbf{z}_i$ where $\mathbf{x}_i$, $\mathbf{y}_i$, $\mathbf{z}_i$ denote the columns of the rotation matrix $\mathbf{R}_i$ that describes the orientation of the frame $\{i\}$ with respect to the robot world inertial frame.
- The projection matrix on the orthogonal complement space of a unit three dimensional vector $\mathbf{a}$ is denoted by $\mathbf{P}(\mathbf{a})$ with $\mathbf{P}(\mathbf{a}) = \mathbf{P}^\top(\mathbf{a})$ and is defined as follows:
$$\mathbf{P}(\mathbf{a}) = \mathbf{I}_3 - \mathbf{a}\mathbf{a}^\top$$
- $\mathcal{I}(\mathbf{b})$ is an element-wise integral of a vector function of time $\mathbf{b}(t) \in \mathbb{R}^n$ over the time variable t, i.e:
$$\mathcal{I}(\mathbf{b}) = \int_0^t \mathbf{b}(\tau) d\tau$$

B. Kinematic model of robot door/drawer opening

We consider a setting in which the end-effector has grasped the handle of a mechanism with a revolute or prismatic joint. Let $\{e\}$ and $\{h\}$ be the end-effector and the handle frame respectively. The two frames are attached on the same kinematically known position e.g. a known point of the end-effector denoted by $\mathbf{p}_e$ and represented by different rotation matrices. The orientation of the end-effector frame is strictly connected to the robot kinematics while the orientation of the handle frame is related to the kinematic constraints of the task. In case of a rotating door (revolute joint) the kinematic constraints are defined by considering a frame $\{o\}$ attached at the unknown center of the circular trajectory of the end-effector while opening the rotating door. The axis $\mathbf{z}_o$ corresponds to the axis of the rotation while $\mathbf{x}_o$, $\mathbf{y}_o$ can be arbitrarily chosen (Fig. 1i).

We make the following assumptions:

Assumption 1. *There is no relative translational motion of the end-effector with respect to the handle, i.e.* ${}^h\dot{\mathbf{p}}_e = 0$.

Assumption 2. *There is no relative translational and rotational motion of the end-effector with respect to the handle, i.e.* ${}^h\dot{\mathbf{p}}_e = 0$ *and* ${}^h\dot{\mathbf{R}}_e = 0$.

Assumption 2 is more restrictive since it implies a fixed grasp of the handle, while Assumption 1 is more general and can accommodate grasps that can be modeled as passive revolute joints. Obviously, Assumption 2 also implies Assumption 1, but in this work the two assumptions will be treated separately, as they correspond to two different grasp types.

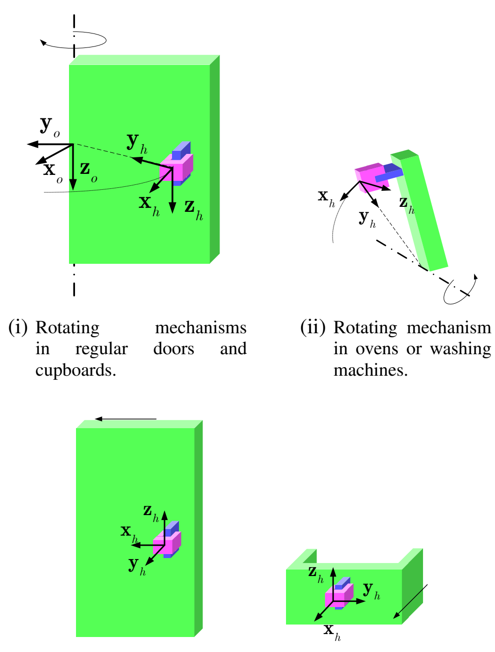
In the following we state a convention in order to define the frame $\{h\}$ in both cases of revolute joints (hinged doors) and prismatic joints (sliding doors, drawers):

a) **Revolute joints**:

- Axis $\mathbf{z}_h$ is equivalent to $\mathbf{z}_o$, i.e. $\mathbf{z}_o \equiv \mathbf{z}_h$
- Axis $\mathbf{y}_h$ is the unit vector along the line connecting the origins of $\{h\}$ and $\{o\}$ with direction towards the hinge.
- Axis $\mathbf{x}_h$ can be regarded as the allowed motion axis; it can be formed as follows: $\mathbf{x}_h = \mathbf{y}_h \times \mathbf{z}_h$

b) **Prismatic joints**: Vector $\mathbf{x}_h$ denotes the allowed motion axis. Axes $\mathbf{z}_h$ and $\mathbf{y}_h$ can be arbitrarily chosen in order to span the two-dimensional surface to which $\mathbf{x}_h$ is perpendicular.

Examples of Fig. 1 illustrate the definition of the $\{h\}$ axes.



(i) Rotating mechanisms in regular doors and cupboards.

(ii) Rotating mechanism in ovens or washing machines.

(iii) Sliding mechanisms in closets.

(iv) Mechanism in drawers.

Fig. 1 : Examples of rotating/sliding doors and drawers with revolute and prismatic joints.

For doors with a revolute joint, we can define the radial vector –which is parallel to $\mathbf{y}_h$– as the relative position of the frames $\{o\}$ and $\{e\}$ (or $\{h\}$):

$$\mathbf{r} \triangleq \mathbf{p}_o - \mathbf{p}_e, \tag{1}$$

and use it in the following equation to describe the first-order differential kinematics:

$$\mathbf{v} = \mathbf{r} \times \boldsymbol{\omega}_h \tag{2}$$

where $\mathbf{v}$ expresses the velocity of the end-effector $\dot{\mathbf{p}}_e$ or the handle velocity $\dot{\mathbf{p}}_h$ given ${}^e\dot{\mathbf{p}}_h = 0$.

Note that $\mathbf{r} = \frac{1}{\kappa}\mathbf{y}_h$ where κ denotes the curvature of the cyclic trajectory for the door opening (the inverse of the distance between the end-effector frame and the center of rotation). Thus, the inner product of (2) with $\mathbf{x}_h$ yields:

$$\omega = \kappa v \tag{3}$$

where $v \triangleq \mathbf{x}_h^\top \mathbf{v}$ denotes the end-effector/handle translational velocity magnitude and $\omega \triangleq \mathbf{z}_h^\top \boldsymbol{\omega}_h$ the rotational velocity of the handle. Although we consider a revolute joint at the hinge of the door, the constraint equation (3) can model cases of sliding doors or drawers represented by prismatic joints. Large values of radius correspond to practically zero curvature i.e. straight line trajectories for opening the mechanism and zero rotational velocity for the handle. Given the mechanism is rigid and Assumption 1 or 2, the remaining constraints regarding the translational velocity are:

$$\mathbf{P}(\mathbf{x}_h)\mathbf{v} = 0 \tag{4}$$

Constraint equation (4) implies that the end-effector/handle velocity can be parameterised as follows:

$$\mathbf{v} = v\mathbf{x}_h \tag{5}$$

Additionally Assumption 2 imposes extra constraints on the end-effector rotational velocities:

$$\boldsymbol{\omega}_e = \boldsymbol{\omega}_h \text{ with } \mathbf{P}(\mathbf{z}_h)\boldsymbol{\omega}_h = \mathbf{0} \tag{6}$$

C. Robot kinematic model

We consider the case of a n-DoF velocity-controlled manipulator satisfying the following assumption:

Assumption 3. *The kinematic structure and the number of DoF are sufficient for generating a 6 DoF movement of the end-effector and hence implementing the velocity for the task defined for a set of constraint's estimates including the actual constraint.*

An anthropomorphic arm with spherical wrist with $n = 6$ DoFs can satisfy Assumption 3.

For a velocity-controlled manipulator a reference generalized velocity $\mathbf{u}_{\text{ref}} \triangleq [\mathbf{v}_{\text{ref}}^\top \; \boldsymbol{\omega}_{\text{ref}}^\top]^\top \in \mathbb{R}^6$ ($\mathbf{v}_{\text{ref}} \in \mathbb{R}^3$ and $\boldsymbol{\omega}_{\text{ref}} \in \mathbb{R}^3$ denote the translational and rotational part respectively) expressed at the inertial frame can be considered as a kinematic controller which is mapped to the joint space in order to be applied at the joint velocity level as follows:

$$\dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q})\mathbf{u}_{\text{ref}} \tag{7}$$

with $\mathbf{q}$, $\dot{\mathbf{q}} \in \mathbb{R}^n$ being the joint positions and velocities respectively and $\mathbf{J}(\mathbf{q})^+ = \mathbf{J}(\mathbf{q})^\top \left[\mathbf{J}(\mathbf{q})\mathbf{J}(\mathbf{q})^\top\right]^{-1}$ being the inverse or the pseudo-inverse of the manipulator Jacobian $\mathbf{J}(\mathbf{q}) \in \mathbb{R}^{6\times n}$ relating the joint velocities $\dot{\mathbf{q}}$ to the end-effector velocities $[\dot{\mathbf{p}}_e^\top \; \boldsymbol{\omega}_e^\top]^\top$. If we consider the typical Euler-Lagrange robot dynamic model, the velocity error at the joint level drives the torque (current) controller $\mathbf{u}_{\boldsymbol{\tau}}(t)$.

Assumption 4. *The actuator has sufficient torque output, external force compensation, and current control loop frequency* *to keep the error between commanded and actual velocity negligible. Also, the inertial dynamics of the door mechanism are sufficiently weak, such that the portion of measured forces arising from accelerating the door mechanism are negligible.*

Cases where Assumption 4 is not valid are treated in the robustness analysis of Section IV-D.

D. Control Objective

The task of controlling the robot to manipulate a door or a drawer, can naturally be described in the handle frame. The desired variables should be defined in the robot inertial (or end-effector) frame to be executable by the robot. Let $\mathbf{f} \in \mathbb{R}^3$ denote the interaction force exerted at the end-effector, $\boldsymbol{\tau} \in \mathbb{R}^3$ the torque around the origin of the end-effector frame and $\mathbf{f}_d$, $\boldsymbol{\tau}_d$ the corresponding desired vectors. Let $v_d(t)$ be the desired velocity along the motion axis of frame $\{h\}$. Then the desired velocity $\mathbf{v}_d(t)$ is defined along $\mathbf{x}_h$, i.e. $\mathbf{v}_d = v_d(t)\mathbf{x}_h$, and the force control objective can be achieved by projecting the desired force on the orthogonal complement space of $\mathbf{x}_h$ (constrained directions) i.e. $\mathbf{P}(\mathbf{x}_h)\mathbf{f}_d$; a small valued or zero vector $\mathbf{f}_d$ corresponds to small forces along the constraint directions. The control objective can be formulated as:

Problem 1. *Design a translational velocity control $\mathbf{v}_{ref}$ such that $\mathbf{P}(\mathbf{x}_h)\mathbf{f} \to \mathbf{P}(\mathbf{x}_h)\mathbf{f}_d$ and $\mathbf{v} \to v_d(t)\mathbf{x}_h$, without knowing accurately the motion axis $\mathbf{x}_h$ and the corresponding constraint directions $\mathbf{P}(\mathbf{x}_h)$.*

When Assumption 2 is valid, the desired rotational velocity can be defined using $v_d(t)$ along the axis $\boldsymbol{\kappa} \triangleq \kappa\mathbf{z}_h$, i.e. $\boldsymbol{\omega}_d(t) = v_d(t)\boldsymbol{\kappa}$. In this case the total interaction torque denoted by $\boldsymbol{\tau} \in \mathbb{R}^3$ is controllable and thus an additional control objective can be formulated as follows:

Problem 2. *Design a rotational velocity control $\boldsymbol{\omega}_{ref}$ to act in parallel to $\mathbf{v}_{ref}$ such that $\boldsymbol{\tau} \to \boldsymbol{\tau}_d$ and $\boldsymbol{\omega} \to \kappa v_d(t)\mathbf{z}_h$ without knowing the axis of rotation $\mathbf{z}_h$ and the variable κ. The rotation control objective is mainly set to achieve identification of $\mathbf{z}_h$ and κ.*

We consider that the opening task is accomplished when the observed end-effector trajectory — which coincides with the handle trajectory — has progressed far enough to enable the robot to perform a subsequent task, like picking up an object in a drawer or passing through a door. Hence, some perception system observing the progress of the opening of the mechanism is additionally required to provide the robot with the command to halt the opening procedure.

IV. Control Design

In this section, we propose a solution to Problems 1 and 2 stated in Section III-D above. When only Assumption 1 holds, the solution to Problem 1 is given by Theorem 1, and when Assumption 2 holds, the solution to Problem 1 and 2 is given by Theorem 1 and 2. Robustness analysis is performed to derive bounds for the estimation error in case of disturbances. Proofs of the propositions and theorems of this section are given in the Appendix. First, we propose a translational velocity reference that can employ two different update laws corresponding to Assumptions 1 and 2 respectively.

A. Translational velocity reference with force feedback

Let $\hat{\mathbf{x}}_h(t)$ denote the online estimate of the motion direction $\mathbf{x}_h$. Dropping the argument t from $\hat{\mathbf{x}}_h(t)$ and $v_d(t)$ for notation convenience, we let $\mathbf{v}_{\text{ref}}$ be given by:

$$\mathbf{v}_{\text{ref}} = v_d\hat{\mathbf{x}}_h - \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f \tag{8}$$

where $\mathbf{v}_f$ is a PI force feedback input defined as follows:

$$\mathbf{v}_f = \alpha_f\tilde{\mathbf{f}} \ + \beta_f\mathcal{I}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right] \tag{9}$$

with $\tilde{\mathbf{f}} = \mathbf{f} - \mathbf{f}_d$ and α_f, β_f being positive control constants. Note that the first term of the reference velocity is the desired velocity along the estimated motion direction and it is not in general consistent with the allowable motion direction. However the second term – which is a force controller – compensates for the inconsistency owing to kinematic uncertainties and renders a reference velocity $v_{\text{ref}} \triangleq \mathbf{x}_h^\top\mathbf{v}_{\text{ref}}$ along the actual motion direction by generating forces along the constrained directions (that can be considered as Lagrange multipliers [32]).

Let $\theta(t)$ denote the angle formed between the actual vector $\mathbf{x}_h$ which is a rotating vector and its online estimate $\hat{\mathbf{x}}_h$ which is time-varying. Given that the estimate $\hat{\mathbf{x}}_h$ is a unit vector, $\cos\theta(t)$ can be defined as follows:

$$\cos\theta(t) \triangleq \mathbf{x}_h^\top\hat{\mathbf{x}}_h = {}^e\mathbf{x}_h^\top {}^e\hat{\mathbf{x}}_h \tag{10}$$

The definition is independent of the frame in which $\mathbf{x}_h$ and $\hat{\mathbf{x}}_h$ are expressed. In general, an online estimate of the vector $\hat{\mathbf{x}}_h$ provided by an adaptive estimator is not unit but in the following we are going to design an update law that produces estimates of unit magnitude. The derivative of $\theta(t)$ depends on both estimation rate and door motion velocity:

$$\frac{d}{dt}\cos\theta(t) = \mathbf{x}_h^\top\left(\dot{\hat{\mathbf{x}}}_h - v\boldsymbol{\kappa}\times\hat{\mathbf{x}}_h\right) \tag{11}$$

When the grasp imposes constraint on the rotation of the end-effector with respect to the handle (Assumption 2), the derivative of $\theta(t)$ is independent of the door motion velocity. The derivative of $\cos\theta(t)$ can be calculated as follows:

$$\frac{d}{dt}\cos\theta(t) = {}^e\mathbf{x}_h^\top {}^e\dot{\hat{\mathbf{x}}}_h \text{ for } {}^e\dot{\mathbf{x}}_h = \mathbf{0} \tag{12}$$

In the following, we drop out the argument of t from $\theta(t)$ for notation convenience.

The velocity error $\tilde{\mathbf{v}} \triangleq \mathbf{v} - \mathbf{v}_{\text{ref}}$ can be decomposed along $\hat{\mathbf{x}}_h$ and the corresponding orthogonal complement space as follows:

$$\tilde{\mathbf{v}} = \mathbf{P}(\hat{\mathbf{x}}_h)(\mathbf{v} + \mathbf{v}_f) + (v\cos\theta - v_d)\,\hat{\mathbf{x}}_h \tag{13}$$

In case of velocity controlled manipulators described by (7) we get $\tilde{\mathbf{v}} = \mathbf{0}$. Since the right-hand side of (13) consists of two orthogonal terms, $\tilde{\mathbf{v}} = \mathbf{0}$ implies the following closed-loop system equations:

$$\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f = -v\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h \tag{14}$$

$$v = \tfrac{1}{\cos\theta}v_d \tag{15}$$

Taking the norm of each side of (14) and substituting (15) gives:

$$\|\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f\| = |v_d \tan\theta| \tag{16}$$

From (14)-(16) it is clear how the estimation error in the axis of motion affects the force errors and the velocity of the end-effector. Note that the higher the uncertainty in the motion axis θ is the higher the velocity v and the estimated constraint forces $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{f}$ can be. In the extreme case of $|\theta(t)| = \pi/2$ which is equivalent to trying to move the mechanism along a direction which is completely mechanically constrained extremely high forces arise. Hence, the update law must at least guarantee $|\theta(t)| \neq \pi/2, \forall t$. Equations (14)-(16) describing the closed loop system link the physical controlled variables like velocities and forces with the uncertainty measure $\theta(t)$ and thus they are instrumental in the design of the update laws for estimating the unknown parameters described in the following subsections.

1) Update Law for the Motion Direction given Assumption 1: We propose the following update law for $\hat{\mathbf{x}}_h$[1]:

$$\dot{\hat{\mathbf{x}}}_h = -\gamma v_{\text{ref}}\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f - v_{\text{ref}}\hat{\mathbf{x}}_h \times \hat{\boldsymbol{\kappa}} \tag{17}$$

where γ is positive control gain for tuning the adaptation rate, $\hat{\boldsymbol{\kappa}}$ is the online estimate of the scaled rotational axis $\boldsymbol{\kappa}$ that is produced by the following appropriately designed update law:

$$\dot{\hat{\boldsymbol{\kappa}}} = \boldsymbol{\Gamma}_\kappa \hat{\mathbf{x}}_h \times \mathbf{v}_{\text{ref}}, \tag{18}$$

with $\boldsymbol{\Gamma}_\kappa \in \mathbb{R}^{3\times3}$ being a positive definite gain matrix, and $v_{\text{ref}} \triangleq \mathbf{x}_h^\top \mathbf{v}_{\text{ref}}$ can be calculated independently of the knowledge of the motion direction for $\hat{\mathbf{x}}_h^\top \mathbf{x}_h > 0$ as follows:

$$v_{\text{ref}} = \operatorname{sgn}\left(\hat{\mathbf{x}}_h^\top \mathbf{v}_{\text{ref}}\right)\|\mathbf{v}_{\text{ref}}\| \tag{19}$$

The use of the update laws (17), (18) is instrumental for the stability analysis and the convergence of the estimated parameters. In the Appendix it is shown that the use of Eq. (17), (18) enables the proof of the following Propositions:

Proposition 1. *Update law (17) ensures that the norm of $\hat{\mathbf{x}}_h(t)$ is invariant, i.e. given $\|\hat{\mathbf{x}}_h(0)\| = 1$, $\|\hat{\mathbf{x}}_h(t)\| = 1, \ \forall t$.*

Proposition 2. *Update laws (17), (18) driven by the reference velocity $\mathbf{v}_{ref}$ given by (8) with a time-varying desired velocity yield to the following nonautonomous (time-dependent) nonlinear system with states θ and $\tilde{\boldsymbol{\kappa}} = \hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}$ which are well defined in the domain $D = \{\theta \in \mathbb{R}, \tilde{\boldsymbol{\kappa}} \in \mathbb{R}^3 : |\theta| < \frac{\pi}{2}\}$:*

$$\dot{\theta} = -\gamma v_d^2(t)\frac{\tan\theta}{\cos\theta} - \frac{v_d(t)}{\cos\theta}\tilde{\boldsymbol{\kappa}}^\top \mathbf{n} \tag{20}$$

$$\dot{\tilde{\boldsymbol{\kappa}}} = v_d(t)\tan\theta \boldsymbol{\Gamma}_\kappa \mathbf{n} \tag{21}$$

with $\mathbf{n}$ being a unit vector perpendicular to the surface defined by $\mathbf{x}_h$ and $\hat{\mathbf{x}}_h$, that implies that the estimation error angle θ stays in D and converges to zero for v_d satisfying the persistent excitation (PE) condition (see [33]), i.e.:

$$\int_t^{t+T_0} v_d^2(\sigma)d\sigma \geq \alpha_0 T_0 \tag{22}$$

$\forall t \geq 0$ and for some $\alpha_0, \ T_0 > 0$.

[1]Details on the design of the update laws can be found in the Appendix.

2) Update Law for the Motion Direction given Assumption 2: Since the relative orientation of the handle frame and the end-effector is constant we can propose a simpler update law by using as a regressor the end-effector frame rotation matrix. We propose the following update law for $\hat{\mathbf{x}}_h$[1]:

$$\hat{\mathbf{x}}_h = \mathbf{R}_e {}^e\hat{\mathbf{x}}_h \tag{23}$$

$${}^e\dot{\hat{\mathbf{x}}}_h = -\gamma v_d \mathbf{R}_e^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f \tag{24}$$

where γ is a positive control gain for tuning the adaptation rate. Proposition 3, 4 describe how the update law (23), (24) produces well-defined estimates that converge to the actual values. In the Appendix it is shown that the use of Eq. (24) enables the proof of the following Propositions:

Proposition 3. *Update law (24) ensures that the norm of ${}^e\hat{\mathbf{x}}_h(t)$ is invariant, i.e. starting with $\|{}^e\hat{\mathbf{x}}_h(0)\| = 1$, $\|{}^e\hat{\mathbf{x}}_h(t)\| = 1, \ \forall t$.*

Proposition 4. *Update laws (23), (24) driven by the reference velocity $\mathbf{v}_{ref}$ given by (8) with a time-varying desired velocity yield to the following nonautonomous (time-dependent) nonlinear system with state θ which is well defined in the domain $D' = \{\theta \in \mathbb{R} : |\theta| < \frac{\pi}{2}\}$:*

$$\dot{\theta} = -\gamma v_d^2(t)\tan\theta \tag{25}$$

which implies that the estimation error angle θ stays in D and converges to zero for v_d satisfying the PE condition given by (22).

3) Summary and force convergence results: Propositions 2 and 4 imply that given the update laws and the controller the estimates are well defined [2] and converge to their actual values. Propositions can be considered as intermediate steps for proving the stability of the overall system including aditionally the internal state introduced by the force integral in the following Theorem for Assumptions 1 and 2 (see Appendix for proof).

Theorem 1. *Consider a velocity controlled manipulator (7), grasping the handle of a sliding/rotating door or a drawer. If the robot is driven by a velocity control input $\mathbf{v}_{ref}$ (8) that uses a PI force feedback input $\mathbf{v}_f$ (9) and:*

1) *the update law (17), (18) given that Assumption 1 is valid, or*
2) *the update law (23), (24) given that Assumption 2 is valid,*

then Problem 1 will be solved, i.e., smooth opening of the moving mechanism and identification of the estimated parameters will be achieved. Analytically, the following convergence results are guaranteed: $\hat{\mathbf{x}}_h \to \mathbf{x}_h$, $\mathbf{v} \to \mathbf{x}_h v_d$, and $\mathbf{P}(\mathbf{x}_h)\tilde{\mathbf{f}} \to 0$, given that v_d satisfies the PE condition given by (22).

B. Rotational velocity reference with torque feedback

In case of Assumption 1, the rotational velocity of the end-effector can be set in order to optimize some performance index such as the manipulability index of the arm while torque cannot be controlled. On the other hand, Assumption 2 implies that the rotational velocity of the end-effector is strictly

[2]The estimates are unit vectors if the initial estimate is unit and $|\theta(t)| \neq \pi/2, \forall t$, is true since $|\theta(t)| < \pi/2$ if $|\theta(0)| < \pi/2$.

connected to the rotational velocity of the mechanism and subsequently to the translational velocity of the end-effector through the constraint (3). Hence the reference rotational velocity should be appropriately designed using the desired translational velocity v_d and exploiting torque feedback in order to fulfill the constraints (3), (6):

$$\boldsymbol{\omega}_{\text{ref}} = v_d \hat{\boldsymbol{\kappa}} - \boldsymbol{\omega}_\tau \tag{26}$$

where $\hat{\boldsymbol{\kappa}}$ is the online estimate of $\boldsymbol{\kappa}$ and it is appropriately designed as follows:

$$\dot{\hat{\boldsymbol{\kappa}}} = -v_d \boldsymbol{\Gamma}_\kappa \boldsymbol{\omega}_\tau \tag{27}$$

with $\boldsymbol{\Gamma}_\kappa$ being a positive definite matrix of update gains, and $\boldsymbol{\omega}_\tau$ is a PI torque feedback input defined as follows:

$$\boldsymbol{\omega}_\tau = \alpha_\tau \tilde{\boldsymbol{\tau}} + \beta_\tau \mathcal{I}(\tilde{\boldsymbol{\tau}}) \tag{28}$$

where $\tilde{\boldsymbol{\tau}} = \boldsymbol{\tau} - \boldsymbol{\tau}_d$.

The design of the update law (27) is instrumental for the proof of the following theorem (see Appendix):

Theorem 2. *Consider a velocity controlled manipulator (7) grasping the handle of a sliding/rotating door or a drawer according to Assumption 2. If the robot is driven by a velocity control input that consists of both $\mathbf{v}_{ref}$ (8) and $\boldsymbol{\omega}_{ref}$ (26) that uses a PI torque feedback input $\boldsymbol{\omega}_\tau$ (28) as well as the update law (27) to estimate the vector $\boldsymbol{\kappa}$, then Problem 2 will be solved, i.e., the following convergence results – additionally to those of Theorem 1 – are guaranteed: $\tilde{\boldsymbol{\tau}} \to 0$, $\mathcal{I}(\tilde{\boldsymbol{\tau}}) \to 0$, $\hat{\boldsymbol{\kappa}} \to \boldsymbol{\kappa}$, $\boldsymbol{\omega}_e \to v_d \boldsymbol{\kappa}$, for v_d satisfying the PE condition given by (22).*

C. Torque-controlled robot manipulators

In the aforementioned results we have considered an ideal velocity-controlled robot manipulator which is connected with the environment through rigid constraints. These assumptions allow the forces to be modeled as Lagrange multipliers related to the the range of uncertainty as shown in (16), similarly to the Lagrange multipliers used for modeling forces in the case of a torque-controlled manipulator that interacts with a rigid environment [34]. In the case of a velocity-controlled manipulator the underlying assumption is that the commanded velocity is achieved adequately fast (Assumption 4), while in the case of a torque-controlled manipulator, the actuator dynamics can be considered negligible.

The adaptive velocity controllers proposed in this paper can be readily modified and applied to the outer loop of a dynamic controller suitable for a torque-controlled manipulator as shown in our previous work [29] where the inner loop is formulated by the superposition of an appropriately designed generalized reference force, a velocity error feedback term and a term compensating for the robot dynamic model. The reference velocity (outer loop) used in the velocity error feedback term is similar to the one proposed in this work but did not use the proportional force errors terms of (8), in order to avoid differentiation of noisy force/torque measurements while calculating the reference acceleration required in the implementation on a torque controlled robot. In general the design of dynamic (torque) controllers of robots may require a term that compensates for the dynamic model of the robot. In case of dynamic uncertainties, adaptive controllers that employ update laws for dynamic parameters' estimation have been proposed [35]. The PE condition is complicated and the joint trajectories have to be properly chosen in order to identify the dynamic parameters online. However dynamic parameter identification is not crucial for guaranteeing the tracking error performance. In contrast to dynamic uncertainties, here we consider uncertainties of the parameters involved in the kinematic constraints. The identification of these parameters – that is prerequisite for achieving the control objectives – depends on the trajectory of the end-effector rather than on the individual joints' trajectories.

In the following section, instead of extending the analysis to the case of a torque-controlled robot manipulator, we present a robustness analysis for the performance of the system under disturbances $\boldsymbol{\delta}(t)$ arising at the velocity level that may also represent errors arising in the inner control loop.

D. Robustness Analysis

In this section, we present a robustness analysis for the performance of the system under disturbances $\boldsymbol{\delta}(t)$ arising at the velocity level, i.e. $\mathbf{v} = \mathbf{v}_{\text{ref}} + \boldsymbol{\delta}(t)$. These disturbances can incorporate delays at the velocity tracking control loop that would be vanishing for the case of a desired constant velocity, as well as disturbances arising due to modeling errors e.g. compliance and deformations at the grasp or at the joint of the mechanism.

The closed loop system equations (14), (15) are now affected by the disturbances as follows:

$$\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f = \mathbf{P}(\hat{\mathbf{x}}_h)\left[-v\mathbf{x}_h + \boldsymbol{\delta}(t)\right] \tag{29}$$

$$v = \tfrac{1}{\cos\theta}\left[v_d + \hat{\mathbf{x}}_h^\top \boldsymbol{\delta}(t)\right] \tag{30}$$

In the following propositions we examine the robustness of the update law (23), (24) in case of disturbances. The proofs of the propositions can be found in the Appendix.

Proposition 5. *The update laws (23), (24) driven by the reference velocity $\mathbf{v}_{ref}$ given by (8) with a time-varying desired velocity in case of disturbances (i.e. eq. (29), (30) hold) yield to the following nonautonomous (time-dependent) nonlinear system with state θ which is well defined in the domain $D' = \{\theta \in \mathbb{R} : |\theta| < \frac{\pi}{2}\}$:*

$$\dot{\theta} = -\gamma v_d^2(t)\tan\theta - \gamma v_d(t)\frac{sgn(\theta)}{\cos\theta}\mathbf{n'}^\top \boldsymbol{\delta}(t) \tag{31}$$

where $\mathbf{n'}$ is a constrained direction (i.e. $\mathbf{n'}^\top \mathbf{x}_h = 0$) lying on the common plane of $\hat{\mathbf{x}}_h$ and $\mathbf{x}_h$.

Proposition 6. *The system (31) is uniformly ultimately bounded with respect to the following region:*

$$\Omega = \{\theta \in D' : \quad |\theta| < \arcsin\lambda(t)\} \tag{32}$$

where $\lambda(t) = \frac{|\mathbf{n'}^\top \boldsymbol{\delta}(t)|}{|v_d|}$ for $v_d \neq 0$.

Variable $\lambda(t)$ denotes the range of the region of angles θ in which the estimation error converges and is well-defined for $|\mathbf{n'}^\top \boldsymbol{\delta}(t)| < |v_d|$. Notice that $\lambda(t)$ can be alternatively written as $\lambda(t) = \cos\varphi(t)\ell(t)$ where $\ell(t)$ is defined as the ratio of the magnitude of the velocity error disturbance and the commanded desired velocity i.e. $\ell \triangleq \frac{\|\boldsymbol{\delta}(t)\|}{|v_d|}$ and $\varphi(t)$ denotes the angle formed between $\mathbf{n'}$ and $\boldsymbol{\delta}(t)$. In the extreme case of a velocity disturbance being aligned with the constrained direction[3], a well-defined $\lambda(t)$ requires that the ratio $\ell(t)$ is smaller than one. If $|\varphi(t)|$ is smaller than 90 deg, the ratio $\ell(t)$ is allowed to take values bigger than one. If the disturbance is aligned with the motion direction (i.e. $\lambda(t) = 0$) convergence of the estimation error to zero is guaranteed irrespective of the magnitude of the disturbance. Notice also that in the case of a vanishing disturbance ($\lambda(t) \to 0$), region Ω shrinks to zero, thus guaranteeing identification of the uncertain motion axis. In the case of persistent disturbances the identification error is comparable to the error arising from estimation based on inaccurate velocity measurements.

If Assumption 1 holds we can achieve a similar result by modifying the update law using a σ-modification [33]. Errors in rotational velocity can be treated in a similar fashion.

E. Discussion

The proposed control scheme produces estimates of the unconstrained motion direction and axis of rotation (in the case of a rotational door) using the update laws (17), (18) (Assumption 1) or (24), (27) (Assumption 2) respectively. In case of Assumption 1, the motion direction estimate converges to the actual direction but there is no proof that the scaled rotation vector converges to the actual one; note however that the rotation vector estimate is not used in the velocity reference (8) and thus its convergence does not affect stability and performance. In this case torques are not controlled and the redundant degrees of freedom can be used to enhance manipulability. In case of Assumption 2, both estimated vectors converge to the actual values and the estimates are used within velocity references (8) and (26). The velocity references enforce the robot to move with a desired velocity while controlling both forces and/or torques along the constrained directions to small values guaranteeing compliant behavior.

By defining the handle frame according to the task constraints and involving the curvature instead of the radius and the center of rotation, the proposed method is applicable to both revolute and prismatic mechanisms. Coupling the estimation with the controller following the adaptive control framework (see e.g. [33]) makes the method inherently on-line, enabling proofs of the convergence of estimated parameters to true values and convergence of force/torque errors.

Note that no projection operators have been used in the update laws design reducing the amount of the required prior knowledge. The main condition for guaranteed performance is that the initial estimate is not perpendicular to the true value i.e. $\theta(0) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. A typical example where this condition is not satisfied could be when opening a drawer with an initial estimate corresponding to a sliding door (c.f. Fig. 2, cases (iv) and (v)). This issue can be overcome by using a moderate deviation in the initial estimate (see Section V). The proposed method alone can not handle the case where the initial estimate is in the opposite direction of the true value, as this would generate a closing motion. This can be handled by an external monitoring system that stops the motion and retries with a different initial estimate if measured forces are too high, similar to a human who first pushes a door, and when it does not open, tries to pull it instead.

In the case of a fixed grasp we can produce explicit estimates of the physical location of the hinge of a revolute door, as reliable estimates of both radial direction and radial distance are available. If we make the assumption that a large enough radial distance (we arbitrarily choose 10 m) implies a prismatic mechanism, Algorithm 1, that can be used at any time instant continuously or in a discrete manner, will identify the hinge position. Given the center of rotation and the estimate of the curvature, we can estimate local variables with respect to the initial position of the end-effector such as the angular state of the door or the translation of the drawer and use them in order to provide internally – and not with an external perception system – a halt command.

Algorithm 1 Reasoning of the type of joint/Calculation of the rotation center

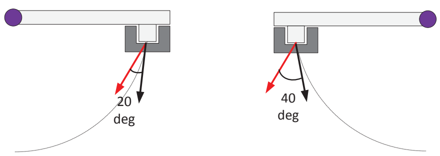
```
while Not Done do
    if ‖κ̂‖ > 0.1 then
        Rotational door
        Calculate the estimated radius ρ̂ := ‖κ̂‖^{-1}
        Calculate the estimated radial direction:
            ŷ_h := ρ̂κ̂ × x̂_h
        Calculate the center of rotation p̂_o := p_e + ρ̂ŷ_h.
    else
        Sliding door or drawer
    end if
end while
```

V. Scenarios and Evaluation

To illustrate and demonstrate the generality of the approach, we evaluate the performance for five different mechanisms in simulation, and three mechanisms in experiments on a physical robot. The simulations consider five different scenarios covering five common cases found in domestic environments, see Fig. 2. All cases are treated with the same initial estimates and controller gains. Cases (i) and (ii) are typical revolute doors with vertical axis, with the hinge to the left or to the right, respectively. Case (iii) models a revolute door with axis of rotation parallel to the floor, such as is common for ovens. The radius of these door are all set to 50 cm. Case (iv) models a sliding door, and case (v) a typical drawer. The common initial estimate used in all cases is that of a prismatic joint, assuming $\hat{\boldsymbol{\kappa}}(0) = \mathbf{0}$. The initial estimate of the unconstrained direction of motion is 30 deg offset from the normal direction to the plane of the door or drawer. The initial estimates are shown as red/gray arrows, and the true direction is shown as black arrows in Fig. 2. The angular values given are

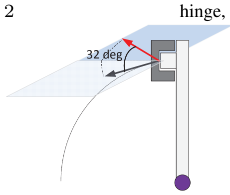
[3] Note that large velocity errors in the constraint directions would be highly unlikely, as the the constraint directions are defined as the directions in which the mechanism has significantly higher stiffness, and high velocities in these directions are effectively blocked.

the initial errors of the estimates. The controller gains are chosen as follows: $\alpha_f = \alpha_\tau = 0.05$, $\beta_f = \beta_\tau = 0.005$, $\gamma = \gamma_\kappa = 2000$. The desired motion velocity is 5 cm/s, given as $v_d = 5(1 - e^{-10t})$ cm/s to avoid sharp initial transients.

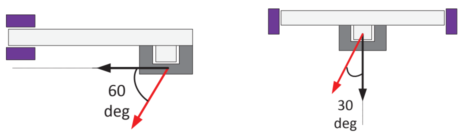


(i) Revolute door, left hinge, $\kappa = 2$

(ii) Revolute door, right hinge, $\kappa = 2$.



(iii) Revolute door, bottom hinge, $\kappa = 2$.



(iv) Prismatic door, $\kappa = 0$.

(v) Prismatic drawer, $\kappa = 0$.

Fig. 2 : Five different simulation cases using the same initial estimate: the angle indicates the initial error.

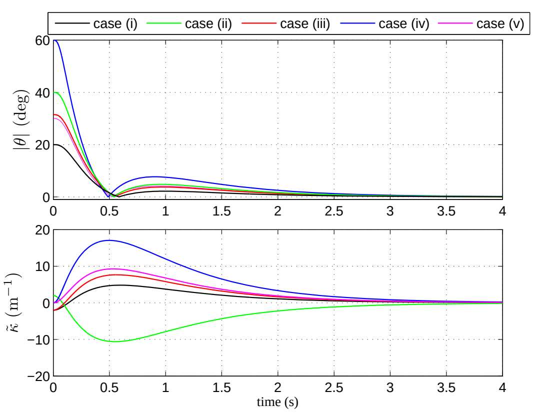


Fig. 3 : Estimation responses for the estimator (17), (18) (Assumption 1): (upper) estimation error response in the orientation of motion axis; (lower) estimation error response for the inverse distance between hinge and end-effector.

Fig. 3, 4(upper) show the response of the motion axis estimation errors for update laws (17), (18) (Assumption 1 - passive revolute joint) and (24), (27) (Assumption 2 - fixed grasp) respectively – convergence to the actual axis is achieved even for larger initial errors. Note that for fixed grasps the settling time is shorter and there is less overshoot as compared to the performance for the revolute grasp, even though the same gains were used. In Fig. 3(upper) the sharp corner in the plot of the absolute value of the angle estimation error at 0.5 s corresponds to an overshoot. Fig. 3, 4(lower) depict the estimation error for the most important element of $\boldsymbol{\kappa}$. In the

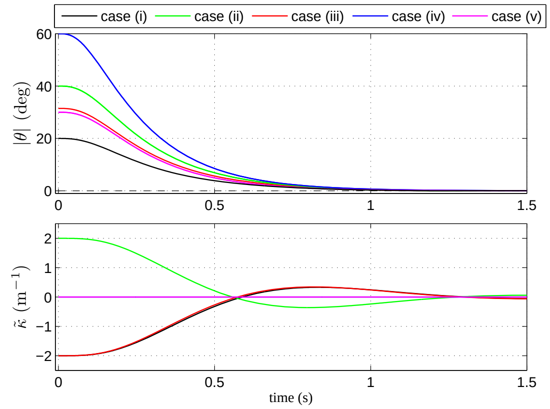
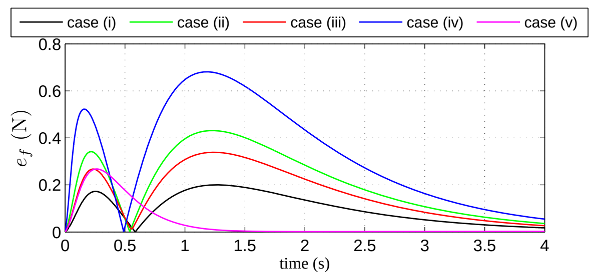
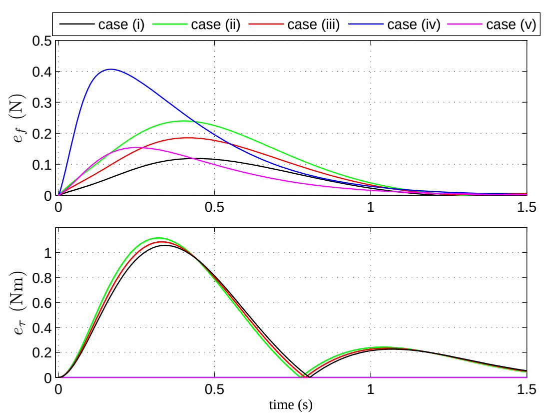


Fig. 4 : Estimation responses in case of the estimator (24), (27) (Assumption 2) : (upper) estimation error response in the orientation of motion axis; (lower) response of the estimation error of the inverse distance between hinge and end-effector.



(i) Assumption 1



(ii) Assumption 2 : (upper) norm of the projected force error, (lower) norm of the torque error

Fig. 5 : Force and torque responses

case of a fixed grasp (Fig. 4), this corresponds to the inverse signed distance κ between the end-effector and the hinge and the estimate $\hat{\kappa}$ is not modified when $\hat{\kappa}(0)$ coincides with κ and it converges to its actual value in all cases. In case of the passive revolute joint (Fig. 3), the convergence to actual value is also achieved, even though it is not proven theoretically, but the convergence time is twice the corresponding time for fixed grasps and a high overshoot is observed. Furthermore, some of the elements of $\boldsymbol{\kappa}$ are initially modified even when the original estimate coincides with the actual parameter, and converge with the rest of the system.

If Assumption 2 holds, it has been theoretically proven that combining estimates of the modulated rotation axis with the motion axis we can calculate the center of rotation of the rotational doors in real time using Algorithm 1; simulation gives errors of approximately 1.4 cm after 1.5 s, which is equivalent of opening the door 7.5 cm. Given the threshold of $\|\hat{\boldsymbol{\kappa}}\| > 0.1$, the revolute doors are identified as such after 0.2 s. The estimation error responses (Fig. 3) show that Algorithm 1 can be used even when only Assumption 1 holds, but the identification procedure is slower (error of 3.4 cm after 4 s). Fig. 5ii shows the responses of the Euclidian norms of force and torque errors ($e_f = \|\mathbf{P}(\mathbf{x}_h)\tilde{\mathbf{f}}\|$ and $e_\tau = \|\tilde{\boldsymbol{\tau}}\|$ respectively) in the case of fixed grasps. Errors converge to zero following the convergence rate of modulated rotation axis and motion axis. The same is true for the force errors in the case of a passive revolute joint grasp, as shown in Fig. 5i.

VI. Experiments

To evaluate the performance of the proposed method under unmodelled system and sensor noise, the performance of the door opening controllers were also tested on a real robot setup. Our setup consists of a 7-DoF manipulator whose joints are velocity controlled. New velocity setpoints are given at 130 Hz, and maintained by internal PID current controllers running at 2 kHz. The manipulator includes a wrist mounted ATI Mini45 6-axis force-torque sensor sampled at 650 Hz, which we use for the force feedback and estimation part of our controllers. Additionally, it is equipped with a two finger parallel gripper which allows us to grasp the doors. The robot is approximately human-sized, and has a mass of approximately 150 kg. See [36] for a detailed description of the system.

As in the simulations, we used the door opening controllers to open and identify the kinematic parameters of doors of three different types of kinematics: a revolute door, a sliding door and a drawer, as shown in Fig. 6. Each of the experiments were initialized with a 30 deg error in the axis of motion on the motion plane. For the three kinds of doors we performed experiments using fixed grasps on the doors (Assumption 2). Two of the doors — the revolute door and the drawer — were of the type typically used for kitchen cupboards, and were light-weight and fitted with significantly less rigid handles. For these, the performance of the controller evaluated both when grasping the rigid fronts of the doors directly and when grasping the less rigid handles. This allowed a test of the robustness of the system to deviations from the assumptions of rigid links in the kinematic chain. The third door was a sliding door of much larger mass, with a very rigid heavy-duty handle.

In the experiments, we set v_d to be constant for the whole trial, and thus the velocity is limited by the parts of the task where the manipulator is close to a singularity, or cannot move fast for some other reason, such as being limited by mass and/or friction of the heavy sliding door. Faster performance can be achived by letting v_d vary over the door opening tasks.

We also constructed alternative cylindrical handles for the revolute door and the drawer which allowed unconstrained rotation of the robot's gripper around the axis of the handle as shown in Fig. 6i. With these handles the generated grasps can be considered as passive revolute joints, and thus we can test our controllers following Assumption 1 of section III-B. The ground truth of the axis of motion of the revolute door was obtained by manually moving the end effector to a series of points while grasping the handle, fitting a circle to the resulting trajectory and calculating the tangent of the circle at each point.

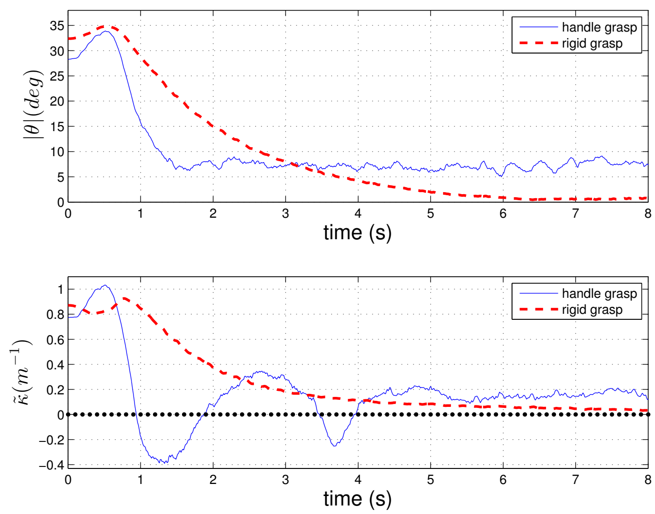


Fig. 7 : Estimation errors for a revolute door with fixed grasp, (upper): motion axis estimation error, (lower): inverse radius of curvature estimation error. Red dashed line show the estimation errors while grasping the door directly in a rigid manner, while the blue solid line shows the performance when grasping the less rigid handle.

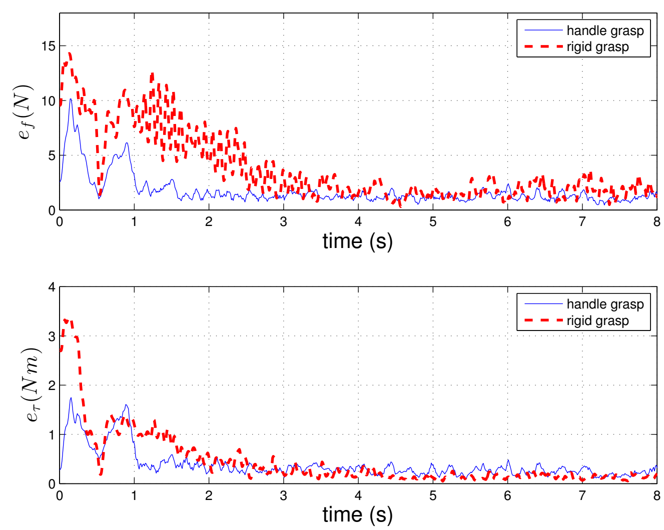


Fig. 8 : Force and torque responses for the revolute door experiment with fixed grasp, upper figure: norm of the projected force error, lower figure: norm of the torque error.

Fig. 7 shows the estimation error of the motion axis while opening a revolute door with a fixed grasp and the estimation error of the curvature $\tilde{\kappa}$, while Fig. 8 shows the corresponding

(i) Revolute door with passive handle. (ii) Sliding door. (iii) Drawer with passive handle.

Fig. 6 : Doors used for experimental evaluation of our controller

norm of the projected force error and the norm of the torque error.

The results of the revolute door experiment while grasping the door directly is shown with a red dashed line. Here we see convergence to small estimation errors; less than 0.8 deg for the motion axis estimate, and 0.033m^{-1} for the curvature. In comparison, the performance when grasping the less rigid handle of the door is shown in solid blue lines in the figures. Even though the controller managed to successfully open the door and regulate the forces and torques, it incurred a 7 deg steady state error in the estimation of the motion axis and a $0.16\ \text{m}^{-1}$ error in the estimation of the inverse radius of curvature κ. In Fig. 8, we can see that the less rigid handle acts as a damper and lets the force controller converge faster.

These observations illustrate how the discrepancy between assumed and actual rigidity of the grasp on the handle and the rigidity of the handle itself with respect to the door affect the performance of our control scheme, as discussed in Section IV-D.

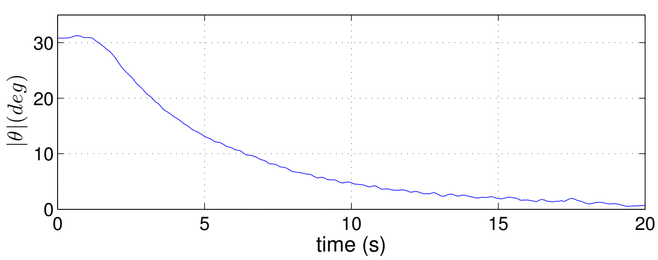


Fig. 9 : Sliding door with fixed grasp: motion axis estimation error.

For the sliding door with a fixed grasp, the controller was able to operate the mechanism and we obtained the motion axis estimation error shown in Fig. 9 and force-torque response shown in Fig. 10. This door, including the handle, was much more rigid than the others, thus the estimation of the motion axis shows convergence with a steady state error of less than 0.2 deg.

Fig. 11 and Fig. 12 show the performance of our controller for a drawer using fixed grasps. We observe that, similar to the revolute door case, the motion axis estimation error increases significantly from a 1.2 deg steady state error to a 6 deg error when applying a less rigid grasp, i.e. on the handle. This result illustrates that even when slightly relaxing the rigid grasp assumption, the robot can still control the doors, but the

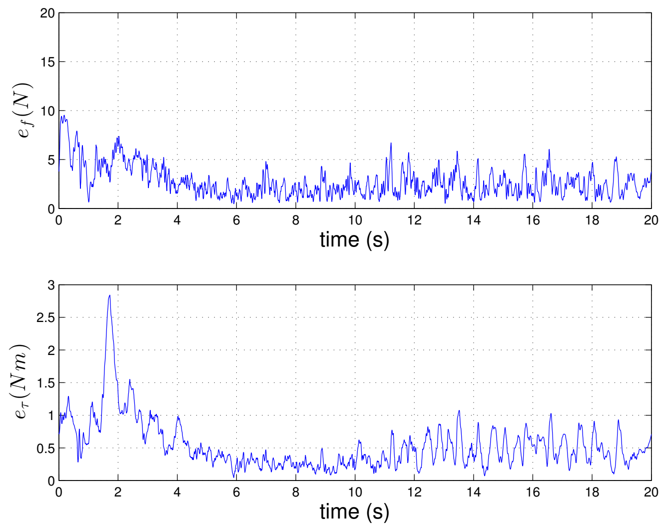


Fig. 10 : Force and torque responses for a sliding door with fixed grasp, upper figure: norm of the projected force error, lower figure: norm of the torque error.

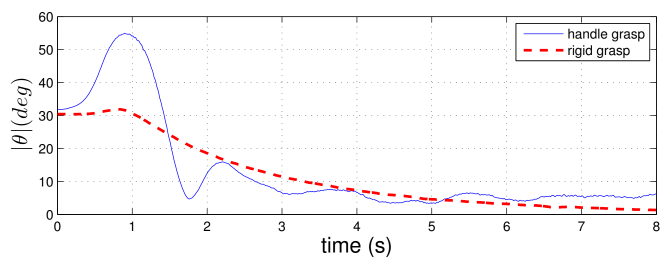


Fig. 11 : Motion axis estimation error for a drawer applying two types of grasps. The red dashed line shows the performance when grasping the drawer directly, and the solid blue line shows the estimation error while grasping the less rigid handle of the drawer.

estimate is affected in terms of transient response and steady state errors.

Fig. 13 - 16 show the performance of our controller for a revolute door and a drawer each with passive revolute handles. Both mechanisms show a similar response in the estimation of the motion axis, with steady state errors of 0.9 deg and 0.4 deg respectively for the revolute door and the drawer. With this type of passive handle the controller does not exert torques on the handle and thus we obtain convergence of the motion

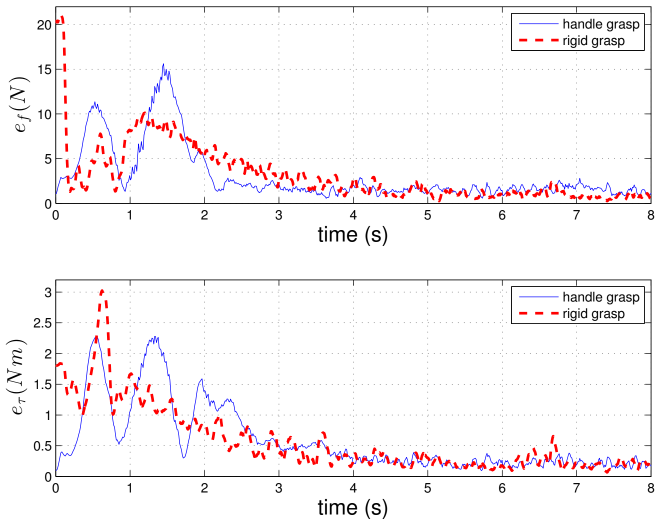


Fig. 12 : Force and torque responses for a drawer with fixed grasp, upper figure: norm of the projected force error, lower figure: norm of the torque error.

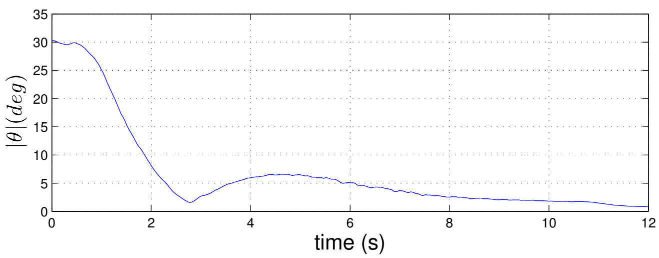


Fig. 13 : Motion axis estimation error for a revolute door with passive revolute handle.

axis estimate with low steady state errors comparable to those obtained when considering fixed grasps, as long as the rigidity assumption is fulfilled in the latter case.

In summary, we observe that our adaptive control scheme performs well as long as the doors are non-deformable and depending on how strongly the rigid grasp assumption is fulfilled in the case of fixed grasps on the doors. For typical between-room doors, the rigidity assumption is fulfilled, while the handles on some cupboard doors and drawers may have too low rigidity for the sensors and actuators on the robot used, causing small errors in the estimates.

VII. Conclusions

We propose a unified method for manipulating different types of revolute and prismatic mechanisms. The method is model-free and can be used to identify the type and geometrical characteristics of one-joint mechanisms. By coupling estimation and action the method is inherently online and can be used in real-time applications. The method consists of a generalized velocity controller using estimates of the motion direction, the axis of rotation and update laws for the estimated vectors. The design of the overall scheme guarantees compliant behavior and convergence of the estimated vectors to their actual values.

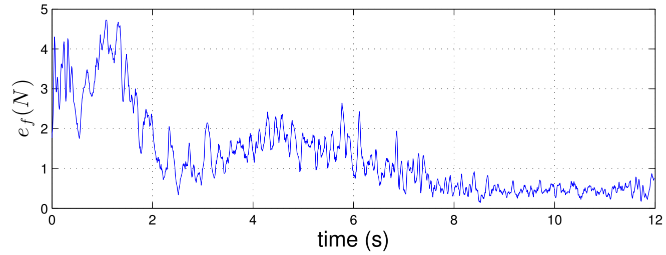


Fig. 14 : Revolute door experiment with passive revolute handle. Force response: norm of the projected force error.

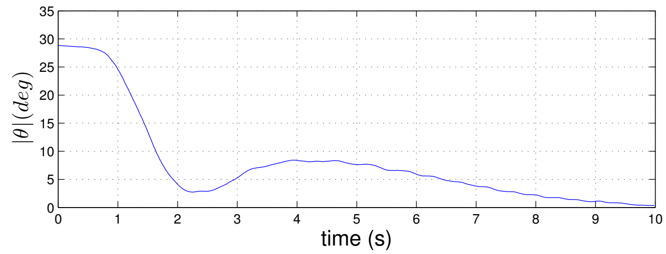


Fig. 15 : Motion axis estimation error for a drawer with passive revolute handle.

Appendix

Proof of Proposition 1: By projecting (17) along $\hat{\mathbf{x}}_h$ yields $\frac{d}{dt}\left(\frac{1}{2}\|\hat{\mathbf{x}}_h\|^2\right) = -\gamma v_{\text{ref}}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\hat{\mathbf{x}}_h\right]^\top \mathbf{v}_f + v_{\text{ref}}(\hat{\mathbf{x}}_h \times \hat{\mathbf{x}}_h)^\top \hat{\boldsymbol{\kappa}} = 0$, since $\mathbf{P}(\hat{\mathbf{x}}_h)\hat{\mathbf{x}}_h = \mathbf{0}$ and $\hat{\mathbf{x}}_h \times \hat{\mathbf{x}}_h = \mathbf{0}$. □

Proof of Proposition 2: By projecting the update law (17) along $\mathbf{x}_h$ and subsequently substituting (11), (14), $\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa} + \tilde{\boldsymbol{\kappa}}$ and $v = v_{\text{ref}}$ (7) we get:

$$-\sin\theta\dot{\theta} = \gamma v_{\text{ref}}^2 \mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h - v_{\text{ref}} \mathbf{x}_h^\top (\hat{\mathbf{x}}_h \times \tilde{\boldsymbol{\kappa}})$$

Taking into account (15), $\mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h = \sin^2\theta$ and $\mathbf{x}_h^\top(\hat{\mathbf{x}}_h \times \tilde{\boldsymbol{\kappa}}) = -\tilde{\boldsymbol{\kappa}}^\top(\hat{\mathbf{x}}_h \times \mathbf{x}_h)$ yields

$$-\sin\theta\dot{\theta} = \gamma v_d^2 \tan^2\theta + \frac{v_d}{\cos\theta}\tilde{\boldsymbol{\kappa}}^\top(\hat{\mathbf{x}}_h \times \mathbf{x}_h) \quad (33)$$

As both $\mathbf{x}_h$ and $\hat{\mathbf{x}}_h$ are unit (see Proposition 1) we can write the cross product $(\hat{\mathbf{x}}_h \times \mathbf{x}_h)$ as $\sin\theta\mathbf{n}(t)$ with $\mathbf{n}(t)$ being a unit vector perpendicular to the plane defined by $\mathbf{x}_h$ and $\hat{\mathbf{x}}_h$. Hence the non-trivial solution of (33) is given by the differential equation (20). Given $\dot{\tilde{\boldsymbol{\kappa}}} = \dot{\hat{\boldsymbol{\kappa}}}$, $\mathbf{v} = \mathbf{v}_{\text{ref}} = \mathbf{x}_h v = \mathbf{x}_h v_{\text{ref}}$ and $(\hat{\mathbf{x}}_h \times \mathbf{x}_h) = \sin\theta\mathbf{n}(t)$ we can easily transform (18) to (21).

In order to examine the stability of the nonlinear nonautonomous system we consider the following positive definite function in the domain D

$$W(\theta, \tilde{\boldsymbol{\kappa}}) = U(\theta) + \frac{1}{2}\tilde{\boldsymbol{\kappa}}^\top \boldsymbol{\Gamma}_\kappa^{-1}\tilde{\boldsymbol{\kappa}}, \; U(\theta) = 1 - \cos\theta \quad (34)$$

Differentiating $W(\theta, \tilde{\boldsymbol{\kappa}})$ with respect to time along the system trajectories (20), (21) we get:

$$\dot{W} = -\gamma v_d^2 \tan^2\theta \quad (35)$$

Since $W(\theta, \tilde{\boldsymbol{\kappa}})$ is locally positive definite and $\dot{W} \leq 0$ which implies $W(\theta, \tilde{\boldsymbol{\kappa}}) \leq W(\theta(0), \tilde{\boldsymbol{\kappa}}(0))$ we get that $\tilde{\boldsymbol{\kappa}}$ is bounded and $U(\theta) \leq W(\theta(0), \tilde{\boldsymbol{\kappa}}(0))$. The latter implies that:

$$\cos\theta \geq \cos\theta(0) - \frac{1}{2}\tilde{\boldsymbol{\kappa}}(0)^\top \boldsymbol{\Gamma}_\kappa^{-1}\tilde{\boldsymbol{\kappa}}(0) \quad (36)$$

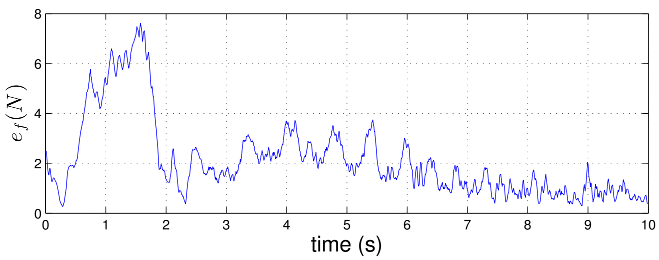


Fig. 16 : Drawer experiment with passive revolute handle. Force response: norm of the projected force error.

which practically means that starting with $\theta(0) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\theta(t) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall t$ if $\mathbf{\Gamma}_\kappa$ is chosen such that:

$$\cos\theta(0) - \frac{1}{2}\tilde{\boldsymbol{\kappa}}(0)^\top \mathbf{\Gamma}_\kappa^{-1}\tilde{\boldsymbol{\kappa}}(0) > 0 \tag{37}$$

Since $\theta(0)$ and $\tilde{\boldsymbol{\kappa}}(0)$ are unknown, appropriately large value for $\mathbf{\Gamma}_\kappa$ can guarantee that the aforementioned condition is satisfied. Consequently, (14) and (15) implies that $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f$ and v are bounded. Furthermore, the boundedness of $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f$ implies that the update law rate $\dot{\hat{\mathbf{x}}}_h$ is bounded and subsequently $\dot{\theta}$ is bounded. Thus $\ddot{W}$ is bounded and according to Barbalat's Lemma $\dot{W} \to 0$. If v_d satisfies the PE condition then $\dot{W} \to 0$ implies $\theta \to 0$. $\square$

Proof of Proposition 3: Projecting (24) along ${}^e\hat{\mathbf{x}}_h$ yields $\frac{d}{dt}\left(\frac{1}{2}\|{}^e\hat{\mathbf{x}}_h(t)\|^2\right) = -\gamma v_d \left[\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{R}_e {}^e\hat{\mathbf{x}}_h\right]^\top \mathbf{v}_f = 0$ (since $\mathbf{P}(\hat{\mathbf{x}}_h)\hat{\mathbf{x}}_h = 0$). Note that $\hat{\mathbf{x}}_h$ has also invariant magnitude since it is derived by expressing ${}^e\hat{\mathbf{x}}_h$ at the robot inertia frame by using the rotation matrix $\mathbf{R}_e$. $\square$

Proof of Proposition 4:

By projecting the update law (24) along $\mathbf{x}_h$ and subsequently substituting (12), (14), (15) we get:

$$-\sin\theta\dot{\theta} = \gamma\frac{v_d^2}{\cos\theta}\mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h \tag{38}$$

Taking into account $\mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h = \sin^2\theta$ we can readily see that the non-trivial solution of (38) is given by the differential equation (25). Differentiating the following positive definite Lyapunov function in the domain D':

$$U(\theta) = -\log(\cos\theta) \tag{39}$$

we get:

$$\dot{U} = -\gamma v_d^2 \tan^2\theta \tag{40}$$

Note that (40) implies $U(\theta) \leq U(\theta(0))$, which implies that starting with $\theta(0) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $U(\theta)$ remains bounded $\forall t$. Since function $U(\theta)$ is a logarithmic barrier function, its boundedness implies that $\theta(t) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall t$. Consequently (14), (15) and (24) implies that $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f$, v and the update law rate $\dot{\hat{\mathbf{x}}}_h$ are bounded. Eq. (40) implies also the exponential convergence of the angle error to zero (details can be found in [31]) given that v_d satisfies the PE condition. $\square$

Proof of Theorem 1: We extend the positive function $W(\theta, \tilde{\boldsymbol{\kappa}})$ (proof of Proposition 2, Eq. (34)) or the function $U(\theta)$ (proof of Proposition 4, Eq. (39)) by adding a quadratic term of $\mathcal{I}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right]$ and we consider the following Lyapunov-like function:

$$V = \alpha_f\beta_f\mathcal{I}^2\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right] + \frac{1}{\gamma}W(\theta, \tilde{\boldsymbol{\kappa}}) \tag{41}$$

By differentiating (41), completing the squares $\alpha_f^2\left\|\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right\|^2$, $\beta_f^2\left\|\mathcal{I}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right]\right\|^2$, substituting (16) and (35) (or (40) in case of U given by (39) is used in (41)) we get:

$$\dot{V} = -\alpha_f^2\left\|\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right\|^2 - \beta_f^2\left\|\mathcal{I}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right]\right\|^2$$

Hence, $V(t) \leq V(0)$, $\forall\ t$ which additionally to the boundedness results of Proposition 2 or Proposition 4 implies that $\mathcal{I}[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}]$ is bounded. The boundedness of $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f$ and $\mathcal{I}[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}]$, implies that $\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}$ is bounded. Differentiating (14), (15) and using the boundedness of $\mathcal{I}[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}]$, $\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f$ and $\dot{\hat{\mathbf{x}}}_h$, it can be easily shown that $\frac{d}{dt}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{v}_f\right]$ is bounded. Hence, the second derivative of V is bounded allowing the use of Barbalat's Lemma in order to prove that $\dot{V} \to 0$ and consequently $\mathcal{I}\left[\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}}\right]$, $\mathbf{P}(\hat{\mathbf{x}}_h)\tilde{\mathbf{f}} \to 0$. Note that the aforementioned convergence results are referred to the estimated motion space defined by $\hat{\mathbf{x}}_h$. Taking into account Proposition 2 or 4 that implies the convergence of θ to zero or $\hat{\mathbf{x}}_h \to \mathbf{x}_h$ for v_d satisfying the PE condition, we get $\mathcal{I}\left[\mathbf{P}(\mathbf{x}_h)\tilde{\mathbf{f}}\right]$, $\mathbf{P}(\mathbf{x}_h)\tilde{\mathbf{f}} \to 0$ $\square$

Proof of Theorem 2: First, we will reform $\boldsymbol{\omega}_{\text{ref}}$ by adding/subtracting the term $\boldsymbol{\kappa}(v - v_d)$, using (15) and substituting $\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa} + \tilde{\boldsymbol{\kappa}}$ as follows:

$$\boldsymbol{\omega}_{\text{ref}} = \boldsymbol{\kappa}v + \tilde{\boldsymbol{\kappa}}v_d - \boldsymbol{\omega}_\tau + v_d\left(\tfrac{\cos\theta - 1}{\cos\theta}\right)\boldsymbol{\kappa} \tag{42}$$

For design purposes we consider the following positive definite function:

$$V = \alpha_\tau\beta_\tau\|\mathcal{I}(\tilde{\boldsymbol{\tau}})\|^2 + \frac{1}{2}\tilde{\boldsymbol{\kappa}}^\top \mathbf{\Gamma}_\kappa^{-1}\tilde{\boldsymbol{\kappa}} + \frac{\xi}{\gamma}U(\theta) \tag{43}$$

with $U(\theta)$ being defined in (39) and ξ being a positive constant. By differentiating (43) with respect to time and substituting $\dot{\tilde{\boldsymbol{\kappa}}} = \dot{\hat{\boldsymbol{\kappa}}}$, $\boldsymbol{\omega} = \boldsymbol{\omega}_{\text{ref}}$ given by (42), (40) and the rotational constraints (3), (6) we get:

$$\begin{aligned}\dot{V} = &-\alpha_\tau^2\|\tilde{\boldsymbol{\tau}}\|^2 - \beta_\tau^2\|\mathcal{I}(\tilde{\boldsymbol{\tau}})\|^2 + v_d\left(\tfrac{\cos\theta-1}{\cos\theta}\right)\boldsymbol{\omega}_\tau^\top\boldsymbol{\kappa} \\ &-\xi v_d^2\tan^2\theta + \tilde{\boldsymbol{\kappa}}^\top\left(\mathbf{\Gamma}_\kappa^{-1}\dot{\hat{\boldsymbol{\kappa}}} + v_d\boldsymbol{\omega}_\tau\right)\end{aligned} \tag{44}$$

In order to cancel the last term of the right side part of (44) we set $\dot{\hat{\boldsymbol{\kappa}}} = -v_d\mathbf{\Gamma}_\kappa\boldsymbol{\omega}_\tau$ which corresponds to the update law (27). By using (27) and the inequality:

$$\boldsymbol{\omega}_\tau^\top\boldsymbol{\kappa}\left(\frac{\cos\theta - 1}{\cos\theta}\right)v_d \leq \frac{\|\boldsymbol{\omega}_\tau\|^2}{4} + \|\boldsymbol{\kappa}\|^2 v_d^2\left(\tfrac{\cos\theta-1}{\cos\theta}\right)^2 \tag{45}$$

we can upper-bound $\dot{V}$ (44) as follows:

$$\begin{aligned}\dot{V} \leq &-\alpha_\tau^2\|\tilde{\boldsymbol{\tau}}\|^2 - \beta_\tau^2\|\mathcal{I}(\tilde{\boldsymbol{\tau}})\|^2 + \frac{\|\boldsymbol{\omega}_\tau\|^2}{4} \\ &-\xi v_d^2\tan^2\theta + \|\boldsymbol{\kappa}\|^2 v_d^2\left(\tfrac{\cos\theta-1}{\cos\theta}\right)^2\end{aligned} \tag{46}$$

Expanding $\|\boldsymbol{\omega}_\tau\|^2$ – using (28)– and setting $\xi > \|\boldsymbol{\kappa}\|^2$, we get:

$$\dot{V} \leq -\frac{\alpha_\tau^2}{2}\|\tilde{\boldsymbol{\tau}}\|^2 - \frac{\beta_\tau^2}{2}\|\mathcal{I}(\tilde{\boldsymbol{\tau}})\|^2 - 2\|\boldsymbol{\kappa}\|^2 v_d^2 \left(\frac{1-\cos\theta}{\cos\theta}\right) \quad (47)$$

Since $\cos\theta \leq 1$ and $\theta(t) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ provided that $\theta(0) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (Proposition 4), the derivative of function V can be upper-bounded as follows:

$$\dot{V} \leq -\frac{\alpha_\tau^2}{2}\|\tilde{\boldsymbol{\tau}}\|^2 - \frac{\beta_\tau^2}{2}\|\mathcal{I}(\tilde{\boldsymbol{\tau}})\|^2$$

Hence, $V(t) \leq V(0)$, $\forall\ t$ which implies that $\mathcal{I}(\tilde{\boldsymbol{\tau}})$ and $\tilde{\boldsymbol{\kappa}}$ are bounded. Hence, by taking into account the constraints (3), (6) for the closed loop system $\boldsymbol{\omega} = \boldsymbol{\omega}_{\text{ref}}$ (42), it is clear that $\tilde{\boldsymbol{\tau}}$ is bounded and hence $\dot{\hat{\boldsymbol{\kappa}}}$ is bounded. Using the aforementioned boundedness results as well as those implied by Proposition 4 and Theorem 1, it can be easily proved by differentiating $\dot{V}$ that $\ddot{V}$ is bounded. Hence, applying Barbalat's Lemma we get that $\tilde{\boldsymbol{\tau}} \to 0$, $\mathcal{I}(\tilde{\boldsymbol{\tau}}) \to 0$. Using the aforementioned convergence results as well as $\theta \to 0$, it can be shown that $\hat{\boldsymbol{\kappa}} \to \boldsymbol{\kappa}$ provided that v_d satisfies the PE condition and hence $\boldsymbol{\omega}_e \to v_d\boldsymbol{\kappa}$. □

Proof of Proposition 5: By projecting the update law (24) along $\mathbf{x}_h$ and subsequently substituting (12), (29), (30) and $\mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h = \sin^2\theta$ we get:

$$-\sin\theta\dot{\theta} = \gamma\frac{v_d^2}{\cos\theta}\sin^2\theta + \gamma v_d\left(\frac{\hat{\mathbf{x}}_h}{\cos\theta} - \mathbf{x}_h\right)^\top \boldsymbol{\delta}(t)$$

Note that vector $\frac{\hat{\mathbf{x}}_h}{\cos\theta} - \mathbf{x}_h$ is perpendicular to $\mathbf{x}_h$ since $\left(\frac{\hat{\mathbf{x}}_h}{\cos\theta} - \mathbf{x}_h\right)^\top \mathbf{x}_h = 0$ while its magnitude is equal to $|\tan\theta|$. By defining a unit vector $\mathbf{n}'(t)$ parallel to $\frac{\hat{\mathbf{x}}_h}{\cos\theta} - \mathbf{x}_h$ we can easily get:

$$-\sin\theta\dot{\theta} = \gamma\frac{v_d^2}{\cos\theta}\mathbf{x}_h^\top \mathbf{P}(\hat{\mathbf{x}}_h)\mathbf{x}_h + \gamma v_d|\tan\theta|\delta(t) \quad (48)$$

The nontrivial solution of (48) is given by (31). □

Proof of Proposition 6: Differentiating (39) with respect to time and substituting (31), we get that $\dot{U}(\theta)$ is upper-bounded as follows:

$$\dot{U}(\theta) \leq -\gamma v_d^2 \frac{|\tan\theta|}{\cos\theta}\left(|\sin\theta| - \lambda(t)\right) \quad (49)$$

From (49) and Theorem 4.18 of [37] regarding uniform ultimate boundedness we can find that the region in which the estimation error converges is given by (32). □

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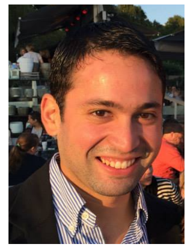

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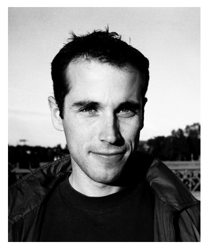

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