KTH Computer Science
and Communication

Multiscale Methods for Wave Propagation Problems

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Akademisk avhandling som med tillstånd av Kungl Tekniska högskolan framlägges till offentlig granskning för avläggande av teknologi doktor i ämnet numerisk analys fredagen den nionde december 2011 klockan 10:00 i E3, KTH Campus, Lindstedtsvägen 3, Stockholm.

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Simulations of wave propagation in heterogeneous media and at high frequencies are important in many applications such as seismic-, electro-magnetic-, acoustic-, fluid flow problems and others. These are classical multiscale problems and often too computationally expensive for direct numerical simulation. The smallest scales must be well resolved over a computational domain represented by the largest scale and this results in a very high computational cost. We develop and analyze numerical techniques based on the heterogeneous multiscale method (HMM) framework for such wave equations with highly oscillatory solutions u^ε where ε represents the size of the smallest scale. In these techniques the oscillatory microscale is approximated on small local microproblems of size ε in spatial and time directions. The solution of the microproblems are then coupled to a global macroscale model in divergence form $u_{tt} = \nabla \cdot F$ where the flux F is obtained from the microproblems. The oscillations can either originate from fluctuations in the velocity coefficients or from high frequency initial and boundary conditions. We have developed algorithms that couple micro and macroscales for both these cases. The choice of macroscale variables is inspired by the analytic theories of homogenization and geometrical optics respectively. In the first case local averages $u \approx u^\varepsilon$ are used on the macroscale. In the second case, phase ϕ and energy are natural macroscopic variables. There are two major goals of this research. One goal is to develop and analyze algorithms for simulating multiscale wave propagation with low computational complexity, and even independent of ε for finite time problems. This is seen in many examples in one, two and three dimensions. The other goal is to use wave propagation as a model to better understand the HMM framework. An example in this direction is simulation with oscillatory wave field over long time. The dispersive effects that then occur is well approximated by a HMM method that was originally formulated for finite time where added accuracy is required but no explicit adjustment to include dispersion, an evidence of the robustness of the method.

Full text: `http://www.csc.kth.se/~holst/`

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