Exploring Connections Between Proof Complexity and Practical Hardness of SAT

Jakob Nordström

KTH Royal Institute of Technology
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Université d'Artois
Lens, France
February 2, 2015

Joint work with Matti Järvisalo, Massimo Lauria, Arie Matsliah, Marc Vinyals, and Stanislav Živný

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Proof Complexity and SAT Solving

Proof complexity

- Satisfiability fundamental problem in theoretical computer science
- SAT proven NP-complete by Stephen Cook in 1971
- Hence totally intractable in worst case (probably)
- One of the million dollar "Millennium Problems"

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- Enormous progress in performance last 15–20 years
- State-of-the-art solvers can deal with real-world instances with millions of variables
- But best solvers still based on methods from early 1960s
- Tiny formulas known that are totally beyond reach

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What makes formulas hard or easy in practice for SAT solvers?

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What (if anything) can proof complexity say about this?

Outline

1. SAT solving and Proof Complexity
   - SAT solving and DPLL
   - Proof Complexity and Resolution
   - Our Results
2. Experiments
   - Benchmark Formulas
   - Set-up
   - Results
3. Directions for Future Research

The General Set-up

Conjunctive normal form (CNF)

ANDs of ORs of variables or negated variables
(or conjunctions of disjunctive clauses)

Example:

$$(x \lor z) \land (y \lor \overline{z}) \land (x \lor \overline{y} \lor u) \land (\overline{y} \lor \overline{u})$$
$$\land\ (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})$$

Proof complexity

Find short certificate that CNF formula is unsatisfiable
(i.e., always work in UNSAT regime)

Some Terminology

- Literal a: variable x or its negation $\overline{x}$
- Clause $C = a_1 \lor \cdots \lor a_k$: disjunction of literals
  (Consider as sets, so no repetitions and order irrelevant)
- CNF formula $F = C_1 \land \cdots \land C_m$: conjunction of clauses
- k-CNF formula: CNF formula with clauses of size $\leq k$
  (where k is some constant)
- All formulas assumed to be k-CNFs in this talk
  (for simplicity of exposition)

The DPLL Method

Based on [Davis & Putnam '60] and [Davis, Logemann & Loveland '62]

Somewhat simplified description:

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- Set $x = 0$ (FALSE), simplify F and try to refute recursively

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- Set $x = 1$ (TRUE), simplify F and try to refute recursively

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- If F contains an empty clause (without literals), then report "unsatisfiable"
- Otherwise pick some variable x in F
- Set $x = 0$ (FALSE), simplify F and try to refute recursively
- Set $x = 1$ (TRUE), simplify F and try to refute recursively
- If result in both cases "unsatisfiable", then report "unsatisfiable"

A DPLL Toy Example

$$
\begin{aligned}
F = \quad & (x \lor z) \land (y \lor \overline{z}) \land (x \lor \overline{y} \lor u) \land (\overline{y} \lor \overline{u}) \\
\land \; & (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})
\end{aligned}
$$

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& \land (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})
\end{aligned}
$$

Visualize execution of DPLL algorithm as search tree

Pick variables in internal nodes; terminate in leaves when falsfied clause found

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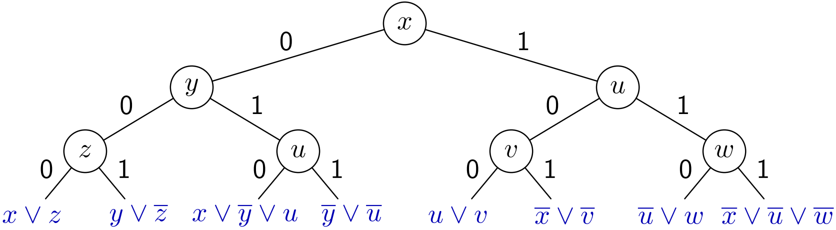

A DPLL Toy Example

$$F = (\quad z) \land (y \lor \overline{z}) \land (\quad \overline{y} \lor u) \land (\overline{y} \lor \overline{u}) \land (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})$$

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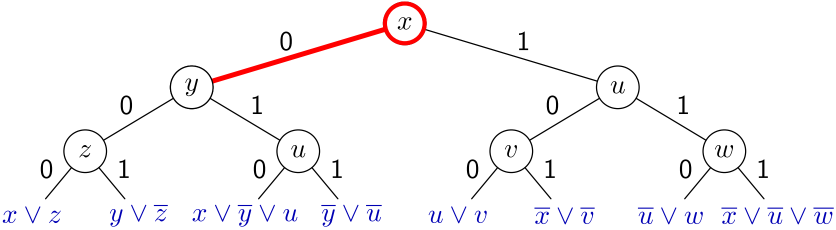

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$$F = \quad (\quad z) \land (\quad \overline{z}) \land (\quad \overline{y} \lor u) \land (\overline{y} \lor \overline{u}) \\ \land (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})$$

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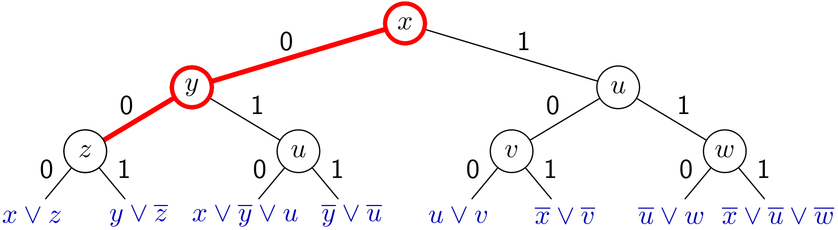

A DPLL Toy Example

$$F = \quad (\qquad) \wedge (\quad \overline{z}) \wedge (\quad \overline{y} \vee u) \wedge (\overline{y} \vee \overline{u})$$
$$\wedge\ (u \vee v) \wedge (\overline{x} \vee \overline{v}) \wedge (\overline{u} \vee w) \wedge (\overline{x} \vee \overline{u} \vee \overline{w})$$

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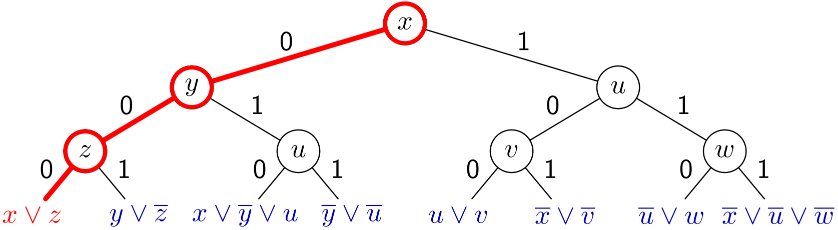

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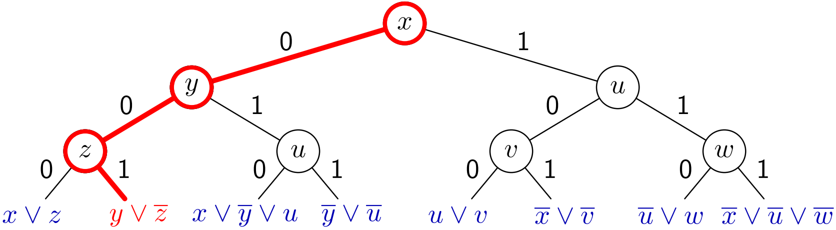

A DPLL Toy Example

$$F = (\quad\quad z) \land (y \lor \overline{z}) \land (\quad\quad\quad\quad u) \land (\quad\quad \overline{u})$$
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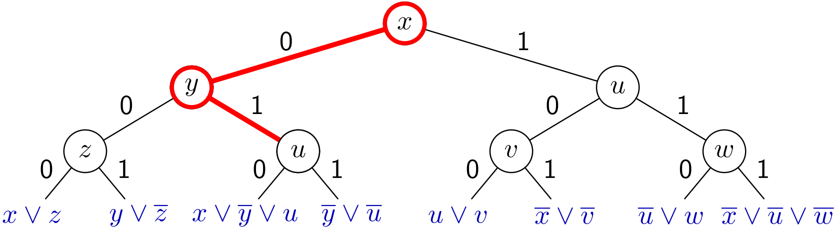

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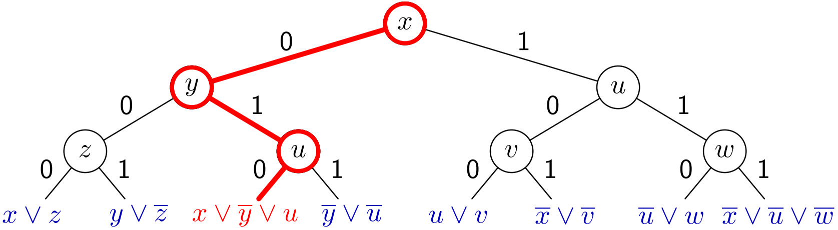

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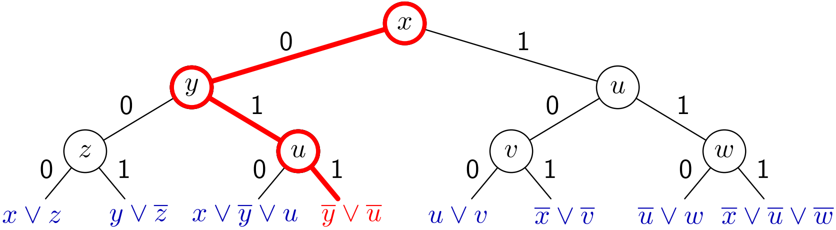

A DPLL Toy Example

$$F = (x \lor z) \land (y \lor \overline{z}) \land (x \lor \overline{y} \lor u) \land (\overline{y} \lor \overline{u}) \land (u \lor v) \land (\overline{x} \lor \overline{v}) \land (\overline{u} \lor w) \land (\overline{x} \lor \overline{u} \lor \overline{w})$$

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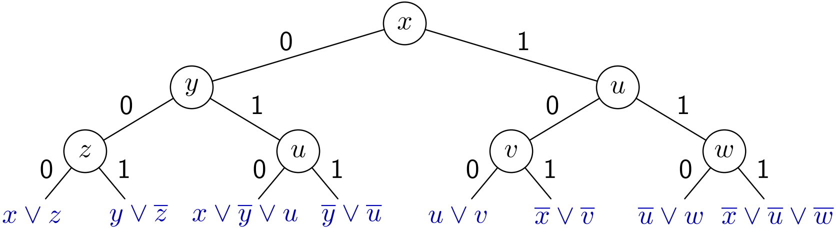

State-of-the-art DPLL SAT solvers

Many more ingredients in modern SAT solvers, for instance:

- Choice of pivot variables crucial
- In particular, always do unit propagation on sole remaining variable in a clause [which our toy example didn't]
- When reaching falsified clause, compute why partial assignment failed — add this info to formula as new clause
  Conflict-driven clause learning (CDCL)
- Can't keep everything learned — prune clause database when it gets too large (but which clauses should be removed?)
- Every once in a while, restart (but save computed info)

Proof Complexity

Proof search algorithm: defines proof system with derivation rules

Proof complexity: study of proofs in such systems

- Lower bounds: no algorithm can do better (even optimal one always guessing the right move)
- Upper bounds: gives hope for good algorithms if we can search for proofs in system efficiently

Resolution

Resolution rule:

$$\frac{B \lor x \quad C \lor \overline{x}}{B \lor C}$$

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Observation

If F is a satisfiable CNF formula and D is derived from clauses $C_1, C_2 \in F$ by the resolution rule, then $F \land D$ is satisfiable.

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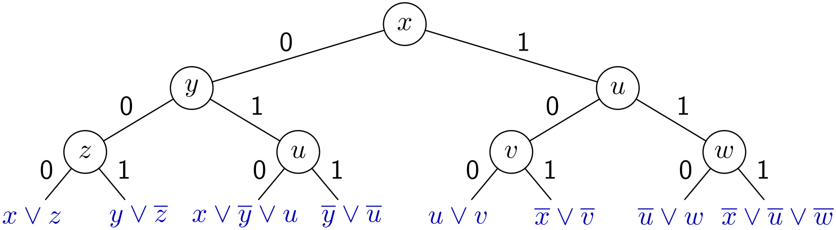
Observation

If F is a satisfiable CNF formula and D is derived from clauses $C_1, C_2 \in F$ by the resolution rule, then $F \land D$ is satisfiable.

Prove F unsatisfiable by deriving the unsatisfiable empty clause $\perp$ from F by resolution

CDCL Solvers Generate Resolution Proofs

Simple example for DPLL:



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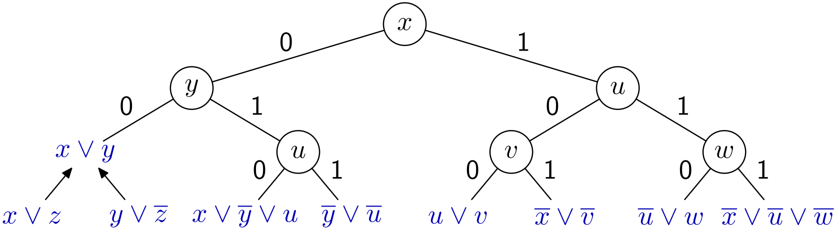

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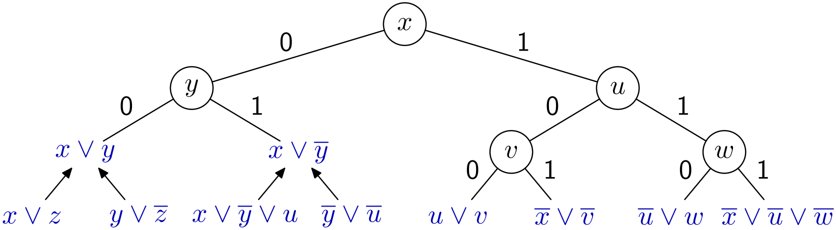

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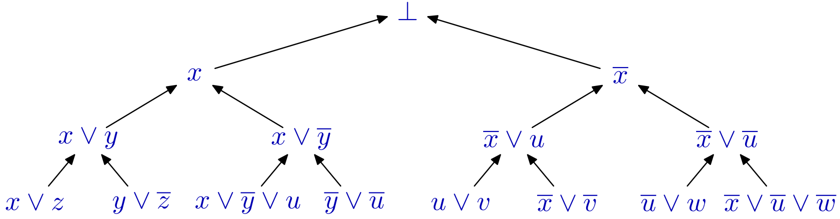

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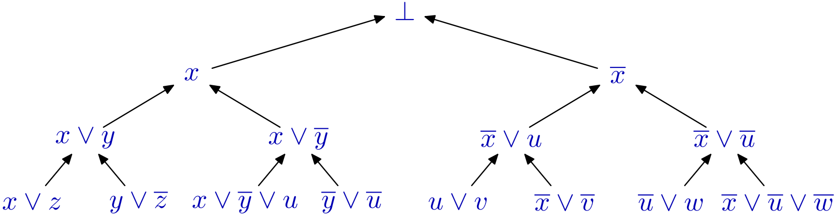


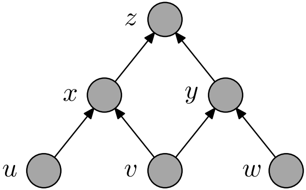
- Conflict-driven clause learning adds “shortcut edges” in tree
- But still yields resolution proof
- True also for (most) preprocessing techniques

The Theoretical Model

- Goal: Refute given CNF formula (i.e., prove it is unsatisfiable)
- Proof system operates with disjunctive clauses
- Proof/refutation is "presented on blackboard"
- Derivation steps:
  - Write down clauses of CNF formula being refuted (axiom clauses)
  - Infer new clauses by resolution rule
  - Erase clauses that are not currently needed (to save space on blackboard)
- Refutation ends when empty clause $\perp$ is derived

Example CNF Formula

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$



Defined in terms of directed acyclic graph (DAG):

- source vertices true
- truth propagates upwards
- but sink vertex is false

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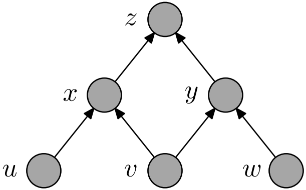

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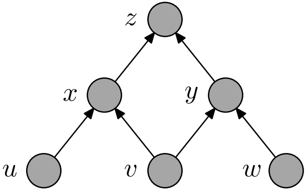

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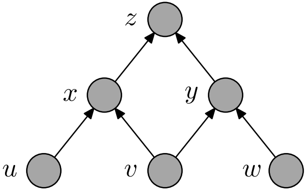

Defined in terms of directed acyclic graph (DAG):

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Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	0
largest clause seen on board	0
max # lines on board	0

Can write down axioms,
erase used clauses or
infer new clauses by resolution rule
(but only from clauses currently on the board!)

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	1
largest clause seen on board	1
max # lines on board	1

u

Write down axiom 1: u

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	2
largest clause seen on board	1
max # lines on board	2

u
v

Write down axiom 1: u
Write down axiom 2: v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	3
largest clause seen on board	3
max # lines on board	3

u

v

$\overline{u} \lor \overline{v} \lor x$

Write down axiom 1: u

Write down axiom 2: v

Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	3
largest clause seen on board	3
max # lines on board	3

u
v
$\overline{u} \lor \overline{v} \lor x$

Write down axiom 1: u
Write down axiom 2: v
Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$
Infer $\overline{v} \lor x$ from
u and $\overline{u} \lor \overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

u
v
$\overline{u} \lor \overline{v} \lor x$
$\overline{v} \lor x$

Write down axiom 1: u
Write down axiom 2: v
Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$
Infer $\overline{v} \lor x$ from
u and $\overline{u} \lor \overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

u
v
$\overline{u} \lor \overline{v} \lor x$
$\overline{v} \lor x$

Write down axiom 2: v
Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$
Infer $\overline{v} \lor x$ from
u and $\overline{u} \lor \overline{v} \lor x$
Erase the clause $\overline{u} \lor \overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

u
v
$\overline{v} \vee x$

Write down axiom 2: v
Write down axiom 4: $\overline{u} \vee \overline{v} \vee x$
Infer $\overline{v} \vee x$ from
u and $\overline{u} \vee \overline{v} \vee x$
Erase the clause $\overline{u} \vee \overline{v} \vee x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

u
v
$\overline{v} \lor x$

Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$
Infer $\overline{v} \lor x$ from
u and $\overline{u} \lor \overline{v} \lor x$
Erase the clause $\overline{u} \lor \overline{v} \lor x$
Erase the clause u

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

v

$\overline{v} \lor x$

Write down axiom 4: $\overline{u} \lor \overline{v} \lor x$

Infer $\overline{v} \lor x$ from

u and $\overline{u} \lor \overline{v} \lor x$

Erase the clause $\overline{u} \lor \overline{v} \lor x$

Erase the clause u

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	4
largest clause seen on board	3
max # lines on board	4

v

$\overline{v} \lor x$

u and $\overline{u} \lor \overline{v} \lor x$

Erase the clause $\overline{u} \lor \overline{v} \lor x$

Erase the clause u

Infer x from

v and $\overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	5
largest clause seen on board	3
max # lines on board	4

v
$\overline{v} \lor x$
x

u and $\overline{u} \lor \overline{v} \lor x$
Erase the clause $\overline{u} \lor \overline{v} \lor x$
Erase the clause u
Infer x from
v and $\overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	5
largest clause seen on board	3
max # lines on board	4

v
$\overline{v} \lor x$
x

Erase the clause $\overline{u} \lor \overline{v} \lor x$
Erase the clause u
Infer x from
v and $\overline{v} \lor x$
Erase the clause $\overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	5
largest clause seen on board	3
max # lines on board	4

v

x

Erase the clause $\overline{u} \lor \overline{v} \lor x$
Erase the clause u
Infer x from
v and $\overline{v} \lor x$
Erase the clause $\overline{v} \lor x$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	5
largest clause seen on board	3
max # lines on board	4

Board:

v

x

Erase the clause u

Infer x from

v and $\overline{v} \lor x$

Erase the clause $\overline{v} \lor x$

Erase the clause v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	5
largest clause seen on board	3
max # lines on board	4

x

Erase the clause u
Infer x from
v and $\overline{v} \lor x$
Erase the clause $\overline{v} \lor x$
Erase the clause v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	6
largest clause seen on board	3
max # lines on board	4

x

$\overline{x} \lor \overline{y} \lor z$

Infer x from
v and $\overline{v} \lor x$
Erase the clause $\overline{v} \lor x$
Erase the clause v
Write down axiom 6: $\overline{x} \lor \overline{y} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	6
largest clause seen on board	3
max # lines on board	4

x
$\overline{x} \lor \overline{y} \lor z$

Erase the clause $\overline{v} \lor x$
Erase the clause v
Write down axiom 6: $\overline{x} \lor \overline{y} \lor z$
Infer $\overline{y} \lor z$ from
x and $\overline{x} \lor \overline{y} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	7
largest clause seen on board	3
max # lines on board	4

x
$\overline{x} \lor \overline{y} \lor z$
$\overline{y} \lor z$

Erase the clause $\overline{v} \lor x$
Erase the clause v
Write down axiom 6: $\overline{x} \lor \overline{y} \lor z$
Infer $\overline{y} \lor z$ from
x and $\overline{x} \lor \overline{y} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	7
largest clause seen on board	3
max # lines on board	4

x
$\overline{x} \vee \overline{y} \vee z$
$\overline{y} \vee z$

Erase the clause v
Write down axiom 6: $\overline{x} \vee \overline{y} \vee z$
Infer $\overline{y} \vee z$ from
x and $\overline{x} \vee \overline{y} \vee z$
Erase the clause $\overline{x} \vee \overline{y} \vee z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	7
largest clause seen on board	3
max # lines on board	4

x
$\overline{y} \vee z$

Erase the clause v
Write down axiom 6: $\overline{x} \vee \overline{y} \vee z$
Infer $\overline{y} \vee z$ from
x and $\overline{x} \vee \overline{y} \vee z$
Erase the clause $\overline{x} \vee \overline{y} \vee z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	7
largest clause seen on board	3
max # lines on board	4

x

$\overline{y} \lor z$

Write down axiom 6: $\overline{x} \lor \overline{y} \lor z$
Infer $\overline{y} \lor z$ from
x and $\overline{x} \lor \overline{y} \lor z$
Erase the clause $\overline{x} \lor \overline{y} \lor z$
Erase the clause x

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	7
largest clause seen on board	3
max # lines on board	4

$\overline{y} \vee z$

Write down axiom 6: $\overline{x} \vee \overline{y} \vee z$
Infer $\overline{y} \vee z$ from
x and $\overline{x} \vee \overline{y} \vee z$
Erase the clause $\overline{x} \vee \overline{y} \vee z$
Erase the clause x

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	8
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$

$\overline{v} \lor \overline{w} \lor y$

Infer $\overline{y} \lor z$ from
x and $\overline{x} \lor \overline{y} \lor z$

Erase the clause $\overline{x} \lor \overline{y} \lor z$

Erase the clause x

Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	8
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$

$\overline{v} \lor \overline{w} \lor y$

Erase the clause $\overline{x} \lor \overline{y} \lor z$
Erase the clause x
Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	9
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$
$\overline{v} \lor \overline{w} \lor y$
$\overline{v} \lor \overline{w} \lor z$

Erase the clause $\overline{x} \lor \overline{y} \lor z$
Erase the clause x
Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	9
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$
$\overline{v} \lor \overline{w} \lor y$
$\overline{v} \lor \overline{w} \lor z$

Erase the clause x
Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{v} \lor \overline{w} \lor y$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	9
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$
$\overline{v} \lor \overline{w} \lor z$

Erase the clause x
Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{v} \lor \overline{w} \lor y$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	9
largest clause seen on board	3
max # lines on board	4

$\overline{y} \lor z$
$\overline{v} \lor \overline{w} \lor z$

Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{y} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	9
largest clause seen on board	3
max # lines on board	4

$\overline{v} \lor \overline{w} \lor z$

Write down axiom 5: $\overline{v} \lor \overline{w} \lor y$
Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{y} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	10
largest clause seen on board	3
max # lines on board	4

$\overline{v} \lor \overline{w} \lor z$

v

Infer $\overline{v} \lor \overline{w} \lor z$ from
$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{y} \lor z$
Write down axiom 2: v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	11
largest clause seen on board	3
max # lines on board	4

$\overline{v} \lor \overline{w} \lor z$

v

w

$\overline{y} \lor z$ and $\overline{v} \lor \overline{w} \lor y$

Erase the clause $\overline{v} \lor \overline{w} \lor y$

Erase the clause $\overline{y} \lor z$

Write down axiom 2: v

Write down axiom 3: w

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	12
largest clause seen on board	3
max # lines on board	4

$\overline{v} \lor \overline{w} \lor z$
v
w
$\overline{z}$

Erase the clause $\overline{v} \lor \overline{w} \lor y$
Erase the clause $\overline{y} \lor z$
Write down axiom 2: v
Write down axiom 3: w
Write down axiom 7: $\overline{z}$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	12
largest clause seen on board	3
max # lines on board	4

$\overline{v} \lor \overline{w} \lor z$

v

w

$\overline{z}$

Write down axiom 2: v

Write down axiom 3: w

Write down axiom 7: $\overline{z}$

Infer $\overline{w} \lor z$ from

v and $\overline{v} \lor \overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

$\overline{v} \lor \overline{w} \lor z$
v
w
$\overline{z}$
$\overline{w} \lor z$

Write down axiom 2: v
Write down axiom 3: w
Write down axiom 7: $\overline{z}$
Infer $\overline{w} \lor z$ from
v and $\overline{v} \lor \overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

$\overline{v} \lor \overline{w} \lor z$
v
w
$\overline{z}$
$\overline{w} \lor z$

Write down axiom 3: w
Write down axiom 7: $\overline{z}$
Infer $\overline{w} \lor z$ from
v and $\overline{v} \lor \overline{w} \lor z$
Erase the clause v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

$\overline{v} \lor \overline{w} \lor z$
w
$\overline{z}$
$\overline{w} \lor z$

Write down axiom 3: w
Write down axiom 7: $\overline{z}$
Infer $\overline{w} \lor z$ from
v and $\overline{v} \lor \overline{w} \lor z$
Erase the clause v

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

$\overline{v} \vee \overline{w} \vee z$
w
$\overline{z}$
$\overline{w} \vee z$

Write down axiom 7: $\overline{z}$
Infer $\overline{w} \vee z$ from
v and $\overline{v} \vee \overline{w} \vee z$
Erase the clause v
Erase the clause $\overline{v} \vee \overline{w} \vee z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

w

$\overline{z}$

$\overline{w} \lor z$

Write down axiom 7: $\overline{z}$

Infer $\overline{w} \lor z$ from

v and $\overline{v} \lor \overline{w} \lor z$

Erase the clause v

Erase the clause $\overline{v} \lor \overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	13
largest clause seen on board	3
max # lines on board	5

w
$\overline{z}$
$\overline{w} \lor z$

v and $\overline{v} \lor \overline{w} \lor z$
Erase the clause v
Erase the clause $\overline{v} \lor \overline{w} \lor z$
Infer z from
w and $\overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

w
$\overline{z}$
$\overline{w} \lor z$
z

v and $\overline{v} \lor \overline{w} \lor z$
Erase the clause v
Erase the clause $\overline{v} \lor \overline{w} \lor z$
Infer z from
w and $\overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

w
$\overline{z}$
$\overline{w} \vee z$
z

Erase the clause v
Erase the clause $\overline{v} \vee \overline{w} \vee z$
Infer z from
w and $\overline{w} \vee z$
Erase the clause w

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \vee \overline{v} \vee x$
5. $\overline{v} \vee \overline{w} \vee y$
6. $\overline{x} \vee \overline{y} \vee z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

$\overline{z}$

$\overline{w} \vee z$

z

Erase the clause v

Erase the clause $\overline{v} \vee \overline{w} \vee z$

Infer z from

w and $\overline{w} \vee z$

Erase the clause w

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

$\overline{z}$

$\overline{w} \lor z$

z

Erase the clause $\overline{v} \lor \overline{w} \lor z$

Infer z from

w and $\overline{w} \lor z$

Erase the clause w

Erase the clause $\overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

$\overline{z}$

z

Erase the clause $\overline{v} \lor \overline{w} \lor z$

Infer z from

w and $\overline{w} \lor z$

Erase the clause w

Erase the clause $\overline{w} \lor z$

Example Resolution Refutation

1. u
2. v
3. w
4. $\overline{u} \lor \overline{v} \lor x$
5. $\overline{v} \lor \overline{w} \lor y$
6. $\overline{x} \lor \overline{y} \lor z$
7. $\overline{z}$

Blackboard bookkeeping	
total # clauses on board	14
largest clause seen on board	3
max # lines on board	5

$\overline{z}$

z

w and $\overline{w} \lor z$
Erase the clause w
Erase the clause $\overline{w} \lor z$
Infer $\bot$ from
$\overline{z}$ and z

Example Resolution Refutation

1. u
2. v
3. w
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Blackboard bookkeeping	
total # clauses on board	15
largest clause seen on board	3
max # lines on board	5

$\overline{z}$

z

$\bot$

w and $\overline{w} \lor z$

Erase the clause w

Erase the clause $\overline{w} \lor z$

Infer $\bot$ from

$\overline{z}$ and z

Complexity Measures for Resolution

Let n = size of formula

Length/size

clauses in refutation — at most $\exp(n)$ [in our example: 15]

Width

Size of largest clause in refutation — at most n [in our example: 3]

Space

Max # clauses one needs to remember when "verifying correctness of refutation on blackboard" — at most n (!) [in our example: 5]

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- Clearly lower bound on running time for any CDCL algorithm (except for preprocessing techniques going beyond resolution)

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  (but slighly idealized theoretical model)
- Better practical hardness measure?

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This work can be viewed as implementing program outlined in [ABLM08]

Result 1: Separation of Space and Tree-like Space

We don't believe in tree-like space as hardness measure

- Tree-like space tightly connected with tree-like length
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We prove first asymptotic separation of space and tree-like space

Theorem

There are formulas requiring space $\mathcal{O}(1)$ for which tree-like space grows like $\Omega(\log n)$

Only constant-factor separation known before [Esteban & Torán '03]

Result 2: Small Backdoor Sets Imply Small Space

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We show connections between (strong) backdoors and space complexity (elaborating on [ABLM08])

Theorem (Informal)

If a formula has a small backdoor set (for some common flavours of backdoors), then it requires small space

Result 3: Correlation Between Hardness and Space?

Recall

$$\log \text{length} \leq \text{width} \leq \text{space} \leq \text{tree-like space}$$

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(*) But such formulas are nontrivial to find

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In particular, well-studied (and well-understood) for pebble games modelling calculations described by DAGs ([Cook & Sethi '76] and others)

- Time needed for calculation: # pebbling moves
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- Time needed for calculation: # pebbling moves
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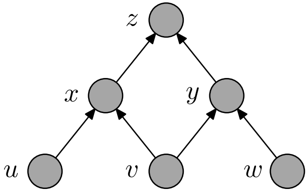
Some quick graph terminology

- DAGs consist of vertices with directed edges between them
- vertices with no incoming edges: sources
- vertices with no outgoing edges: sinks

The Black-White Pebble Game

Goal: get single black pebble on sink vertex z of G



# moves	0
Current # pebbles	0
Max # pebbles so far	0

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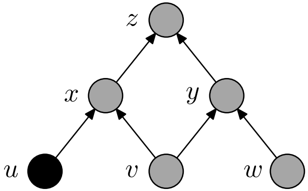

# moves	1
Current # pebbles	1
Max # pebbles so far	1

1. Can place black pebble on (empty) vertex v if all predecessors (vertices with edges to v) have pebbles on them

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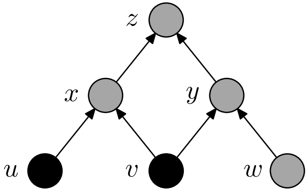

# moves	2
Current # pebbles	2
Max # pebbles so far	2

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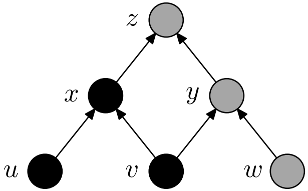

# moves	3
Current # pebbles	3
Max # pebbles so far	3

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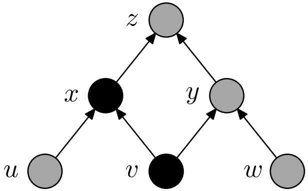

# moves	4
Current # pebbles	2
Max # pebbles so far	3

1. Can place black pebble on (empty) vertex v if all predecessors (vertices with edges to v) have pebbles on them
2. Can always remove black pebble from vertex

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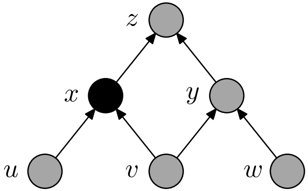

# moves	5
Current # pebbles	1
Max # pebbles so far	3

1. Can place black pebble on (empty) vertex v if all predecessors (vertices with edges to v) have pebbles on them
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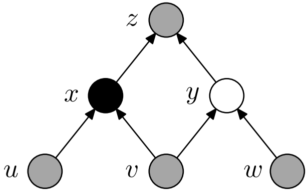

# moves	6
Current # pebbles	2
Max # pebbles so far	3

1. Can place black pebble on (empty) vertex v if all predecessors (vertices with edges to v) have pebbles on them
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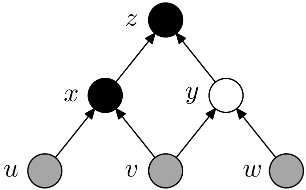
Goal: get single black pebble on sink vertex z of G



# moves	7
Current # pebbles	3
Max # pebbles so far	3

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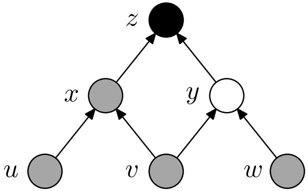

# moves	8
Current # pebbles	2
Max # pebbles so far	3

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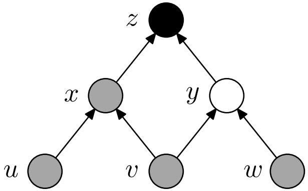

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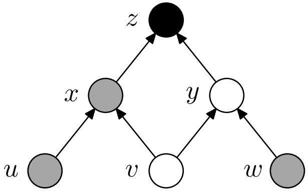

# moves	9
Current # pebbles	3
Max # pebbles so far	3

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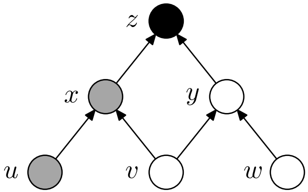

# moves	10
Current # pebbles	4
Max # pebbles so far	4

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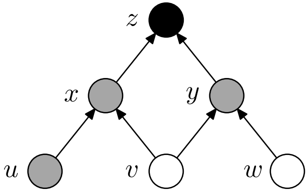

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Current # pebbles	3
Max # pebbles so far	4

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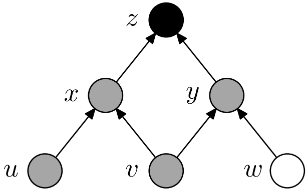

# moves	12
Current # pebbles	2
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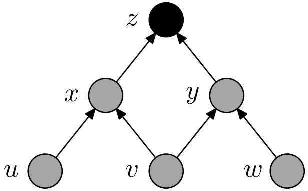

# moves	13
Current # pebbles	1
Max # pebbles so far	4

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Use Pebbling Formulas...

CNF formulas encoding so-called pebble games on DAGs

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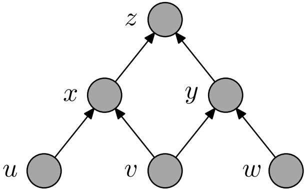

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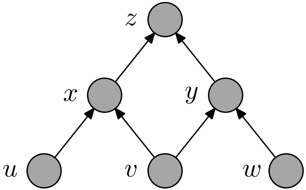

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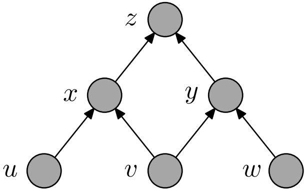

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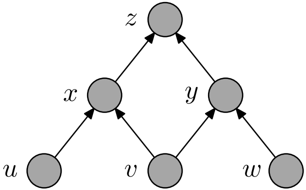

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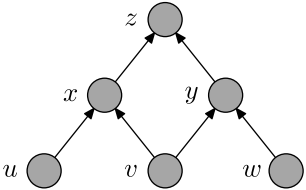

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Extensive literature on pebbling time-space trade-offs from 1970s and 80s

Pebbling formulas studied by [Bonet et al. '98, Raz & McKenzie '99, Ben-Sasson & Wigderson '99] and others

Hope that pebbling properties of DAG somehow carry over to resolution refutations of pebbling formulas.

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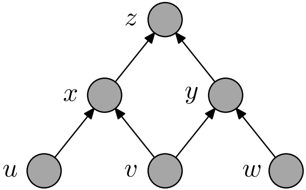

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- truth propagates upwards
- but sink is false

Extensive literature on pebbling time-space trade-offs from 1970s and 80s

Pebbling formulas studied by [Bonet et al. '98, Raz & McKenzie '99, Ben-Sasson & Wigderson '99] and others

Hope that pebbling properties of DAG somehow carry over to resolution refutations of pebbling formulas. **Except. . .**

. . . with Functions Substituted for Variables. . .

Won't work — pebbling formulas solved by unit propagation, so supereasy

Make formula harder by substitution of Boolean functions for variables

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Example 1: Exclusive or

$$
\begin{aligned}
x &\leftarrow (x_1 \oplus x_2) &= (x_1 \lor x_2) \land (\overline{x}_1 \lor \overline{x}_2) \\
\overline{x} &\leftarrow \neg(x_1 \oplus x_2) &= (x_1 \lor \overline{x}_2) \land (\overline{x}_1 \lor x_2)
\end{aligned}
$$

Example 2: Not-all-equal

$$
\begin{aligned}
x &\leftarrow NAE(x_1, x_2, x_3) &= (x_1 \lor x_2 \lor x_3) \land \\
&& (\overline{x}_1 \lor \overline{x}_2 \lor \overline{x}_3) \\
\overline{x} &\leftarrow \neg NAE(x_1, x_2, x_3) &= (x_1 \lor \overline{x}_2) \land (\overline{x}_1 \lor x_2) \land \\
&& (x_1 \lor \overline{x}_3) \land (\overline{x}_1 \lor x_3) \land \\
&& (x_2 \lor \overline{x}_3) \land (\overline{x}_2 \lor x_3)
\end{aligned}
$$

...and Expand to Get New CNF Formula

Example with substituting every variable x with $x_1 \oplus x_2$:

$$\overline{y} \vee z$$

$$\Downarrow$$

$$\neg(y_1 \oplus y_2) \vee (z_1 \oplus z_2)$$

$$\Downarrow$$

$$\big((y_1 \vee \overline{y}_2) \ \wedge \ (\overline{y}_1 \vee y_2)\big) \ \vee \ \big((z_1 \vee z_2) \ \wedge \ (\overline{z}_1 \vee \overline{z}_2)\big)$$

$$\Downarrow$$

$$\begin{aligned} & (y_1 \vee \overline{y}_2 \vee z_1 \vee z_2) \\ \wedge\ & (y_1 \vee \overline{y}_2 \vee \overline{z}_1 \vee \overline{z}_2) \\ \wedge\ & (\overline{y}_1 \vee y_2 \vee z_1 \vee z_2) \\ \wedge\ & (\overline{y}_1 \vee y_2 \vee \overline{z}_1 \vee \overline{z}_2) \end{aligned}$$

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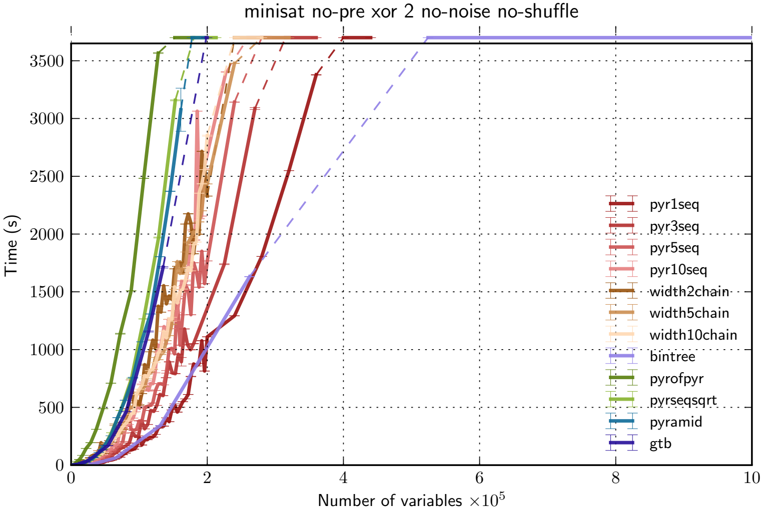
Now CNF formula inherits pebbling graph properties!

(Also works for other functions with “right” properties, like NAE)

About the Experiments

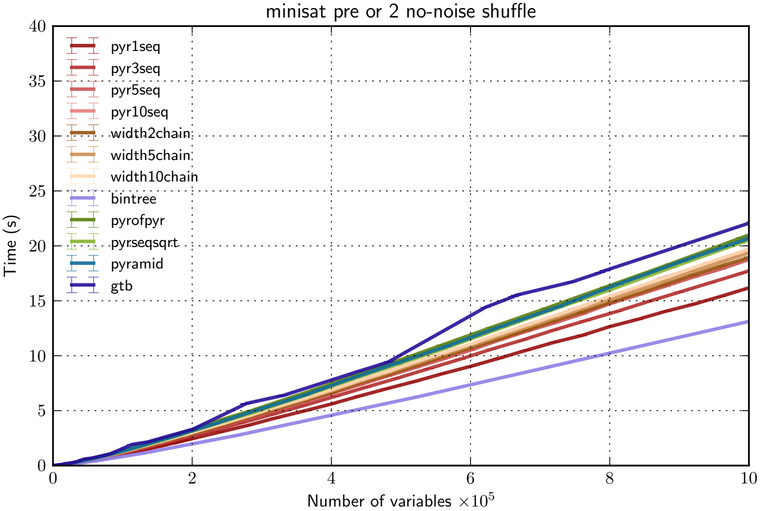
- 12 graph families with varying space complexity
- 12 different functions used to obtain CNF formulas from graphs
- Total of 144 formula families with around 50 instances per family
- CDCL solvers Minisat 2.2, Glucose 2.2, and Lingeling ala
- Experiments
  - with and without preprocessing
  - with and without random shuffling of formulas
- AMD Opteron 2.2 GHz CPU (2374 HE) with 16 GB of memory
- Time-out 1 hour per instance
- Massive amounts of data...

Example Results for MiniSat Without Preprocessing



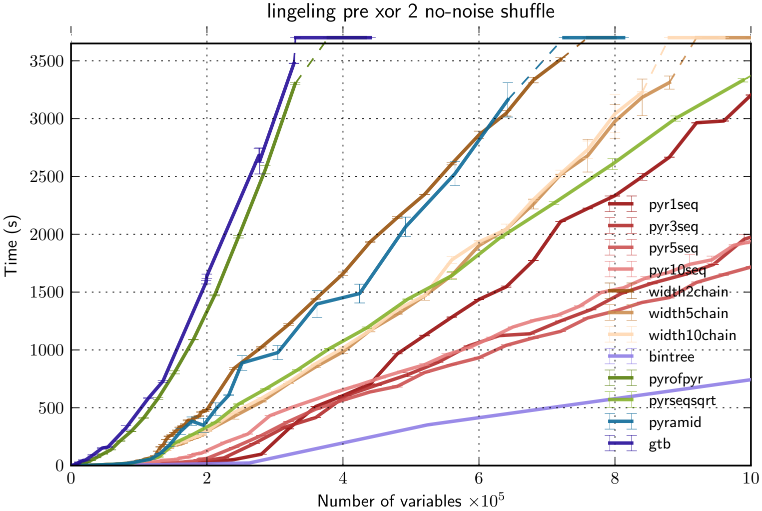
Looks nice... “Easy” formulas solved faster than “hard” ones

Example Results for MiniSat with Preprocessing



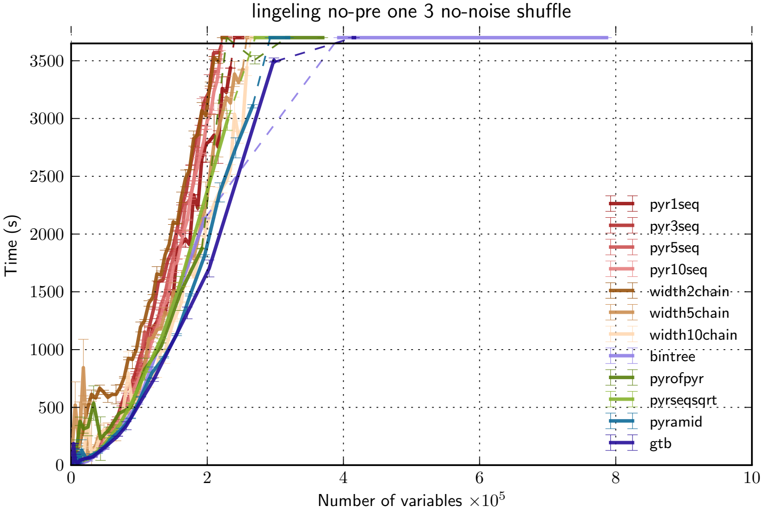
Preprocessing makes formulas much easier; order still mostly right

Example Results for Lingeling with Preprocessing



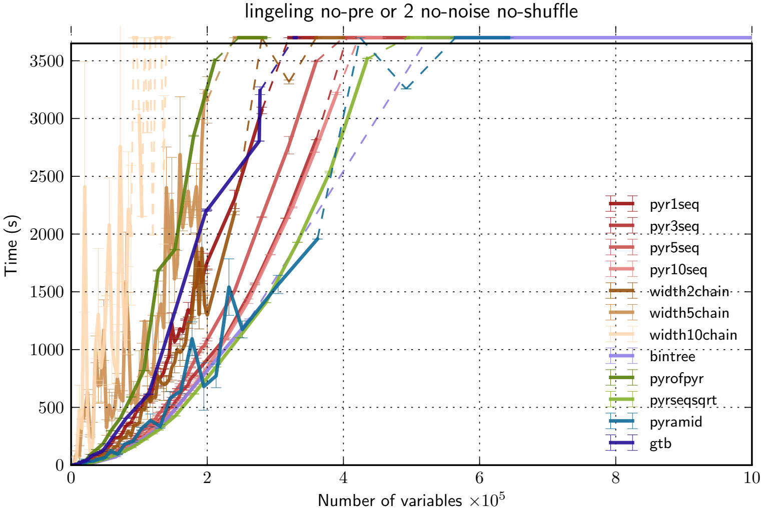
And sometimes clear differences even after preprocessing

Less Nice Example for Lingeling Without Preprocessing



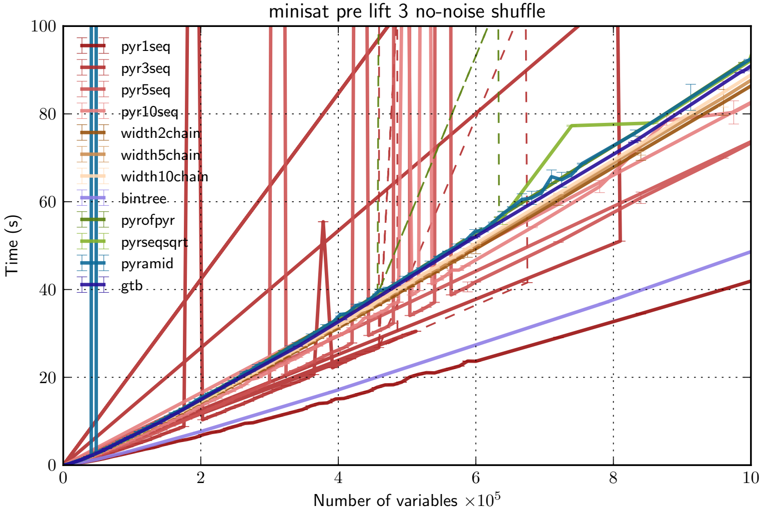
Hardness of formulas is in opposite order of that expected...

Second Example for Lingeling Without Preprocessing



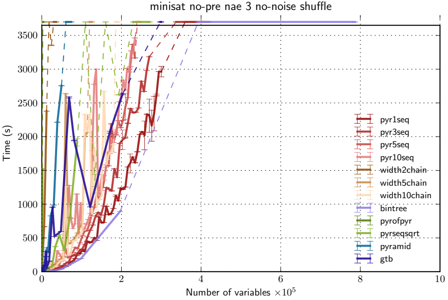
"Easy" formulas are too hard and running time oscillates?!

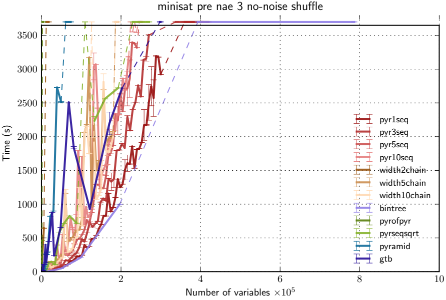
Crazy Example with MiniSat



What is going on with formulas generated from pyramids?!

For Some Functions Preprocessing Really Doesn't Help...

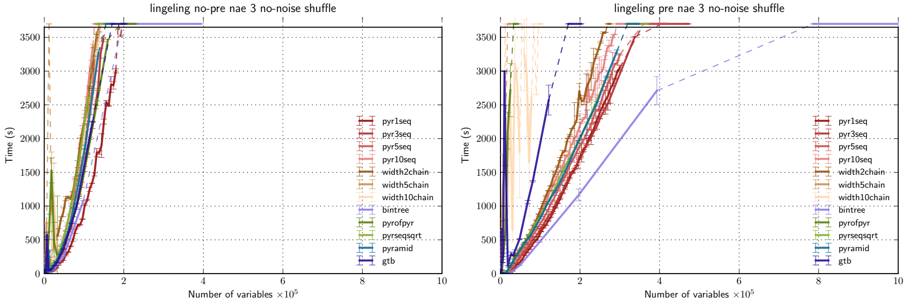




For the NAE_3 substitution function the MiniSat preprocessor doesn't seem to help at all [left: without preprocessing; right: with preprocessing]

For other functions, though, preprocessing can decrease running times by orders of magnitude, as we just saw

. . . Or Sometimes Even Hurts



Same experiments for NAE_3 substitution function but run on Lingeling [left: without preprocessing; right: with preprocessing]

Note that all of these formulas have very short resolution proofs

But some of them seem totally infeasible for Lingeling!

Discussion (1/3): Theory vs. Practice

Dependence on substitution functions

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The problem of easy benchmarks

- All formulas easy by design — very short proofs in small width
- By design: Want to isolate space complexity as the relevant parameter
- But means SAT solvers can "get lucky"

Discussion (2/3): Behaviour of Different SAT Solvers

MiniSAT and Glucose

- Similar behaviour
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Effects of preprocessing

- Always improves running time, but much more significantly for MiniSAT/Glucose (and dampens correlation with space complexity)
- Not surprising — formulas amenable to preprocessing by construction
- Also, space measure doesn't capture what happens during preprocessing

Discussion (3/3): Criticism of Benchmarks

Artificial benchmarks

- True, but the only formulas where we know how to control space
- In general, computing space complexity probably PSPACE-complete
- And computing width complexity EXPTIME-complete [Berkholz '12]

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. . . Or finding a better measure than both width and space. . .

- Maybe some other property of formulas captures hardness better?
- Is there even a clean mathematical measure that can get close to capturing messy real-world hardness?

Some Open Problems

- Is width complexity a better measure of hardness in practice?
- Or is there some other mathematical measure that can explain practical CDCL hardness?
- Do theoretical time-space trade-offs turn up in practice for CDCL solvers?
- Can we build better SAT solvers based on algebra or geometry?

Summing up

- Modern CDCL SAT solvers amazingly successful in practice
- But poorly understood which formulas are easy or hard
- We study space as candidate measure of hardness in practice
- We see no conclusive evidence, but present some intriguing results...
- We believe that connections between proof complexity and SAT solving would be worth further exploration

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Thank you for your attention!