Numerical studies of flow over a circular cylinder at $Re_D=3900$

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Flow over a circular cylinder at Reynolds number 3900 is studied numerically using the technique of large eddy simulation. The computations are carried out with a high-order accurate numerical method based on $B$-splines and compared with previous upwind-biased and central finite-difference simulations and with the existing experimental data. In the very near wake, all three simulations are in agreement with each other. Farther downstream, the results of the $B$-spline computations are in better agreement with the hot-wire experiment of Ong and Wallace [Exp. Fluids 20, 441–453 (1996)] than those obtained in the finite-difference simulations. In particular, the power spectra of velocity fluctuations are in excellent agreement with the experimental data. The impact of numerical resolution on the shear layer transition is investigated. © 2000 American Institute of Physics.

I. INTRODUCTION

Flow over a cylinder has been studied extensively, both experimentally and numerically, for several decades, as reflected in the reviews by Morkovin,$^1$ Berger and Wille,$^2$ Norberg,$^3$ and Beaudan and Moin.$^4$ One of the most recent and comprehensive reviews is given by Williamson.$^5$ The simplicity of the geometry and the abundance of interesting flow features continue to make this flow subject of many current studies.$^5$ Recent increase in computer power has prompted detailed numerical studies of this flow at higher Reynolds numbers.

The flow over a cylinder exhibits vastly different behaviors as the Reynolds number, based on the free-stream velocity, cylinder diameter, and kinematic viscosity, increases from zero to large values. Steady laminar flow exists at Reynolds numbers between 190 and 260. Recently, Barkley and Henderson$^6$ performed three-dimensional Floquet stability analysis of the cylinder wake and suggested the critical value of the Reynolds number, $Re_D=188.5\pm 1.0$, for the onset of this regime, which is characterized by dominant spanwise scales having wavelength of approximately four cylinder diameters. At a Reynolds number of approximately 260, the flow experiences transition to finer scale three-dimensionalities, known as mode B instability, with spanwise characteristic length of around one cylinder diameter. With increasing Reynolds number, the three-dimensional cylinder wake becomes more chaotic, which is believed to lead to the reduction in the base suction coefficient$^5$ defined as the negative of the base pressure coefficient. The shear layers separating from the cylinder become unstable at Reynolds number around 1200, according to Prasad and Williamson.$^7,8$ However, the value of this critical Reynolds number varies in the literature from 300 to 3000.$^7$ It is believed that the separating shear layers are very sensitive to various experimental factors such as free-stream turbulence level,$^9$ acoustic noise,$^{10}$ cylinder vibrations,$^{11}$ end boundary conditions,$^7,8$ and aspect ratio.$^{12}$

A dramatic decrease in the base suction coefficient occurs between Reynolds numbers $2\times 10^5$ and $3.5\times 10^4$; this is associated with a complicated flow regime with laminar separation, transition to turbulence, reattachment, and another separation further downstream along the cylinder surface. At postcritical Reynolds numbers beyond $3.5\times 10^6$, the boundary layer on the cylinder becomes turbulent before separation. This regime is characterized by a lower base suction coefficient and lower drag due to later separation of the turbulent boundary layer.

The presence of various regimes in the flow over a cylinder makes computations of this flow an interesting and challenging task. Two-dimensional calculations have long been routine and are often used to validate numerical techniques and computer codes. However, it has been recognized that at Reynolds numbers higher than $Re_D=250$, two-dimensional calculations yield incorrect values of flow parameters such as drag and lift coefficients. Three-dimensional effects are important at these Reynolds numbers, and three-dimensional calculations are needed for accurate predictions of flow statistics. Over the last seven years, several three-dimensional studies have been carried out, mostly at low Reynolds numbers.$^{13-16}$ Direct numerical simulations have provided valuable information on the physics of the flow over a cylinder in the transitional regime, but they are still limited to flows at low Reynolds numbers. Computations at higher Reynolds numbers have been performed with Reynolds-averaged turbulence models, but these have experienced difficulties in predicting the mean forces...
on the cylinder and the near-wake mean flow statistics (see Ref. 4 for relevant references).

Beaudan and Moin\textsuperscript{4} were the first to attempt a comprehensive large eddy simulation study of a flow over a circular cylinder. The flow at a subcritical Reynolds number $Re_D=3900$ (based on cylinder diameter and free-stream velocity) was chosen mainly because of the existence of the particle image velocimetry (PIV) experimental data of Lourenco and Shih.\textsuperscript{17} Motivated by the direct simulation results of Rai and Moin,\textsuperscript{18} Beaudan and Moin\textsuperscript{4} used high-order upwind-biased schemes for the numerical simulations of the compressible Navier–Stokes equations. The simulations were performed on O-type grids with different resolutions to establish mesh independence of the solution in the vicinity of the cylinder. The profiles of mean velocity and Reynolds stresses obtained in these simulations were in reasonable agreement with the experimental data. However, inside the recirculation region, the streamwise velocity profiles differed in shape from those observed in the experiment.\textsuperscript{17} These differences were attributed to the experimental errors as manifested in the large asymmetry of the experimental data.\textsuperscript{4}

A new experiment at the same Reynolds number was carried out by Ong and Wallace\textsuperscript{19} and provided the mean flow data at several locations in the near wake of the cylinder downstream of the recirculation region. Even though fair agreement between the simulations of Beaudan and Moin\textsuperscript{4} and the experiment was observed in the mean velocity profiles, turbulence intensities at several downstream locations did not match the experimental data. It was concluded that the high-order accurate, upwind-biased numerical schemes exhibited significant numerical dissipation which affected small turbulence scales in the wake. This was especially pronounced in the wake region beyond seven diameters from the cylinder where the simulated power spectra of velocity fluctuations exhibited severe damping at high wave numbers, and the Reynolds stresses were noticeably underpredicted when compared to the experimental data. Simulations with the dynamic subgrid-scale (SGS) model were in better agreement with the experiments than those with the Smagorinsky model and without a SGS model. However, the effect of the model did not appear to be very significant.

Mittal and Moin\textsuperscript{20} used a second-order conservative central-difference scheme for large eddy simulations of the same flow. A Fourier-spectral method was employed in the spanwise direction in conjunction with periodic boundary conditions. Simulations were performed on a C-type curvilinear grid. The results of that simulation were in reasonable agreement with the upwind-difference computations\textsuperscript{4} and the experiments. Examination of the power spectra of velocity fluctuations showed that the small turbulent scales were more energetic than those in the upwind-biased simulations, and were in agreement with the experiment over a wider range of wave numbers. It was concluded that nondissipative methods are more suitable for large eddy simulations (LES). However, there were still noticeable discrepancies between numerical and experimental results in the profiles of the Reynolds stresses at several downstream locations, probably due to the low-order scheme employed. These new simulations did not shed any light on the source of the differences between Beaudan and Moin’s LES and Lourenco and Shih’s experiment\textsuperscript{17} in the region close to the cylinder. In this region, both simulations agreed with each other and, in particular, showed a shape of the streamwise velocity profile inside the recirculation region different from that observed in the experiment of Lourenco and Shih.\textsuperscript{17}

Numerical and modeling aspects of large eddy simulations of the flow past a circular cylinder at $Re_D=3900$ were studied by Breuer,\textsuperscript{21} who performed computations with five different numerical schemes and with the dynamic and Smagorinsky subgrid-scale models. That work confirmed the findings of the previous numerical studies described above

\begin{table}[h]
\centering
\caption{Flow parameters from cylinder flow computations at $Re_D=3900$. $-C_p$ is from Ref. 40 at $Re_D=4020$, $St$ from Ref. 19 at $Re_D=3900$; $L_{re}/D$ from Ref. 10 at $Re_D=3900$; $\theta_{sep}$ from Ref. 41 at $Re_D=5000$, $U_{\text{mean}}$ from Ref. 17 at $Re_D=3900$; upwind is simulation of Ref. 4; central is simulation of Ref. 20.}
\begin{tabular}{cccccccc}
\hline
Data from & $C_D$ & $-C_p$ & $St$ & $\theta_{sep}$ & $L_{re}/D$ & $U_{\text{mean}}$
\hline
Expt & 0.99±0.05 & 0.88±0.05 & 0.215±0.005 & 86.0°±2 & 1.4±0.1 & −0.24±0.1
Upwind & 1.00 & 0.95 & 0.203 & 85.8° & 1.36 & −0.32
Central & 1.00 & 0.93 & 0.207 & 86.9° & 1.40 & −0.35
Present & 1.04 & 0.94 & 0.210 & 88.0° & 1.35 & −0.37
\hline
\end{tabular}
\end{table}
and showed that simulations with central difference schemes were in better agreement with the experimental data than those performed with dissipative methods. Low-order upwind schemes were not able to predict correctly the base suction coefficient, separation angles, and the size and structure of the recirculation region behind the cylinder. The best agreement with the experimental data was achieved with a central difference scheme in conjunction with the dynamic subgrid-scale model which is in agreement with the studies of Beaudan and Moin\(^4\) and Mittal and Moin.\(^20\) Again, the influence of the subgrid-scale model was not found to be significant. Most of the simulations by Breuer\(^21\) were performed with the same spanwise domain size as those done by Beaudan and Moin\(^4\) and Mittal and Moin.\(^20\) It was reported that doubling the domain in the spanwise direction while preserving the spanwise resolution did not affect the results significantly.

The numerical issues raised in the previous large eddy simulations prompted us to attempt simulations of the flow over a circular cylinder using a newly developed, accurate numerical method based on B-splines.\(^22\) In particular, one of our objectives was to clarify some of the discrepancies that exist between numerical simulations and the experiment of Lourenco and Shih\(^17\) in the recirculation region.

II. GOVERNING EQUATIONS AND NUMERICAL METHOD

The governing equations in this study are incompressible Navier–Stokes and continuity equations,

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} + H &= -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u}, \\
\nabla \cdot \mathbf{u} &= 0,
\end{align*}
\]

where \(Re\) is the Reynolds number, \(\mathbf{u}=(u,v,w)\) is the velocity vector, \(p\) is the pressure, and \(H=(H_u,H_v,H_w)\) is the convective term.

We chose a divergence-free implementation of the Galerkin B-spline method\(^22\) for a numerical solution of Eq. (1). In this approach, the velocity vector, \(\mathbf{u}\), is represented in terms of two distinct classes of divergence-free vectors\(^23\) which is equivalent to independently representing two components of the velocity with the third determined by the continuity equation,

\[
\mathbf{u}(x,y,z,t) = \sum_{k_z} \sum_m \left[ \hat{a}_m^+(t,k_z) \mathbf{q}^+_m(x,y,z) + \hat{a}_m^-(t,k_z) \mathbf{q}^-_m(x,y,z) \right],
\]

where a caret indicates a Fourier transform, \(k_z\) is the spanwise wave number, \(\mathbf{q}_{m,k_z}(x,y,z)\) are expansion vectors, and \(\hat{a}_m^+(t,k_z)\) are expansion coefficients. A weak form of the incompressible Navier–Stokes equations is

\[
\int_\Omega \psi \cdot \frac{\partial \mathbf{u}}{\partial t} d\Omega = -\int_\Omega \psi \cdot (H + \nabla p) d\Omega - \frac{1}{Re} \int_\Omega (\nabla \times \psi) \cdot (\nabla \times \mathbf{u}) d\Omega,
\]

where \(Re\) is the Reynolds number, \(p\) is the pressure, \(H\) is the convective term, \(\psi\) is the vector of weight functions, and \(\Omega\) is the computational domain. The last term in Eq. (3) was obtained using integration by parts and the identity \(\nabla^2 \mathbf{u} = -\nabla \times \nabla \times \mathbf{u}\) which is valid for \(\mathbf{u}\) satisfying \(\nabla \cdot \mathbf{u} = 0\).

Evaluating the integrals in (3), we obtain a system of ordinary differential equations for each wave number \(k_z\),

FIG. 2. (Color) Instantaneous streamwise velocity in the \((x-z, y=0)\) plane in the wake of a circular cylinder at \(Re_d=3900\). There are 52 contours from \(-1.5\) to \(1.5\). The thick solid line shows the contour with \(u=0\).

FIG. 3. (Color) Instantaneous cross-flow velocity in the \((x-z, y=0)\) plane in the wake of a circular cylinder at \(Re_d=3900\). There are 52 contours from \(-1.5\) to \(1.5\). The thick solid line shows the contour with \(v=0\).
solved for the expansion coefficients \( \alpha \) operators have to be stored. These are the expressions for the nonlinear terms. Due to decoupling in the \( z \) direction only matrices for two-dimensional operators have to be stored.

The system of ordinary differential equations (4) can be solved for the expansion coefficients \( \alpha^+ \) and \( \alpha^- \) with a standard time-stepping algorithm. In our study, the time advancement is performed with Crank–Nicolson and third-order Runge–Kutta schemes for the viscous and advection terms, respectively. The mass and viscous matrices are symmetric and positive definite and, therefore, the conjugate gradient method can be used for the iterative solution of the resulting linear system of equations at each time step. The method is capable of incorporating zonal embedded grids providing finer mesh resolution in physically important flow regions. The details of the numerical technique are given in Ref. 22.

The accuracy of a numerical technique based on piecewise polynomials, such as B splines, has been studied in the past. For example, it has been shown that, for a method based on B splines of a particular order \( k \), the convergence rate of \( L2 \) errors of numerical solutions is equal to \( k + 1 \). Convergence studies for two-dimensional problems solved with B-spline based methods are given by Shariff and Moser. It is also worth mentioning that the good properties of B-spline based methods are the behavior of their modified wave number curves and the absence of aliasing errors in nonlinear problems.

III. NUMERICAL SIMULATIONS

The length scales of the cylinder boundary layer, separating shear layers and streamwise vortical structures were considered in designing grids for our calculations (see Fig. 1). The thickness of the boundary layer on the cylinder was taken from Beaudan and Moin, who estimated it from the thickness of the vorticity layer. The number of grid points used in our study to resolve the velocity gradients in the cylinder boundary layer at \( \theta = 170^\circ \) (close to the front stagnation point) and \( \theta = 90^\circ \) (close to the separation point) is given in Table I. The required spanwise domain size and the spanwise resolution are estimated from the prior knowledge of the sizes of the streamwise vortex structures. As reported in the experimental studies by Mansy et al. and Williamson et al., the wavelength of the streamwise structures in the near wake of a circular cylinder scale as

\[
\lambda_z/D \sim 25 \text{Re}_D^{0.5}.
\]

In the study of Mansy et al., the coefficient in Eq. (5) was 20. At Reynolds number \( \text{Re}_D = 3900 \), the estimated wavelength is \( \lambda_z/D \sim 0.4 \). Farther downstream, Williamson et al. and Chyu and Rockwell have reported the existence of larger scale structures with wavelengths,

\[
\lambda_z/D \sim 1.
\]

We use 48 grid points over the spanwise length of \( \pi D \) to...
account for the structures observed in the experiments. This is the same spanwise resolution as in the simulations of Beaudan and Moin and Mittal and Moin. In the plane normal to the cylinder axis, Beaudan and Moin used an O-type grid with 144 × 136 grid points. Simulations of Mittal and Moin were performed on a C-type mesh with 401 × 120 points in the plane normal to the cylinder axis, with 140 points on the cylinder surface and 129 points on the wake center line. Overall, the present simulations have slightly higher resolution in the circumferential direction and lower resolution in the radial direction at the cylinder surface than that in the simulations of Beaudan and Moin and Mittal and Moin. We are able to use 20%–40% finer grid spacings downstream of the cylinder, at x/D > 4, due to the zonal grid structure. The total number of grid points in case 2 is about 30% higher than that in the simulations of Beaudan and Moin and 50% lower than that in the simulations of Mittal and Moin.

Most of the results presented are from case 2. Simulations of case 1 with a coarser grid and of case 3 with a finer grid were performed to establish grid independence. Initially, coarse grid simulations with approximately 200,000 grid points were performed. The flow field was initialized with the potential flow solution and advanced in time until a statistically steady vortex shedding was established. Then, the flow field was interpolated on the fine grid and advanced in time for approximately ten shedding cycles to allow all transients to exit the computational domain. Thereafter, the statistics were accumulated over approximately seven additional shedding cycles.

Some of the important flow parameters from our computations, the results of the two previous large eddy simulations, and the available experimental data are summarized in Table II. The experiments of Lourenco and Shih and Ong and Wallace did not provide values for the mean drag coefficient, base suction coefficient, and separation angle, and we obtained these values from the other experimental studies listed in Table II. The table also lists the length of the mean recirculation region taken from Cardell, instead of that from the experiment of Lourenco and Shih for reasons to be discussed later. The mean drag coefficient, base suction coefficient, recirculation length, separation angle, and Strouhal shedding frequency are found to be in fairly good agreement with the experimental data and the results of the two previous simulations.

FIG. 7. Isosurfaces of instantaneous streamwise vorticity in the flow over a circular cylinder at ReD = 3900: (a) x-y-plane view; (b) x-z-plane view. Positive vorticity, ω/D/U∞ = 5.5—dark; negative vorticity, ω/D/U∞ = −5.5—light.

FIG. 8. (Color) One-dimensional frequency spectra at x/D = 5.0: black, experiment of Ong and Wallace (Ref. 19); red, B splines; green, central FD; blue, upwind FD; ––, grid cutoff; ---, −5/3 slope.
A. Instantaneous flow field

Instantaneous streamwise, cross-flow, and spanwise velocity fields in the first ten diameters downstream of the cylinder are shown in Figs. 2, 3, and 4, respectively. An unsteady recirculation region in Fig. 2 and alternating regions of positive and negative cross-flow velocity corresponding to Karman vortices in Fig. 3 can be clearly observed. The wake flow at this Reynolds number appears to be highly turbulent and three dimensional. Note the presence of both large and small structures in the wake. The flow structures tend to increase in size with increasing streamwise distance, as observed in Fig. 4 which shows instantaneous spanwise velocity. However, small scale fluctuations are present even ten diameters downstream. This is in contrast with the simula-

![Graphs showing one-dimensional frequency spectra at x/D=7.0 and x/D=10.0](http://pof.aip.org/about/rights_and_permissions)
tions of Beaudan and Moin,4 who did not observe small-scale turbulence several diameters downstream because of the inherent numerical dissipation of the upwind finite-difference method. Moser et al.28 observed the significant presence of small-scale turbulence as well as large scale structures in their direct numerical simulations of a time-developing wake at a momentum thickness Reynolds number of Re_D = 2000, which is qualitatively comparable to the present computations.

Contours of instantaneous vorticity magnitude are shown in Fig. 5. Two long shear layers separating from the cylinder and the development of the Karman vortex street are clearly seen in Fig. 5. The vortices arising from the instabilities of the shear layers mix in the primary Karman vortices before propagating downstream, which is consistent with the observations of Chyu and Rockwell11 in their PIV experiments. However, at this Reynolds number, these spanwise vortices are not as clear as at higher Reynolds numbers.11 The bottom laminar shear layer separating from the cylinder and its transition to turbulence is displayed in Fig. 6, which shows an isosurface of the vorticity magnitude. Long separating shear layers with longitudinal extent of about one cylinder diameter is the distinct feature of the flow at this Reynolds number. This observation has been confirmed by several recent experiments.8,10,11 The lengths of the shear layers and the size of the recirculation region are known to be very sensitive to external disturbances, such as free-stream turbulence, cylinder vibrations, acoustic noise, etc.10,11,29 Isosurfaces of streamwise vorticity in two plane views are shown in Fig. 7. The figure shows quasiperiodic streamwise vortical structures in the near-wake region of the cylinder. These structures are similar in shape and size to those described in several experimental studies.26,27 Typical spanwise length scales in Fig. 7 are of the order of one-fourth to one-third of the cylinder diameter, which is in good agreement with the scales estimated from the experimental correlation, Eq. (5).

B. One-dimensional energy spectra

Figures 8, 9, and 10 show one-dimensional frequency spectra at three downstream locations on the centerline of the wake. The Lomb periodogram technique30 with an oversampling factor of 4 was used to perform the spectral analysis of unevenly sampled data. About 13,000 samples at each of Nz = 48 spanwise locations of streamwise and cross-flow velocities were collected over a time interval T U_∞ / D = 35. The spectra calculated from these time series were then av-
eraged in the spanwise direction to increase the statistical sample.\textsuperscript{20,31,32} The frequency is nondimensionalized by Strouhal shedding frequency.

The power spectra in Figs. 8–10 are displayed together with the experimental results of Ong and Wallace,\textsuperscript{19} the upwind-biased finite-difference,\textsuperscript{4} and central finite-difference\textsuperscript{20} calculations. The experiment has small peaks at the shedding frequency and at the first harmonic in the $u$ spectra at $x/D = 5$. These peaks are visible in the $B$-spline simulation spectra. At centerline locations of the wake, streamwise velocity oscillates predominantly at twice the Strouhal frequency (while cross-flow velocity oscillates at Strouhal frequency). This is simply due to velocity distribution in the Karman vortex street. The overall agreement between the $B$-spline calculation and the experimental spectra is excellent at all three locations. The current simulations are also able to reproduce the inertial subrange observed in the experiment. The power spectra are consistent with the instantaneous flow visualizations, where the presence of small scales was clearly observed. The good agreement between numerical and experimental spectra at high wave numbers indicates correct representation of small scales in the $B$-spline simulations. Earlier, Mittal and Moin\textsuperscript{20} reported that nondissipative finite-difference calculations were able to match the experimental data over a larger range of wave numbers than both finite-difference simulations. Note that the streamwise grid spacing, which defines the maximum resolvable frequency, according to Taylor’s hypothesis, is slightly larger in the current simulations than that in the previous two. We were able to use a finer grid at the downstream stations without a substantial increase in the overall number of grid points owing to the zonal grid structure.

C. Mean flow and turbulence statistics

The flow statistics from the current simulations, together with the results of the two previous numerical studies, are shown in Figs. 11–17. Statistics were accumulated over approximately seven vortex shedding cycles, ($T = 35D/U_\infty$), which is approximately the same averaging time interval used by Beaudan and Moin.\textsuperscript{4} The flow quantities were also averaged over the periodic spanwise direction. A variable time step with fixed CFL number was used in all simulations. The $B$-spline simulations were performed with the time step of about $0.005R_c/U_\infty$.

The pressure distribution on the cylinder is shown in Fig. 11. The experimental data of Norberg at approximately the same Reynolds number ($Re_D = 4020$) and the surface pressure profiles from the previous LES are also displayed. All three simulations predict the pressure distribution on the cylinder within the experimental accuracy.

Mean velocities and turbulent Reynolds stresses from the three simulations are compared with the results of the particle image velocimetry experiment of Lourenco and Shih\textsuperscript{17} in the very near wake (up to $x/D = 4$) and with the hot-wire measurements of Ong and Wallace\textsuperscript{19} in the downstream region (from $x/D = 3$ to $x/D = 10$). The $B$-spline simulations are in very good agreement with the upwind-biased and central finite-difference simulations in the first five diameters of the wake. Minor differences between the simulations are insignificant to distinguish one technique from another in this region. However, there are some impor-
tant differences between the results of the three simulations and those of the experiment of Lourenco and Shih. All simulations develop a U-shape profile for the streamwise velocity while the experiment shows nearly a V-shape profile inside the recirculation region immediately behind the cylinder, at $x/D = 1.06$. In the simulations, the mean streamwise velocity develops a V-shape profile farther downstream, near the edge of the recirculation region, e.g., at $x/D \approx 1.54$. The size of the recirculation region is noticeably smaller in the experiment as indicated in Fig. 12. The source of these differences is discussed in Sec. IV. There, it is hypothesized that this particular experiment suffered from some external disturbances that contributed to an earlier transition in the separating shear layers which affected the size of the recirculation region and the shape of the velocity profiles in the very near wake of the cylinder. Note that the two experiments, which are used for the comparison here, were performed at the same Reynolds number, $Re_D = 3900$, but do not agree with each other in the overlap region.

The differences between the three simulations appear farther downstream, at locations $x/D > 5.0$, where the calculations are compared to the hot-wire measurements of Ong and Wallace. In this region of the wake, the grid is fairly coarse and the shortcomings of the numerical techniques become apparent. Upwind-biased calculations are affected by their inherent numerical dissipation and show low levels of turbulent fluctuations in Fig. 16. Nondissipative central difference calculations show better agreement with the experiment but underpredict the peaks of the streamwise velocity fluctuations especially at $x/D = 10.0$. Truncation error of this low-order scheme is known to affect the results of large eddy simulations on coarse grids. On the other hand, the results of the B-spline simulations are in very good agreement with the experiment which is also consistent with the frequency spectra shown earlier.

In order to establish grid independence of the results, we performed computations on a fine grid (case 3) and compared the results with cases 1 and 2. The grids in the vicinity of the cylinder are shown in Fig. 1 for all three cases. The total number of grid points in case 3 was increased by approximately 85% as compared to that of case 2. Also, the diameter of the computational domain in the refined case was decreased by 30% providing additional effective grid refinement. The results of case 2 provided an estimate of the thickness and the length of the separating shear layers which enabled us to cluster grid points in the important regions using a combination of grid stretching and zonal grid embedding. For case 3, the grid resolution in the free shear layers was increased by a factor of 2 in both streamwise and cross-flow directions compared to that in case 2. The number of grid points in the spanwise direction was 48 in all cases. Comparison of mean streamwise velocities and turbulence intensities in the vicinity of the vortex formation region is shown in Fig. 18. The overall agreement between the computations of cases 2 and 3 is good. Note that the U shape of the streamwise velocity profile is preserved in the fine grid computations confirming our earlier conclusion that the simulations predict the flow correctly at this location.

The profiles of mean velocities and turbulence intensities obtained in the coarse grid simulations of case 1 deviate significantly from the results of the finer grids 2 and 3. Even though the grid resolution in case 1 is adequate in the laminar boundary layer, the mesh spacing becomes too large downstream, and the separating shear layers are not well resolved. As a result, the shear layers are shorter and the recirculation region is smaller. Consequently, the development of the flow downstream is different in case 1. Interestingly, the profiles of mean velocity in the under-resolved simulation (case 1) agree with the experimental measurements of Lourenco and Shih.

D. Comparison of simulations with and without a SGS model

To assess the effect of the subgrid-scale model, we performed simulations with and without a subgrid scale model on the grids of cases 1 and 2. The profiles of mean streamwise velocity and streamwise fluctuations for case 2 are shown in Figs. 19–20. We chose to evaluate most of the results downstream of the recirculation region where the flow is well developed and the effect of the model is expected to be pronounced. We find insignificant differences between the mean velocity profiles from simulations with and without a SGS model. However, the differences appear in the rms velocity fluctuations. No-model simulations do not reproduce the double-hump shape of the profile of the rms streamwise velocity fluctuations. Major differences between LES and simulations without a model appear in the profiles of one-dimensional spectra shown in Fig. 21. Spectra at one location, $x/D = 3.0$, are shown. Simulations without a model show a slower decay of the spectra and more energy at the high wave numbers. These trends are observed in both coarse (case 1) and fine (case 2) grid simulations. Faster decay of energy spectra in LES as compared to that in the no-model simulations is consistent with the dissipative nature of the dynamic Smagorinsky SGS model. The differences in the spectra between simulations with and without a model are larger at the locations farther downstream. The model is expected to be more active in the regions where the turbulence is more developed and where the grid is coarser.

The relatively small influence of the subgrid-scale model on simulations of the flow over a cylinder at $Re_D = 3900$ have been previously reported by Beaudan and Moin and by Breuer. These authors, and also Rodi, pointed out the need to perform simulations at a higher Reynolds number to fully assess the effect of subgrid-scale models in LES of flows over bluff bodies.

IV. IMPACT OF GRID RESOLUTION

In Sec. III, we pointed out the differences between the results of the experiment by Lourenco and Shih and large eddy simulations of the flow over a circular cylinder at $Re_D = 3900$. Even though the three numerical simulations employed different numerical techniques and different formulations of the governing equations, in the very near wake, they were in agreement with each other but differed from the experimental data of Lourenco and Shih. The major differences were in the shape and size of the recirculation region.
The streamwise length of the recirculation region obtained in numerical simulations was \( L_r/D = 1.35 \) compared to \( L_r/D = 1.18 \) in the experiment. The shape of the streamwise velocity profile inside the recirculation region also appeared to be different. The simulations showed a U-shape profile at the location \( x/D = 1.06 \) while a V-shape was observed in the experiment as shown in Fig. 13. The size and the form of the recirculation region is directly related to the length of the vortex formation region and the dynamics of the downstream flow. Therefore, it is very important to find a possible source of the differences.

Recently, Xia and Karniadakis\(^{35}\) performed coarse “direct numerical simulations” of the flow over a cylinder at \( \text{Re}_D = 3900 \) using a spectral element method. The surprising result of that study was good agreement between the experimental data of Lourenco and Shih\(^{17}\) and the numerical calculations performed with just two spectral modes in the spanwise direction, i.e., four grid points. These findings provoked our own study of the effect of grid resolution at subcritical Reynolds numbers.

Five calculations were carried out on the same grid in the plane normal to the cylinder axis but with different grid resolutions and domain sizes in the spanwise direction. The grid parameters for the cases considered are summarized in Table III. Case 1 is a two-dimensional calculation. Case 2 has the same coarse spanwise grid resolution as case 4, but half the spanwise domain size. Similar relationships in the spanwise resolutions and domain sizes exist between cases 3 and 5 performed on fine spanwise grids. In cases 2 and 4, four and eight grid points in the spanwise direction is not sufficient to capture the important streamwise vortices. The estimates for the spanwise spacing of these vortices were given in Sec. III. Obviously, the domain size of \( L_z = 1.57D \) severely constrains the development of these structures. It was found that such a limited domain does not alter significantly the statistics in the very near wake (\( x/D < 2 \)) and, therefore, is appropriate for the present analysis. Table III gives the values of global flow quantities in all cases and the experiment.

The streamwise mean velocity profiles from the three-dimensional cases are shown in Fig. 22. The experimental data of Lourenco and Shih\(^{17}\) are shown with square symbols. Like the results of Xia and Karniadakis,\(^{35}\) the mean velocities in the simulations on coarse spanwise grids (cases 2 and 4) appear to be in better agreement with the experiment than those from the refined calculations (cases 3 and 5). In both the experiment and coarse grid simulations, the streamwise mean velocity shows U-shape profiles immediately behind the cylinder and develops V-shape profiles farther downstream. The streamwise mean velocity profile of the fine grid simulations retains the U shape at \( x/D = 1.06 \) and shows a V-shape at \( x/D = 1.54 \). The shape of the mean velocity profile is directly related to the level of velocity fluctuations and, consequently, to the transition in the shear layers. For ex-
FIG. 21. One-dimensional spectra $E_{11}$ at $x/D=3.0$ and $y=0$ from simulations with the dynamic SGS model (---) and without a model (--).  

TABLE III. Grid parameters and global flow quantities in the cylinder flow computations at $Re_D=3900$. The values of $C_D$, $-C_{P_{\infty}}$, $St$, and $L_{rec}/D$ are from different experiments, notated in parentheses: (1) Ref. 40, (2) Ref. 41 at $Re_D=5000$, (3) Ref. 10, (4) Ref. 19, (5) Ref. 17.

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_z$</th>
<th>$L_z$</th>
<th>$C_D$</th>
<th>$-C_{P_{\infty}}$</th>
<th>$St$</th>
<th>$L_{rec}/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1.65</td>
<td>1.5</td>
<td>0.230</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>$\pi D/2$</td>
<td>1.36</td>
<td>1.21</td>
<td>0.190</td>
<td>1.04</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>$\pi D/2$</td>
<td>1.07</td>
<td>0.97</td>
<td>0.212</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>$\pi D$</td>
<td>1.38</td>
<td>1.23</td>
<td>0.193</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
<td>$\pi D$</td>
<td>1.04</td>
<td>0.93</td>
<td>0.210</td>
<td>1.35</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
<td>$\pi D$</td>
<td>1.07</td>
<td>0.98</td>
<td>0.206</td>
<td>1.04</td>
</tr>
<tr>
<td>Expt</td>
<td>⋯</td>
<td>⋯</td>
<td>0.99±0.05 (1)</td>
<td>0.88±0.05 (1)</td>
<td>0.215±0.005 (2), (3)</td>
<td>1.33±0.05 (3)</td>
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<td></td>
<td></td>
<td>0.21±0.005 (4)</td>
<td>1.18±0.05 (5)</td>
</tr>
</tbody>
</table>

Coarse grid in radial and circumferential directions.

FIG. 22. Profiles of streamwise velocity at three locations downstream of a cylinder: (a)—, case 3, $N_z=24$, $L_z=\pi D/2$; ⋯, case 2, $N_z=4$, $L_z=\pi D/2$; (b)—, case 5, $N_z=48$, $L_z=\pi D$; ⋯, case 4, $N_z=8$, $L_z=\pi D$; □, experiment of Lourenco and Shih (Ref. 17).

FIG. 23. Profiles of streamwise velocity fluctuations at three locations downstream of a cylinder: (a)—, case 3, $N_z=24$, $L_z=\pi D/2$; ⋯, case 2, $N_z=4$, $L_z=\pi D/2$; (b)—, case 5, $N_z=48$, $L_z=\pi D$; ⋯, case 4, $N_z=8$, $L_z=\pi D$; □, experiment of Lourenco and Shih (Ref. 17).
ample the streamwise velocity fluctuations, shown in Fig. 23, are noticeably higher in the coarse grid cases.

Figures 24 and 25 show a series of instantaneous streamwise velocity profiles for coarse grid cases 2 and 5, respectively, at the location \( x/D = 1.06 \). Twenty instantaneous velocity profiles separated by \( \Delta Tu_x/D = 0.5 \) are shown together on one plot. The mean velocity profile shown with the thick black line develops the V shape when the level of turbulent fluctuations is high, as in case 2, and the U shape when the level of turbulent fluctuations is relatively low, as in case 5. In the latter case, the V-shape profile develops farther downstream, as shown in Fig. 26. The level of turbulent fluctuations is related to the transition process in the separating shear layers. In the fine grid simulation, the laminar shear layers appear to be longer. The extent of the shear layers can be visualized with contour plots of vorticity magnitude. Instantaneous contours of vorticity magnitude in the four three-dimensional cases are shown in Figs. 27 and 28. The separating laminar shear layers are substantially shorter in the simulations with coarse spanwise resolution than in the fine grid calculations. In other words, in the coarse grid simulations, transition to turbulence in the separating shear layers occurs closer to the cylinder and leads to the development of the V-shape profile and shorter vortex formation region.

It should be pointed out that shorter shear layers were also observed in simulations with a fine spanwise grid but inadequate resolution in the radial and circumferential directions. Figure 29 compares the results from simulations of case 5 (fine grid) and case 6 (coarse grid). We used 48 grid points in the spanwise direction for both cases. The grid in case 6 immediately at the cylinder was comparable to that of case 5 but, due to high stretching factor, it quickly diverged and was inadequate to resolve the thin separating shear layer farther downstream. This led to shorter shear layers. Consequently, the vortex formation length in case 6 was smaller than that in case 5. Again, the mean velocity and turbulence fluctuations from the simulations of case 6 agreed fairly well with the experimental data of Loureno and Shih.17

Early transition in the shear layers probably occurred in the experiment of Loureno and Shih,17 which explains its agreement with the under-resolved simulations. Even though the fine three-dimensional simulations do not agree well with the experiment of Loureno and Shih17 in the near-wake region, they are in good agreement with the experiment at this Reynolds number carried out by Ong and Wallace,19 which was limited to downstream locations. The size of the recirculation region in the fine grid simulations agrees with the experiment of Cardell.10 There is also qualitative agreement with the results of Prasad and Williamson,8 who carried out an experiment at lower Reynolds number of \( Re_D = 2600 \) and observed a U-shape profile of the streamwise velocity at \( x/D = 1.0 \). Long separated shear layers were observed in the experiments of Chyu and Rockwell,11 who used high-image-density particle image velocimetry to visualize the vorticity field. These experiments were performed at Reynolds numbers ranging from \( Re_D = 5000 \) to \( Re_D = 10000 \).

Earlier transition to turbulence in the separating shear layers and, consequently, a shorter vortex formation region

![Fig. 24](image-url) Profiles of streamwise velocity at \( x/D = 1.06 \) in case 2. The black line indicates the mean velocity, and the gray lines are the instantaneous velocity profiles separated by \( \Delta Tu_x/D = 0.5 \).

![Fig. 25](image-url) Profiles of streamwise velocity at \( x/D = 1.06 \) in case 5. The black line indicates the mean velocity, and the gray lines are the instantaneous velocity profiles separated by \( \Delta Tu_x/D = 0.5 \).

![Fig. 26](image-url) Profiles of streamwise velocity at \( x/D = 1.54 \) in case 5. The black line indicates the mean velocity, and the gray lines are the instantaneous velocity profiles separated by \( \Delta Tu_x/D = 0.5 \).
observed in the experiment of Lourenco and Shih\textsuperscript{17} can result from many factors. The range of critical Reynolds number for the shear layer instability reported in the published literature spans from $Re_D = 300$ to $Re_D = 3000$.\textsuperscript{7} One of the possible reasons for such a discrepancy can be attributed to various levels of free-stream turbulence present in different experiments. Gerrard\textsuperscript{36} pointed out that, on increasing the free-stream turbulence level more than 1%, the size of the vortex formation region shrinks. Shorter mean recirculation region causes base suction coefficient to rise. This is consistent with our observations. The base suction coefficient is significantly higher and the mean recirculation region is smaller in the simulations with poor grid resolution. Wu et al.\textsuperscript{37} have also reported that high turbulence background may cause a reduction in the critical Reynolds number for the onset of shear layer transition. The transitional Reynolds number in that study varied from $Re_D = 1000$ to $Re_D = 3000$ and was dependent upon the background disturbance.

Shorter formation lengths of the mean recirculation region can also result from cylinder vibrations. Chyu and Rockwell\textsuperscript{11} observed a dramatic decrease in the formation length and shorter shear layers when the cylinder was perturbed at the inherent instability frequency of the shear layer and its subharmonics. Even at a very small amplitude of oscillations, such as 0.1% of the cylinder diameter, the location of the shear layer breakup moved noticeably closer to the cylinder in their experiments.

Cardell\textsuperscript{10} has found a large effect of acoustic forcing on the shear layer instability and the near-wake structure. Acoustic forcing led to reductions in the length of the separation bubble and affected the values of $u'/U_\infty$, $-C_{pb}$, and $St$. Cardell\textsuperscript{10} also speculated that accidental forcing of the shear layers was a significant factor in the observed large scatter in cylinder flow measurements.

Prasad and Williamson,\textsuperscript{8} Norberg,\textsuperscript{12} and Szepessy and Bearman\textsuperscript{38} studied the effects of various geometric parameters in the experiments on the development of the near-wake flow. Prasad and Williamson\textsuperscript{8} reported that, even in the absence of the free-stream turbulence variations, the spanwise end conditions significantly affect the onset of shear layer instability. Norberg found that cylinder aspect ratios as large as $L/D = 60–70$ were needed for independent experimental conditions at $4000 < Re_D < 10,000$. Note that the aspect ratio in the experiment of Lourenco and Shih\textsuperscript{17} was $L/D = 21$. Szepessy and Bearman\textsuperscript{38} saw variations in the size of the vortex formation length for different aspect ratios. The effect of aspect ratio found in experiments is not applicable to numerical simulations which are performed with periodic boundary conditions in the spanwise direction.

Obviously, the agreement between the under-resolved simulations and the experiment of Lourenco and Shih\textsuperscript{17} is fortuitous. However, it indicates that simulations of flow over a cylinder and perhaps other flows with free shear layers
require careful grid design that take into consideration all essential flow scales. Establishing grid independence, especially in the near-wake region, is important for trustworthy simulations.

V. SUMMARY

In this study, we carried out numerical simulations of flow over a circular cylinder at a subcritical Reynolds number, \( Re_D = 3900 \). The presence of the thin laminar boundary layer, separating shear layers and small scales in the turbulent wake imposes severe grid restrictions. The technique of large eddy simulation with the dynamic subgrid-scale model was used to account for turbulence scales that are not resolved by the grid. The laminar regions are well resolved.

In the vicinity of the cylinder, the present simulations agree well with the two previous computations of this flow and are in a fair agreement with the experimental data of Lourenco and Shih. In the region of the wake from six to ten diameters downstream of the cylinder, the results obtained from the B-spline computations are in better agreement with the experimental data of Ong and Wallace than those of upwind and central finite-difference simulations. This is particularly evident in the profiles of one-dimensional energy spectra at several locations in the wake.

In the studies of the influences of numerical resolution and the spanwise domain size on the three-dimensional simulations, we found that inadequate grid resolution can cause early transition in the shear layers separating from the cylinder which leads to inaccurate predictions of the near-wake flow statistics. Therefore, the agreement of such calculations with experiments that inhibit early shear layer transition due to external disturbances is fortuitous.

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26H. Mansy, P.-M. Yang, and D. R. Williams, "Quantitative measurements.


40C. Norberg, (private communication).