An aero-acoustic study of an in-duct flexible plate

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Abstract
In the application of mixing urea with diesel exhaust gases a triangular plate is used to generate turbulence. The present work treats the effect of having a yielding plate on the scattering properties, as well as the source generation of the plate. The acoustics of the plate are modeled by a 2-port model, and the results are compared with strain gauge measurements. Earlier developed over-determination methods have been applied for the determinations of the pressure amplitudes and the scattering matrix. For the source cross spectrum matrix an over-determination method is suggested and applied. The improvements of all methods are studied, and great improvements are indeed observed. The effect of inserting the plate is small when the scattering matrix is considered, however the source cross spectrum matrix shows that the plate generates a broad band component as well as a dipole, Strouhal tone. The effect of the yielding plate is observed to be a decrease in the broad band component of about $2\, dB$ at the Mach number $0.213$, and a disruption of the mechanism behind the tonal component. It is suggested for future work that a complete structural dynamic investigation of the yielding plate should be made simultaneously with sound spectra measurements at more, narrower flow velocity increments in order to further analyze the effects of the yielding plate.
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<td>complex conjugate</td>
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<td>†</td>
<td>Hermitian transpose</td>
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<td>$u, v, w$</td>
<td>transportation matrices</td>
</tr>
<tr>
<td>$V$</td>
<td>x, y and z-components of the flow velocity</td>
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<td>$W$</td>
<td>acoustic power</td>
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Background
According to the World Health Organization Europe [1], the average life expectancy of European citizens is decreased almost a year by the everyday exposure to air pollutants. For this reason, even though the emission standards for diesel engines differ between regions, it can be expected that the restrictions of the emission of hazardous particles will be set lower by each new standard introduced, regardless of global region. It is therefore of utter importance for the industry to develop new cleaner diesel engine systems.

Among the hazardous particles produced by diesel combustion are nitrogen oxide - NOₓ (NO and NO₂), which in the presence of sunlight react with volatile organic compounds to form smog, an air pollutant which among other things damages lung tissue. A standard method of dealing with NOₓ is selective catalytic reaction using ammonia. However, since ammonia is in itself hazardous to handle, the much more stable component urea is used, which is a solid in room temperature but in temperatures over 152 °C it decomposes into ammonia and isocyanic acid, which in turn can react with water to form more ammonia and carbon dioxide. The efficiency and the amount of byproducts of the selective catalytic reaction depends on the temperature and of the ratio between the components.

The problem inherited in this procedure is that e.g. vehicle exhaust systems are made as compact as possible, which greatly reduces the pipe length where mixing between the chemical components can occur, thus making it difficult to achieve the proper ratio between the components. A method of intensifying the mixing process is therefore needed. Such a method is the generation of turbulence by a triangular shaped plate, placed upstream of the mixing section of the pipe. However, sound is generated by turbulence and other mechanisms associated by the flow over the plate, and the reflection and transmission of sound in the exhaust system will be affected. The emission standards for sound pollution, i.e. noise are for a manufacturer just as important to fulfill as the standards for particle emission. The influence on the acoustics of the mixer plate in a pipe with an air flow is therefore important to investigate, and is the main scope of this thesis.

Main Scope of the Thesis
The main scope of the thesis is to analyze how the stiffness of the mixer plate affects the acoustics of the plate at various flow speeds. In practice this is realized by obtaining a complete 2-port model of the plate for each of the three plate thicknesses 0.5, 1.5 and 3 mm, as well as measuring the relative vibration level of the plates. For reference the 2-port of an empty duct is also obtained. All four cases are obtained at the axial velocities 25, 52, 78 and 89 m/s.

Earlier work suggests various noise suppression techniques in the process of obtaining the 2-port, and a sub task of the present work is set to improve the accuracy of the measurement methods, e.g. by introducing new methods of noise suppression.

Also, in parallel to this work other projects have been carried out, e.g calculations of the influence on the flow field of the plate using computational fluid dynamics (CFD). Future
projects are planned in which attempts will be made to compare the results from this work with acoustic results obtained from CFD. For this reason a second sub task is to obtain pressure loss data, from which some non-dimensional factors can be determined in order to insure that both works are treating the same problem.

**Aero-acoustic Theory**

Sound waves are small disturbances of pressure propagating in a medium, which usually is air. In media not subjected to shear stress, e.g. liquids and gases, the media particle displacement will be in the same direction as the propagation of the wave, i.e. a longitudinal wave. In order to obtain the governing equations for this wave it is instructive to consider a wave front propagating in quiescent air, see figure 1.

![Figure 1](image)

**Figure 1** Schematic of a sound wave, moving from left to right, with a coordinate system fixed to the wave front.

If the coordinate system is set fixed at this wave front, then the air will seem to move towards the wave on the left hand side at a velocity $u$, whilst on the right hand side it will move away with a different velocity $u + u'$. If the change in velocity is large, the wave will be a *shock wave* with a propagation velocity higher than the local speed of sound, but if the velocity disturbance is small, the wave will be a *sound wave*. It is the assumption of small disturbances which allow for the linearization of the governing fluid mechanical equations, leading to the acoustic equations. For simplicity the derivations are made writing the local velocity, density and pressure as

\[
\begin{align*}
\rho &= \rho_\infty + \rho' \\
u &= u_\infty + u' \\
p &= p_\infty + p'
\end{align*}
\]

where the subscript $\infty$ refers to an upstream quantity. Put into the differential forms of the continuity and momentum equations, the following nonlinear equations follows for the $x$-
direction. In order to obtain the governing equations of wave motion the physical principles of conservation of mass, momentum and energy must be applied. These equations comes in different forms for different purposes. Here they are expressed as the differential forms of the continuity equation, the momentum equation and the entropy equation [5]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = m \]  

\[ \rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \mathbf{f}_V + \mathbf{f}_S \]  

\[ T \frac{Ds}{Dt} = \frac{De}{Dt} + p \frac{D}{Dt} \left( \frac{1}{\rho} \right) \]  

where \( m \) is the rate of mass production per unit volume, \( s \) is the entropy and \( e \) the internal energy. \( \mathbf{f}_V \) is a term corresponding to body forces acting on the fluid, e.g. gravitation and \( \mathbf{f}_S \) corresponds to forces acting on the surface of a fluid element. \( \mathbf{V} \) is the velocity vector, \( T \) the temperature, \( \nabla \) the nabla operator and \( D/Dt \) the convective derivate. If no injection of mass is present, the continuity equation (4) in the x direction is

\[ \frac{\partial \rho'}{\partial t} + \rho_\infty \frac{\partial u'}{\partial x} + \rho' \frac{\partial \mathbf{u'}}{\partial x} + u' \frac{\partial \rho'}{\partial x} = 0 \]  

Since \( \rho \) is a state variable it can be expressed as a function of two other state variables as

\[ \rho = \rho(p,s) \]  

Implying

\[ \rho' = \rho' \left( \frac{\partial \rho}{\partial p} \right)_s + s' \left( \frac{\partial \rho}{\partial s} \right)_p \]  

where the subscripts \( s \) and \( p \) refers to isentropic and isobaric relations. If now the process during which the disturbances occur is said to be inviscid and adiabatic, which is true for a sound wave propagating in a quiescent medium, the entropy will turn out to be constant. Also, if the forces are excluded from the momentum equation, the resulting equations will be

\[ \frac{\partial}{\partial t} \left( \frac{p'}{c^2} \right) + \rho_\infty \frac{\partial u'}{\partial x} + \rho' \frac{1}{c^2} \frac{\partial u'}{\partial x} + u' \frac{\partial \left( \frac{p'}{c^2} \right)}{\partial x} = 0 \]
\[
\left( \rho_\infty + \frac{p'}{c^2} \right) \frac{\partial u'}{\partial t} + \left( \rho_\infty + \frac{p'}{c^2} \right) u' \frac{\partial u'}{\partial x} = - \frac{\partial p'}{\partial x}
\]

where
\[
c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s
\]

These equations are nonlinear, and thus it is convenient to get rid of all small terms, e.g. products of the disturbances and of their derivatives. Also, introducing the Taylor expansion of \( c^2 \) around \( \rho_\infty \) gives
\[
c^2 = c^2_\infty + \rho' \left( \frac{\partial c^2}{\partial \rho} \right)_s + \ldots
\]

After discarding all small terms the final result is then
\[
\frac{1}{c^2_\infty} \frac{\partial p'}{\partial t} + \rho_\infty \frac{\partial u'}{\partial x} = 0
\]

(14-1)
\[
\rho_\infty \frac{\partial u'}{\partial t} = - \frac{\partial p'}{\partial x}
\]

(14-2)

These equations are commonly referred to as the acoustic equations. Differentiating (14-1) with respect to \( t \), (14-2) with respect to \( x \) and subtracting the result, the classical wave equation is obtained
\[
\frac{1}{c^2_\infty} \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0
\]

(15)

Again, the approximations made during the derivation of this equation are that only small disturbances are considered, that all processes associated with them are isentropic and that no forces or mass injections are present. This means that the classical wave equation can only describe propagation of sound waves, not actual generation of sound itself, since amongst what have been discarded are the sources of sound. The general solution of (15) will have the form of
\[
p' = F_1(c_\infty t - x) + F_2(c_\infty t + x)
\]

(16)

where \( F_1 \) describes a wave propagating in the positive \( x \)-direction and \( F_2 \) describes a wave propagating in the negative \( x \)-direction. It can now be seen that the constant \( c \) is in fact the speed of propagation, i.e. the speed of sound. Recalling (12) and using the ideal
For air at the temperatures $3 \, K < T < 600 \, K$ it is sufficient just to consider the rotational and translational modes of the molecules, and thus for air at room temperature $\gamma = 1.4$ [3], whilst if energy from the vibration modes were taken into account the ratio would be $1.28$. This is of importance when hot exhaust gases are considered, however they also consist of a completely different mixture of gases than the ordinary air of outdoors, and will not be treated in this work. Continuing with the ideal gas law:

$$\frac{P}{\rho} = \mathcal{R}T$$

(18)

where $\mathcal{R}$ is the specific gas constant. For air at room temperature $\mathcal{R} = 287 \, J/kgK$. The resulting speed of sound is thus

$$c_\infty = \sqrt{\mathcal{R}T\gamma}$$

(19)

In reality $\gamma$, $\mathcal{R}$, and the accuracy of the ideal gas law varies as functions of temperature and pressure, but for an acoustician it is almost always sufficient to consider the ideal gas law to be true, and to use the values of the specific constants as given above. The existence of foreign particles in the air mixture will also affect the speed of sound, e.g. extreme humidity and snow. For the purpose of estimating the temperature dependence, the speed of sound as a function of temperature in the narrow threshold of the temperature fluctuations experienced during the measurements is shown in figure 2.
Figure 45. *The isentropic speed of sound as a function of temperature.*
\[ \gamma = 1.4, \ \Re = 287 \text{ J/kgK}. \]

Returning to the general solution of the wave equation it can be noted that the functions $F$ and $G$ are arbitrary, but according to Fourier theory, any function can be built up by a summation of harmonic functions. Consider a package of disturbances traveling along a path. Although the package may have an arbitrary wave form, all such forms can be built up by superposition of harmonic waves with different time periods, i.e. different frequencies. In acoustics the sound field is characterized by the energy content per frequency carried by the sound waves. Thus it is common to represent the acoustic field by a summation of harmonic waves, and the equation (16) is written as

\[ p' = \exp(i(c_\infty t - x)) + \exp(i(c_\infty t + x)) \]  

(20)

If the arguments in this solution are divided by the speed of sound and multiplied by the angular frequency, the solution per frequency can be written as

\[ p' = \exp(i(\omega t - kx)) + \exp(i(\omega t + kx)) \]  

(21)

where $k$ is the wave number and defined as

\[ k = \frac{\omega}{c_\infty} \]  

(22)

For the remainder of the report the speed of sound will be considered constant, and thus the subscript $\infty$ is dropped. Also, the superscript of the acoustic pressure is dropped, and all pressures should be interpreted as acoustic pressures unless otherwise stated.
The acoustic source terms

In order to describe the generation of sound, the classical wave equation is modified into an inhomogeneous differential equation

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = s(x_i, t)
\]

The right hand side represents the acoustic sources. The solution to this equation is obtained by a superposition of the derived solution from a point source [10], i.e., by describing the sources as a sum of Dirac delta functions. The resulting equation is an integration of all sources contained inside a control volume \(V\)

\[
p(x,t) = \iint_V \frac{s(y,t-r/c)}{4\pi r^2} dV_y
\]

where \(y\) is the position of the source, and \(dV_y\) refers to a spatial integration with respect to \(y\). It can be noted that the distribution of sources uniquely determines the sound pressure outside of the control volume, while for a given sound pressure there exists an infinite number of source distributions. This implies that just by knowing the sound field information about the source cannot be uniquely obtained. However, by splitting the sound field and the source field into two separate spaces and rewriting the continuity equation and momentum equation into a wave equation (in much the same way as the homogeneous wave equation was derived), Lighthill obtained a model for the sources. Using the sound pressure as the acoustic variable the resulting equation is

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i \partial x_i} = \frac{\partial}{\partial t} \left( m + \frac{1}{c^2} \frac{\partial}{\partial t} (p - c^2 \rho) \right) - \frac{\partial f_V}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j} (\rho V_j V_j - \tau_{ij})
\]

where \(\tau_{ij}\) is the shear stress, and the term \(p - c^2 \rho\) represents deviations from the adiabatic processes, e.g., heat injections in I.C. engines. The shear stresses are of importance when attenuation is considered, but their contribution to the generation of sound compared to that of the transport of momentum is smaller by the order of the Reynolds number. Considering the sound generation in the test rig both these terms can be neglected. Also, Lighthill sorted the terms into three groups by the order of the spatial derivative. The three groups are called monopole, dipole and quadrupole source terms. This grouping is important for instance when the scaling of the sound power with velocity is of interest. It can be shown that the contribution to the total sound power of the three terms will scale with the Mach number as

\[
W_m : W_d : W_q \propto M^{1+\text{Dim}} : M^{3+\text{Dim}} : M^{5+\text{Dim}}
\]

where the superscript \(\text{Dim}\) refers to the dimensionality of the wave propagation, e.g.
\[ \text{Dim} = 1 \] for plane wave propagation in ducts. If there is an object placed in the fluid, the sound generation will depend on the interaction between the fluid and the object. An equation describing the sound generated by a solid object in a flow exists and is called Curles equation

\[
p(x, t) = \iiint_{V} \frac{\partial^{2}}{\partial x_{i} \partial x_{j}} \left[ \frac{\rho V_{i} V_{j}}{4\pi r} \right] dV - \iiint_{S} \frac{\partial}{\partial t} \left[ \frac{p n_{i} + \rho V_{i} V_{j} n_{j}}{4\pi r} \right] dS_{y} + \iiint_{S} \frac{\partial}{\partial t} \left[ \frac{\rho V_{i} V_{j} n_{i}}{4\pi r} \right] dS_{y}
\]

where all integrals are evaluated for fields captured at the time \( t_{e} \). The different terms in the equation correspond to different physical phenomena; the volume integral is due to the turbulence downstream of the object, the first surface integral corresponds to unsteady pressure and a transport of momentum, i.e. forces, both applied at the surface of the object, and the last surface integral corresponds to volume flow due to vibrations of the object. These vibrations are excited by the flow, and are in most situations negligible. However, one of the main issues of the project is to measure the sound pressure generated for plates where the thickness i.e. the bending stiffness differ. It is possible that lowering the bending stiffness will alter the flow field in such a way that the turbulence sound decreases, while instead the change of volume flow starts to have a significant impact on the total sound level. From experience it is known that lowering the bending stiffness of a plate will decrease the radiated sound power level, for instance a flag swaying in the wind will be much more quiet than a steel plate.

When a source generated sound spectrum is considered, it is of interest to scale the frequency with a quantity such that the flow generated sound coalesce into a similar value of the new, dimensionless quantity. This dimensionless quantity or number, is called the Strouhal number and is defined as

\[
St = \frac{2\pi L}{u}
\]

where \( L \) is a typical length associated with the unsteady flow process. Without knowing the exact points of the vortex shedding on the plate, it is difficult to give an exact value of \( L \), however in this work \( L \) has been chosen such that

\[
2\pi L = 0.2 \ m \Rightarrow L \approx 0.0318 \ m
\]

**Duct acoustics**

When sound propagates in a pipe or a duct, the acoustic field will highly differ from the acoustic free field, due to the boundary conditions at the wall. The wave fronts will propagate in the shapes of the acoustic modes of the duct cross section. Each of the modes are associated with a cut-on frequency, below which no acoustic waves can propagate in the wave front pattern of the mode. The first mode shapes above the rigid mode are for a circular cross section shown in figure 3, [8].
Below the cut-on frequency of the first elastic mode, the wave fronts will be plane, implying that the 1-dimensional wave propagation solution can be used in order to describe the sound field inside the duct. Thus the frequency range of interest is limited by

\[ f < \frac{1.841c}{\pi D} \]  \hspace{1cm} (30)

Taking a closer look at the equation describing 1-dimensional propagation it is evident that if the time, angular frequency, spatial coordinate and wave number are all real valued, the waves will propagate without any attenuation. In order to take the attenuation of the amplitude into account at least one of these quantities must be complex. For the case where the air inside the duct is quiescent the attenuation can be represented by

\[ k = k_0 (1 + i\delta) \]  \hspace{1cm} (31)

where \( \delta \) is the damping or attenuation term. In the work of Allam [9] several author’s models for taking the attenuation into account are listed, among which Kirschoff describes the attenuation term as an effect of visco-thermal losses at the duct walls, and for a wide duct he found the solution to be

\[ k = \frac{\omega}{c} k_0 = \frac{\omega}{c} \left[ \frac{(1-i)}{k_s \sqrt{2}} \left(1 + \frac{(\gamma - 1)}{\sqrt{Pr}} \right) - \frac{i}{k_s^2} \left(1 + \frac{(\gamma - 1)}{\sqrt{Pr}} - \frac{\gamma (\gamma - 1)}{2Pr} \right) \right] \]  \hspace{1cm} (32)

where \( Pr \) is the Prandtl number.
\[ Pr = \frac{\rho V c_p}{\kappa_{th}} \]  

(33)

which for air at room temperature is about 0.7. \( \kappa_{th} \) is the thermal conductivity and \( c_p \) is the specific heat at constant pressure, given by

\[ c_p = \frac{\gamma R \gamma}{\gamma - 1} \]  

(34)

and \( k_s \) is the shear wave number

\[ k_s = r \sqrt{\frac{\omega}{v}} \]  

(35)

According to Pierce the contribution to the attenuation from the visco-thermal losses in the air itself is of two orders of magnitude less than the losses at the walls, and can be neglected. However it should be noted that for other gas mixtures these losses can come to dominate over those at the walls. Also, the \( k_s^2 \) term in (32) is normally omitted since it is very small.

**Acoustics in a duct with flow**

If flow is present inside the duct, the acoustic field will be affected by the flow field. Considering a one-dimensional flow, if the coordinate system is set fixed with the flow, the walls of the duct will appear to move. A sound wave propagating in the media will then need longer time to travel a certain distance when propagating in the same direction as the walls appear to move, than it will need when propagating in the opposite direction. In a coordinate system where instead the walls are fixed, which is the actual case, the sound waves will appear to propagate at a lower speed upstream than downstream relative to the duct, and thus the distance between each propagating wave peak will be shorter upstream and longer downstream, i.e. the frequency of the upstream propagating sound waves will be higher and the frequency of the downstream propagating sound waves will be lower than if no flow was present. This shift in frequency is known as the Doppler shift, and it shows up in the expression of the wave numbers

\[ k_+ = \frac{k}{(1 + M)} \]  

\[ k_- = \frac{k}{(1 - M)} \]  

(361,2)

where the subscripts \(+\) and \(−\) refers to downstream and upstream propagating waves, and \( M \) is the Mach number, here written as:
If $u$ is known, the wave number is thus readily calculated. The problem is that $u$ varies over the cross section of the duct, due to the boundary layers at the walls. The boundary condition of no slip between the fluid and the walls requires the flow velocity to be zero at the walls. Even though air and most other fluids have a very small capability to carry shear stresses, these have an impact on the flow field. Due to the shear stresses the flow velocity cannot be discontinuous, and thus there will be a layer of flow where the velocity is increased from zero at the wall to the maximum velocity at the axis of the duct. Generally the mean velocity over a cross section is used for the acoustic calculations, which is calculated by multiplying the maximum velocity by a factor. This factor depends on the Reynolds number, which is given by [3]

$$\text{Re} = \frac{D u_{\text{mean}}}{\nu}$$

(38)

where $\nu$ is the kinematic viscosity. The limit between Reynolds numbers for which the flow is either laminar or turbulent is vague since it depends on the inlet conditions of the disturbances of the flow, but if the Reynolds number is above $10^4$ the flow is definitely turbulent. The kinematic viscosity is temperature dependent, and at room temperature of 20 °C it is approximately $1.49 \cdot 10^{-5}$ m² s⁻¹, which for a duct with the inner diameter of 90 mm and a mean velocity of 50 m/s yields a Reynolds number of approximately $3 \cdot 10^5$. Thus the flow can be considered turbulent everywhere in the pipe where (38) is a correct measure of the Reynolds number. In Slichting [3] an empirical relation between the mean and maximum velocity inside a smooth pipe is presented, obtained from the experiments carried out by Nikuradse.

$$\frac{u_{\text{mean}}}{u_{\text{max}}} = \frac{2n^2}{(n+1)(2n+1)}$$

(39)

where $n$ varies slightly with the Reynolds number, see table 1.

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$4 \cdot 10^3$</th>
<th>$2.3 \cdot 10^4$</th>
<th>$1.1 \cdot 10^5$</th>
<th>$1.1 \cdot 10^6$</th>
<th>$2.0 \cdot 10^6$</th>
<th>$3.2 \cdot 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>6.0</td>
<td>6.6</td>
<td>7.0</td>
<td>8.8</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1  
$n$ as function of Reynolds number.

The relation (39) is presented in table 2.

<table>
<thead>
<tr>
<th>$n$</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{\text{mean}}/u_{\text{max}}$</td>
<td>0.791</td>
<td>0.817</td>
<td>0.837</td>
<td>0.852</td>
<td>0.865</td>
</tr>
</tbody>
</table>

Table 2  
Mean/Max flow velocity ratio as function of $n$. 

Now, if the Reynolds number is known, it is possible to approximate the ratio between the mean and maximum velocity. However, from the tables above it can be roughly estimated that a change by one order of magnitude in the range $10^3-10^5$ of Reynolds numbers corresponds to a change of about 3 % of the velocity ratio. For this work the Reynolds number is of the order $10^5$ and thus the ratio is set to 0.82.

When there is a flow present, the attenuation of the acoustic waves will be different. Allam [9] describes several models of these attenuations. The model proposed by Dokumaci is a slight modification to equation (361.2), which does not take the effect of turbulence into account, and the most extensive is the model by Howe. The model by Dokumaci is described by

$$k_\pm = \frac{\omega}{c} \frac{K_0}{(1 \pm K_0 M)}$$  \hspace{1cm} (40)

Allam found that the model proposed by Dokumaci can be used when the acoustic boundary layer thickness is smaller than the viscous boundary layer thickness, whilst the model by Howe agrees well with experiments for an acoustic boundary layer thickness of three to four times the viscous boundary layer thickness. The model by Howe is more complicated and contains zero and first order Hankel equations, and is not used in the present work, instead the wave number by Dokumaci is used for the final results.

Also, it can be noted that when a flow is present, the cut-on frequency of higher order elastic modes of the duct cross section will be shifted downward in frequency. This has however not been taken into account in the present work since the shift will not exceed 10 % (in the present work the measurements are made up to 2 kHz, while the 1st cut-on of a higher order mode without flow is approximately 2.23 kHz).

**The Acoustic 2-port Model**

In order to check the impact on the noise level the mixer plate has, one could argue that it is just a matter of measuring the sound pressure with and without it fitted in the real system, i.e. the insertion loss. However, this information will be useless for the purpose of further reduce the sound level. This is when you need a model, in which values of parameters can be altered and the outcome readily studied. The approximations previous mentioned corresponds to one model of reality, and those approximations are implemented in another model; the acoustic 2-port, see figure 4.
Figure 4  

A general picture of the acoustic 2-port model.

This model has some advantages and some drawbacks. First of all, it is a linear time invariant system, implying that linear algebra is applicable. The drawback is that any non linear behaviour observed in reality will not be seen using this model, and as a direct consequence the results that can be observed will be erroneous. Second, the model works like a black box, with inputs and outputs. Any processes taking place inside the 2-port model element will not be observable, only the results of these processes. Thus, for a given input the output is readily obtained, but what effect a change inside the 2-port would have on the output might not be possible to study. These characteristics of the 2-port stems from the fact that it is based upon the theory of Lighthill, which states that for a given acoustic field, the number of different source distributions which could have generated the acoustic field is infinite. Lighthill’s approach of superposing the acoustic field with the source field is consequently found when considering the equation system defining the 2-port. The pressure amplitudes in the model can be related via [2]

\[
\begin{bmatrix}
  p_{a+} \\
  p_{b+}
\end{bmatrix}
= 
\begin{bmatrix}
  \rho_a & \tau_b \\
  \tau_a & \rho_b
\end{bmatrix}
\begin{bmatrix}
  p_{a-} \\
  p_{b-}
\end{bmatrix}
+ 
\begin{bmatrix}
  p_{a+}^s \\
  p_{b+}^s
\end{bmatrix}
\tag{41}
\]

where \(a\) and \(b\) refers to the cross sections of the duct defining the length of the model. The superscript \(s\) refers to source generated sound pressure amplitudes. The vector with source generated pressures is the source part, or active part of the model. Furthermore the following notation is introduced

\[
S = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
= 
\begin{bmatrix}
  \rho_a & \tau_b \\
  \tau_a & \rho_b
\end{bmatrix}
\tag{42}
\]

where \(S\) is the scattering matrix, also referred to as the passive part of the model. Note that all quantities in (41) are complex quantities and are functions of frequency. The model is therefore evaluated at a certain frequency at a time, and throughout the
remainder of the report any relation between 2-port elements should be interpreted as the relation evaluated at a certain frequency.

**Passive Part**

If there are no sources inside the 2-port element, then the relations between the incoming and outgoing pressure waves will be constant. Putting the source generated pressure amplitude to zero in (41) the resulting equation will be:

\[
\begin{pmatrix}
    p_{a+} \\
    p_{b+}
\end{pmatrix} =
\begin{bmatrix}
    \rho_a & \tau_b \\
    \tau_a & \rho_b
\end{bmatrix}
\begin{pmatrix}
    p_{a-} \\
    p_{b-}
\end{pmatrix}
\]

(43)

Apparently the above mentioned relations are described by the scattering matrix. Each element in the matrix have a physical meaning, which is easy to interpret if the special case of no reflections at the end terminations of the pipes connected to the model is studied. This implies:

\[
p_{b-} = 0 \Rightarrow \rho_a = \frac{p_{a+}}{p_{a-}}
\]

(44, 45)

\[
p_{a-} = 0 \Rightarrow \tau_b = \frac{p_{a+}}{p_{b-}}
\]

Since \(p_{a+}\) is the outgoing wave and \(p_{a-}\) the incoming wave at cross section \(a\), then \(\rho_a\) is obvious the reflection coefficient at \(a\). Further, since \(p_{b-}\) is the incoming wave at cross section \(b\), \(\tau_b\) must be the transmission coefficient for a wave entering the 2-port element at \(b\) and exits at \(a\). In the same way \(\tau_b\) can be interpreted as the transmission coefficient from \(a\) to \(b\), and \(\rho_b\) as the reflection coefficient at \(b\).

These four elements are all characteristics of the acoustic 2-port, and are four of the six parameters which needs to be determined in order to obtain a complete model, but clearly much information are given by these four parameters alone. For instance, the transmission loss, which is the ratio between the incoming to the transmitted sound power, can be written as:

\[
D_{TL} = 10 \cdot \log_{10} \left( \frac{W_i}{W_f} \right) = 10 \cdot \log_{10} \left( \frac{p_{a-}^2}{p_{b+}^2} \right) = 20 \cdot \log_{10} \left( \frac{1}{\tau_a^2} \right)
\]

(46)

where \(\tilde{p}\) should be interpreted as the r.m.s. value of \(p\). The transmission loss is a very important quantity in duct acoustics, since the purpose of almost all acoustical components is to reduce the transmitted sound.
**Active Part**

If there is a source inside the 2-port element, the scattering properties of the model will be the same, so the afore mentioned reasoning is still valid. The difference is that the scattered acoustic field will be superposed a source generated field.

The source is in (41) represented by the amplitude of an harmonic wave propagating away from the 2-port element in both directions. For tonal sources this is a sufficient model, but in the case of turbulent sound generating processes the measured signal will be random. A better representation of the source is therefore the auto spectrum of the pressure amplitude, which gives information about the energy as a function of frequency. However the phase difference between the source sound pressure on each side is also of interest, and therefore the cross spectrum between these pressures are also used. For these reasons, the source generated sound is presented as the source cross spectrum matrix:

\[
\mathbf{G}^s = \begin{bmatrix}
G_{aa}^s & G_{ab}^s \\
G_{ba}^s & G_{bb}^s
\end{bmatrix}
\]  

(47)

where

\[G_{ij} = p_j p_i^*\]  

(48)

The notation \(^*\) refers to a complex conjugated quantity. In (47) the diagonal elements of the source cross spectrum matrix represents the single sided auto spectra of the source generated sound pressures in the directions \(a\) and \(b\), and thus for a point source in a duct the two spectra should be equal apart from a possible phase difference. The cross diagonal elements are the single sided cross spectra between the sound pressures, which tells something about how correlated the source generated sound fields on both sides are. In theory the cross diagonal elements should be identical.

In order to implement the source cross spectrum matrix in a network, it is necessary to write it on a vector form. This is done by using the relation

\[(r.m.s.)^2_a = \int_0^\infty G_{aa}^s \, df\]  

(49)

A source vector can then be formed containing the r.m.s. - values of the pressures

\[\mathbf{p}_s^s(\Delta f) = \begin{bmatrix}
G_{aa}^s (\Delta f)^{1/2} \\
G_{ba}^s (\Delta f)^{1/2}
\end{bmatrix}\]  

(50)

where \(\Delta f\) is a narrow frequency band at which the integral in (49) is evaluated, e.g. a multiple of the frequency step used in the measurements. It should be noted here that both elements in (50) are real quantities. The absolute phase of any of the two elements is irrelevant, but the relative phase difference is important if reactive effects such as
cancellation of the amplitudes of the sound waves are to be considered. The relative phase information is contained in the cross diagonal elements. However, if there are several 2-port elements containing sources, the relative phase of the different sources must also be known if reactive effects are to be considered. Whether or not this information can be obtained depends on the sound generation mechanisms associated with the different sources. For instance if all sources consists of fans driven by the same electrical input signal, the phase difference might be known. Another approach is to consider all sources to be uncorrelated, implying that the reactive effects between the sound waves emitted from the different sources can be omitted. For flow generated noise sources this approach might be very close to reality.

Cascade Networks

Since one of the pros of the 2-port model is the possibility to put it in a network, an overview of 2-port networks will be presented though they have not been used in the present work.

The obtained 2-port model of the acoustic component can be connected to other 2-port models, e.g. straight pipes, mufflers and end terminations. This is when the linear approximations become powerful, though generally the implementation is not straightforward. For simple systems with elements serial connected, i.e. cascade networks, there exist a different 2-port model, commonly referred to as the transfer matrix formulation, see figure 5.

![Acoustic 2-port model with upstream and downstream acoustic pressures and acoustic volume flow velocities as unknowns.](image)

This is in matrix formalism written as:

\[
\begin{pmatrix}
q_a \\
p_a
\end{pmatrix}
= T \cdot \begin{pmatrix}
q_b \\
p_b
\end{pmatrix}
\]

where \(q\) is the acoustic volume flow velocity and \(T\) is a 2 \(\times\) 2 matrix. In a cascade network with a source only at the inlet of the system, the downstream quantities of the first acoustic element will be the upstream quantities of the next, and since every 2-port is a linear time invariant system the quantities at the outlet of the network system can be related to those at the inlet by linear algebra:
\[
\begin{pmatrix}
    p_{in} \\
    q_{in}
\end{pmatrix}
= \prod_{n=1}^{N} T_n \begin{pmatrix}
    p_{out} \\
    q_{out}
\end{pmatrix}
\]  

(52)

This simple relation between the input and output of the total system enables fast evaluations using the model, and various acoustic quantities of the system can easily be calculated, e.g. the transmission loss and the insertion loss. However the drawback of the model is that the acoustic volume flow velocity is not readily measured. Hence \( q \) must be expressed in terms of other measurable quantities, e.g. the acoustic pressures at two different cross sections.

**Arbitrary Networks**

For more general networks with multiple connections between 2-port models and with arbitrary source arrangements, the scattering matrix model investigated in this work is better suited than the aforementioned transfer matrix formalism. R. Glav and M. Åbom [4] presented a general formalism for treating complex networks using the scattering matrix formulation of the 2-ports. The strength of their formalism apart from the generality is that no matrix inversions are necessary. A brief description of the method will be given here. Consider a network as seen in figure 6.

![Figure 6](image)

**Figure 6**  
Schematic view of a network, where the white, large circles represent 2-ports and the black, small circles represents nodes. The flow direction into each 2-port is indicated by arrows. The figure is borrowed from [4].

In this formulation, each 2-port element is connected to a joint. Each node is in turn modeled as a multiport, with the positive direction in opposite to that of the other 2-ports, see figure 7.

![Figure 7](image)

**Figure 7**  
Definitions of directions for nodes and 2-ports. The figure is borrowed from [4].
Since all ports connected to a node are 2-ports, each consisting of 2 equations and 4 unknowns, a node consists of $2m$ equations relating $4m$ unknowns, where $m$ is the number of 2-ports connected to the node. The system of equations is written as:

$$S_n^p p_n^+ = S_n^n p_n^+ + p_n^{ns} \quad (53)$$

where $p_n^+$ and $p_n^{ns}$ are $m_n \times 1$ column vectors, with elements representing wave amplitudes coming from the connected 2-ports, and $S_n^n$ are $m_n \times m_n$ matrices. The difference between (53) and the traditional scattering matrix formulation is the introduction of these two scattering matrices, one associated with each direction of propagation. At a first glance this may seem like an unnecessary complication, but it is this very convenient feature that in the end will yield a solution without using matrix inversions. Next, the unknown wave amplitudes are put into two column vectors

$$p_+^c = \begin{bmatrix} p_{+a} \\ p_{+b} \\ \vdots \\ p_{+a} \\ p_{+b} \end{bmatrix}_{M} \quad p_-^c = \begin{bmatrix} p_{-a} \\ p_{-b} \\ \vdots \\ p_{-a} \\ p_{-b} \end{bmatrix}_{M} \quad (54,55)$$

The system of equations for the complete network can then be written as:

$$p_+^c = S^c p_+^c + p_+^{cs} \quad (56)$$

where $S^c$ is the scattering matrix of the cascade network and contains the scattering matrices of the individual 2-ports:

$$S^c = \begin{bmatrix} S_1 & 0 & \ldots & 0 \\ 0 & S_n & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & S_M \end{bmatrix} \quad (57)$$

In order to combine (53) and (56) an index matrix relating global to local pressures at the nodes is required:

$$p_{n/+}^n = G^n p_{n/+}^c \quad (58)$$

where $G^n$ is the index matrix for node $n$. Using (58) in (53):

$$S_n^n G^n p_+^c = S_n^n G^n p_+^c + p_n^{ns} \quad (59)$$

It is now possible to combine (59) with (56), yielding:
\[
(S^n G^n - S^n G^n S^c) p^- = S^n G^n p^c + p^{ns}
\]  
(60)

Note that \(p^+\) has been eliminated. Then using a topology matrix containing information about which end of the 2-ports are connected to which node, an assembling process over all \(n\) nodes is written, yielding \(p^-\) without using matrix multiplications. Using \(p^-\) and (59) \(p^+\) can be solved for, and thus the complete system is known. Although more complex than the solution procedure for the cascade network, this method is still easy to apply, and the generality speaks for this method and of the scattering matrix formulation of the 2-port.

**Test Object**

The physical object represented by the acoustic 2-port element is a triangular plate, see figure 8.

![Figure 8](image)

*Figure 8*  
The test object: a triangular plate. The red square with wires attached is a strain gauge used for vibration measurements. Visible as well is the clay used for air sealing.

The angle of attack of the plate is 32 degrees, and it is supported by a plastic frame on the outside of the duct and through a slit in the duct wall, see figure 9.
Figure 9  The mounting of the plate with an attack angle of 32 degrees.

Due to the circular shape of the duct cross section the actual object is in fact a triangular plate with an oval base, which projected in the duct axis direction has the same radius at bend as the duct. For dimensions and calculations of the area see the appendix.

Test Rig

The test rig consists of three serial connected pipes, which in turn is connected to a sub-sonic wind tunnel at the upstream end, and to a large laboratory hall at the downstream end. To minimize reflections mufflers are put both at the outlet end and in the junction between the inlet end and the wind tunnel, see figures 10 and 11.

Figure 10  Schematic of the test rig.
The three pipes consist of one plastic pipe, in which the test object is placed, one upstream aluminum pipe and one downstream aluminum pipe. All pipes have an outer diameter of 100 mm and an inner diameter of 90 mm. For other dimensions see Appendix. For reference measurements an empty plastic pipe is used, identical to the test object pipe apart from the slit for the plate mounting in the latter.

**Measurement setup**

The vibration measurement setup and the measurement setup for determining the drag coefficient are standard and will not be described in detail; the vibrations of the plate are measured by a strain gauge connected to a signal conditioning amplifier which in turn is connected to a signal acquisition system. Pressure measurements are made using a digital pressure difference meter, which for the static pressure loss is connected via rubber hoses to holes at the duct wall, and for the velocity measurements to a Prandtl tube. The acoustic measurement chain is shown in figure 12.
The sound pressures inside the duct are measured using microphones “flush”-mounted at the duct wall and aimed perpendicular to the axis of the duct. The microphone signals are transferred via a short cable to a multichannel signal conditioning amplifier, which filters and amplifies the signals. The conditioning amplifier is connected via 5 m long coaxial cables to a multichannel signal acquisition system, which filters, samples and quantifies the signals. The acquisition system is connected to a pc, used for data storage and control of the acquisition system.

The loudspeakers used for the measurements of the passive properties are driven by a signal generated by the acquisition system, and amplified by a regular home electronic amplifier. Also, the signal is processed by an equalizer in order to increase the power in frequencies where the signal to noise ratio is low. The input signal to the equalizer is measured in order to form transfer functions between that signal and the sound pressures.

All post calculations such as transforming the time data to frequency spectra, calculating auto and cross spectra, as well as averaging are computed using the pc.

**Instrumentation**

The software used in order to control the signal acquisition system is the SigLab GUI. Each SigLab system has 4 inputs channels and 2 output channels, however for the acoustic measurements 2-3 systems are connected using sync cables forming a multichannel acquisition system with 12 input channels and 6 output channels. For the vibration measurements only one SigLab system is used.
**Acoustic measurement instrumentation**

Since during the work, several types of computer bugs were experienced when calculating cross spectra using 10 channels, the number of microphones used simultaneously was cut short to 8. Also, the maximum number of reference channels for cross spectra calculations possible to set in the SigLab software was 4.

Since the microphone spacing criteria are different for the active and the passive measurements, different microphones positions were used for the two cases. The serial numbers, distances from the plastic pipe and the label for the remainder of this work for all microphones used are shown in table 3.

<table>
<thead>
<tr>
<th>B &amp; K Microphones model 4938 (¼ inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upstream Microphones</strong></td>
</tr>
<tr>
<td>Label</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

Table 3  **Microphones used during the measurements.**

Since only 8 microphones were used simultaneously only 2 signal conditioning amplifiers were used, each with 4 channels, see table 4.

<table>
<thead>
<tr>
<th>B &amp; K Signal Conditioning Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial number amplifier 1</td>
</tr>
<tr>
<td>Serial number amplifier 2</td>
</tr>
</tbody>
</table>

Table 4  **Signal Conditioning Amplifiers used during measurements.**

The acquisition systems each have 4 input channels, implying that 3 systems are needed, since 8 channels are required for the microphone signals and 1 channel for the input signal to the loudspeakers, which also requires 1 output channel. The acquisition systems are listed in table 5.

<table>
<thead>
<tr>
<th>SigLab Signal Acquisition System model 20-42</th>
</tr>
</thead>
<tbody>
<tr>
<td>System 1, serial number</td>
</tr>
<tr>
<td>System 2, serial number</td>
</tr>
<tr>
<td>System 3, serial number</td>
</tr>
</tbody>
</table>

Table 5  **Signal Acquisition Systems used during measurements.**
The amplifier and equalizer used for the input signal to the loudspeakers are listed in table 6.

<table>
<thead>
<tr>
<th>Equalizer</th>
<th>Technics SH8065, serial number: MB3126B127</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifier</td>
<td>NAD Electronics LTD Model 319, serial number: G7931900102</td>
</tr>
</tbody>
</table>

**Table 6**  
*Equalizer and Amplifier used for the input signal to the loud speakers.*

The loudspeakers consists of high quality, 8 Ω speaker elements, mounted in boxes constructed by the MWL laboratory staff.

**Vibration measurement instrumentation**

The strain gauge was mounted on the plate by thin layer of tape. In order to minimize the disturbance on the flow field, the signal cable was drawn through a small slit in the mounting device of the plate. Details of the strain gauge used are listed in table 7.

<table>
<thead>
<tr>
<th>Strain Gauge SHOWA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Gauge Length</td>
</tr>
<tr>
<td>LOT Number</td>
</tr>
</tbody>
</table>

**Table 7**  
*The strain gauge used for the vibration measurements.*

The signal processing systems are shown in table 8.

<table>
<thead>
<tr>
<th>Signal processing systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain Gauge Signal Conditioner</td>
</tr>
<tr>
<td>SigLab Acquisition System 20-42</td>
</tr>
</tbody>
</table>

**Table 8**  
*Signal processing systems used for the vibration measurements.*

**Static and total pressure instrumentation**

In order to measure the flow velocity, a Prandtl tube was aimed towards the flow in the axis of the duct. The Prandtl tube was connected by short rubber hoses to an air speed probe, converting the pressures into electric signals. The signals are then processed by a digital manometer instrument, which calculates the flow velocity from the difference in static and total pressure.

<table>
<thead>
<tr>
<th>Air speed probe SWA 07</th>
<th>Serial number: 378709</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWEMA 3000 digital manometer</td>
<td>Serial number: 672319</td>
</tr>
</tbody>
</table>

**Table 9**  
*Static and total pressure instrumentation used.*

The same setup is used for pressure loss measurements, but then the Prandtl tube is replaced by two static pressure holes in the duct wall, between which the static pressure...
loss is obtained. As can be seen in the figures describing the dimensions in the test rig, see appendix, the pipe length over which the static pressure loss is measured is 3081 mm.

**Acoustic Measurement method**

First of all, it should be noted that the scattering of acoustic waves inside the duct depends on the flow field, and thus the scattering effects will differ if the flow velocity changes. This implies that the measurement procedure described in this section must be repeated for every flow velocity step investigated. The main measurement procedure is as follows: first the scattering effects of the 2-port element is measured. Then using the measured data from the scattering measurements the reflection data of the test rig can be calculated. Third the source generated sound is measured, and reflections in the test rig together with the scattering effects of the 2-port element are then taken into account.

**Scattering matrix measurement method**

Consider the equation 41, repeated here for convenience:

\[
\begin{pmatrix}
  p_{a+} \\
  p_{b+}
\end{pmatrix} = 
\begin{bmatrix}
  \rho_a & \tau_b \\
  \tau_a & \rho_b
\end{bmatrix}
\begin{pmatrix}
  p_{a-} \\
  p_{b-}
\end{pmatrix} + 
\begin{pmatrix}
  p_{a+}^s \\
  p_{b+}^s
\end{pmatrix}
\]  

(41)

The four constants in the scattering matrix is what defines the passive part, and thus during the scattering measurements these are the unknowns, implying that the pressures are the variables which should be measured. It is now assumed that during the scattering matrix measurements the source vector can be put to zero, which will result in an equation system with 2 equations and the elements in the scattering matrix as 4 unknowns. Consequently two measurements of the complex pressures are needed in order to form 4 equations. Furthermore a unique solution requires the equations to be linearly independent, which implies that the measurements must be made using two independent acoustic loads. However before going any further, consider again the equation (41). Sound will propagate in both directions at both sides of the 2-port at all times, and thus a measured sound pressure at a given point will consist of contributions from both the upstream and the downstream propagating wave. A method for resolving these contributions and thereby obtaining the pressures in the equation from the measured quantities is required. Such a method is the two-microphone wave decomposition method, which is described in numerous references.

**The two-microphone wave decomposition method**

Consider a harmonic planar wave propagating in the positive \( x \)-direction as seen in figure 13.
Then the sound pressure inside the duct can be described by

\[ p(x, t) = p_+ \exp(i\omega t - ik_+x) \]  \hspace{1cm} (61)

If the sound pressure is measured at two different cross sections at \( t = 0 \), the two measured sound pressures will be related to the amplitude of the wave via a spatial relation only:

\[ p_i(x_i) = p_+ \exp(-ik_+x_i) \]  \hspace{1cm} (62)

where the subscript \( i \) corresponds to the index of the microphones. If now a wave propagating in the negative direction is introduced, (62) will be superposed by this wave

\[ p_i = p_+ \exp(-ik_+x) + p_- \exp(ik_-x) \]  \hspace{1cm} (63)

Rearranging (63) and considering the microphone 1 gives

\[ p_+ = p_1 \exp(ik_+x_1) - p_- \exp(ix_1(k_- + k_+)) \]  \hspace{1cm} (64)

Putting this into (63) for microphone 2

\[ p_2 = (p_1 \exp(ik_+x_1) - p_- \exp(ix_1(k_- + k_+))) \exp(-ik_+x_2) + p_- \exp(ik_-x_2) \]  \hspace{1cm} (65)

Rearranging (65):

\[ p_- = \frac{p_2 \exp(ik_-x_2) - p_1 \exp(ik_+x_1)}{\exp(ix_1(k_- + k_+)) - \exp(ix_1(k_- + k_+))} \]  \hspace{1cm} (66)

In a similar way:
\[ p_+ = \frac{p_2 \exp(-ikx_2) - p_1 \exp(-ikx_1)}{\exp(-ix_k(k_+ + k_-)) - \exp(-ix_1(k_+ + k_-))} \]  

(67)

Note that the upstream pressure amplitudes \( p_{a+} \) and \( p_{a-} \) are calculated from upstream microphone data, and the downstream pressure amplitudes \( p_{b+} \) and \( p_{b-} \) from downstream microphone data. If the two-microphone method is put in matrix form, a weakness of the method is found.

\[
\begin{bmatrix}
\exp(-ikx_1) & \exp(ikx_1) \\
\exp(-ikx_2) & \exp(ikx_2)
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix} =
\begin{bmatrix}
p_+ \\
p_-
\end{bmatrix}
\]

(68)

This system is solvable only when the matrix inverse exists, which is only true when the determinant is non-zero, i.e. when

\[ \exp(i(k_-x_2 - k_+x_1)) - \exp(i(k_-x_1 - k_+x_2)) \neq 0 \]

(69)

For the case with zero mean velocity the wave numbers are equal and thus:

\[ \exp(ik(x_2 - x_1)) - \exp(-ik(x_2 - x_1)) \neq 0 \]

(70)

Implying

\[ (x_2 - x_1) \neq \frac{n\pi}{k} \quad \text{or} \quad (x_2 - x_1) \neq \frac{n\lambda}{2}, \quad n = 0, 1, 2... \]

(71)

That is, the method will fail when the microphone distance is equal to multiples of half the wave length. This can be seen as a spatial analogy to Nyquist-Shannon’s classical time sampling theorem, stating that the sampling frequency must be double the frequency of the wave in order to resolve it without errors.

**Over determination of the pressure amplitudes**

The equation (63) is obviously valid for any number of microphones. If the number of microphones is \( N > 2 \) it is possible to reformulate (68) into an over determined system of equations:

\[
\begin{bmatrix}
\exp(-ikx_1) & \exp(ikx_1) \\
\exp(-ikx_2) & \exp(ikx_2) \\
\vdots & \vdots \\
\exp(-ikx_N) & \exp(ikx_N)
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_N
\end{bmatrix} =
\begin{bmatrix}
p_+ \\
p_2 \\
\vdots \\
p_N
\end{bmatrix}
\]

(72)

The equation can be solved for using the Moore-Penrose pseudo-inverse, which is the best solution in a least square sense. This over determination method is in principle the same method as the two-microphone method, however here the first frequency where the
wave decomposition will fail corresponds to the limits achieved when putting the shortest microphone distance into (71). The reason why the frequency limit is set by the smallest distance is because as long as two rows are independent in (72), the matrix inverse will exist and thus the system will be solvable.

**Determination of the scattering matrix**

If two independent sets of data exist it is now possible to calculate the scattering matrix, setting up the equation

\[
\begin{bmatrix}
(p_{a+})^l & (p_{a+})^n \\
(p_{b+})^l & (p_{b+})^n
\end{bmatrix}
= \begin{bmatrix}
\rho_a & \tau_b \\
\tau_a & \rho_a
\end{bmatrix}
\begin{bmatrix}
(p_{a-})^l & (p_{a-})^n \\
(p_{b-})^l & (p_{b-})^n
\end{bmatrix}
\tag{73}
\]

where the superscripts \( I \) and \( II \) refers to the two independent sets of measurements. Note that (73) corresponds to 4 equations and 4 unknowns, a system which is possible to solve by hand without using any matrix inversions. The result is

\[
\rho_a = \frac{p_{a+}^l p_{b-}^n - p_{a+}^n p_{b-}^l}{p_{a+}^l p_{b-}^n - p_{a+}^n p_{b-}^l}
\]

\[
\tau_b = \frac{p_{a+}^l p_{a-}^n - p_{a+}^n p_{a-}^l}{p_{a+}^l p_{a-}^n - p_{a+}^n p_{a-}^l}
\]

\[
\tau_a = \frac{p_{b+}^l p_{b-}^n - p_{b+}^n p_{b-}^l}{p_{b+}^l p_{b-}^n - p_{b+}^n p_{b-}^l}
\]

\[
\rho_b = \frac{p_{b+}^l p_{a-}^n - p_{b+}^n p_{a-}^l}{p_{b+}^l p_{a-}^n - p_{b+}^n p_{a-}^l}
\]

Recall now that the equations above were derived for the case when the source vector was zero. Even if the source is a fan which can be turn off, in the real measurement situation the source vector is hardly zero since the presence of flow through the 2-port element will result in a contribution to the source vector. In the case of a 2-port model of the flow induced sound generation of a plate, this contribution is in fact the source vector itself. Furthermore flow induced noise will be generated from all components in the test rig. However, it is possible to suppress the source vector and the flow induced noise by the use of loudspeakers connected to the duct. These can typically be made to emit a higher sound pressure level than the flow induced sound mechanisms, and thus the effect of the source generated sound might be neglected. Another possibility is to generate an acoustic field which is uncorrelated with the source generated sound field and with the flow noise. In the present work both methods are applied simultaneously.
To use the fact that the source and the flow noise are uncorrelated to the loudspeakers, the transfer function between the pressure and the input signal to the amplifier of the loudspeakers is formed

\[ H_{lepi} = \frac{p_i}{le} \]  

(75)

where \( le \) refers to the loudspeaker electrical signal and \( i \) refers to a microphone index. In order to get rid of noise in the measured sound pressure, the transfer function estimate used is

\[ H_{lepi} = \frac{G_{lepi}}{G_{ele}} \]  

(76)

Then the pressure \( p_i \) is replaced by the corresponding transfer function. In a modern signal acquisition system these transfer functions are calculated directly and averaged over. The two independent sets of data \( I \) and \( II \) can be obtained by using two different setups of loudspeakers, for instance loudspeakers connected at the two different sides of the 2-port. Since the transfer function is a linear function, it is obvious that using two sets of loudspeaker gain levels will not yield two different transfer functions, implying that the independent sets of data \( I \) and \( II \) cannot be obtained this way.

**Over determination of the scattering matrix**

If \( N>II \) sets of independent measurements are available, the equation (73) can be reformulated into an over determined system of equations:

\[
\begin{bmatrix} p_{a+}^I & p_{a+}^H & \ldots & p_{a+}^N \\ p_{b+}^I \\ p_{b+}^H \\ \vdots \\ p_{b+}^N \end{bmatrix} = \begin{bmatrix} \rho_a & \tau_b & \rho_a & \tau_b & \ldots & \rho_a & \tau_b & \rho_a & \tau_b & \ldots \\ \rho_a & \rho_a & \rho_a & \rho_a & \ldots & \rho_a & \rho_a & \rho_a & \rho_a & \ldots \end{bmatrix} \begin{bmatrix} p_{a-}^I \\ p_{a-}^H \\ \ldots \\ p_{a-}^N \\ p_{b-}^I \\ p_{b-}^H \\ \ldots \\ p_{b-}^N \end{bmatrix}
\]  

(77)

which can be solved for using a pseudo-inverse of the matrix containing the negative propagating pressure amplitudes

\[ S = P_+ P_-^{-1} \]  

(78)

where again the inversion should be interpreted as the Moore-Penrose pseudo-inverse.

**Source cross spectrum matrix measurement method**

If the loudspeakers are turned off and the sound pressure is measured at a certain cross section in the duct, the sound pressure at that cross section will consist of flow noise, the source generated sound pressure and reflected sound waves. In order to extract the source generated sound pressure, it is necessary to calculate the contribution from reflections
and transmissions in the test rig, which means that both the scattering matrix of the 2-port element and the passive properties of the test rig itself are required. The latter are represented by reflection coefficients defined for incoming waves towards each duct end termination. In order to achieve this an expression relating the source pressure vector to measureable quantities are required. This expression can easily be derived using the equations

\[ p = p_+ + p_- \]  \hspace{1cm} (63) \\
\[ p_+ = S p_- + p_+^s \]  \hspace{1cm} (41) \\
\[ R p_+ = p_- \]  \hspace{1cm} (79)

where the matrix \( R \) relating the pressure amplitudes contains the reflection coefficients

\[
R = \begin{bmatrix}
R_a & 0 \\
0 & R_b
\end{bmatrix}
\]  \hspace{1cm} (80)

Combining (41) and (79) yields

\[
(E - SR)p_+ = p_+^s
\]  \hspace{1cm} (81)

(63) put into 79) yields

\[
p_+ = (E + R)^{-1} p
\]  \hspace{1cm} (82)

Finally combining (81) and (82) yields the sought relation:

\[
(E - SR) (E + R)^{-1} p = p_+^s
\]  \hspace{1cm} (83)

These reflection coefficients are defined as the ratio between a reflected wave to an incoming wave:

\[
R_a = \frac{p_{a-}}{p_{a+}}, \quad \text{and} \quad R_b = \frac{p_{b-}}{p_{b+}} \]  \hspace{1cm} (84)

Following the reasoning leading to the equations (44) and (45) it is now very tempting to write the reflection coefficients as the inverse of \( \rho_a \) and \( \rho_b \) respectively. However, this would lead to an erroneous result since that reasoning was only done for a test rig where for the determination of the reflection coefficient \( \rho_a \), the reflected wave \( p_{a-} \) was zero, and for \( \rho_b \), the reflected wave \( p_{b-} \) was zero. The actual \( \rho_a \) and \( \rho_b \) are calculated according to equations (7414). In order to calculate the reflection coefficients \( R_a \) and \( R_b \) using (84), it is required that \( p_a \) is the reflected wave of the incoming wave \( p_{a+} \) towards the upstream end
termination. In practice this means that for the measurement leading to the quantities $p_a$ and $p_{a+}$ from which $R_a$ is to be determined, every $p_a$ wave must be a reflection of a $p_{a+}$ wave, implying that every $p_{a+}$ wave must originate from somewhere downstream of the cross section for which $R_a$ is determined, e.g. the downstream side of the 2-port. This is achieved by only using the loudspeakers at side $b$ when calculating $R_a$ and consequently; only loudspeakers at side $a$ when calculating $R_b$. Equation (32) is therefore modified into

$$R_a = \frac{p_{a+}^{LB}}{p_{a+}^{LA}}, \quad \text{and} \quad R_b = \frac{p_{b+}^{LA}}{p_{b+}^{LB}}$$

(85)

where the superscript $LA$ and $LB$ refers to measurements made with speakers turned on only at side $a$ and $b$ respectively. If the two sets of independent data $I$ and $II$, needed in order to determine the scattering matrix are obtained by using the loudspeakers at one side a time, then those measurements can obviously be used for (85) as well.

As discussed previous in this work both the amplitude and the relative phase between the source generated sound on both sides are relevant, and this information is contained in the auto spectra and the cross spectra, represented by the source cross spectrum matrix. Introducing the notation

$$G_{aa}^s \quad G_{ab}^s \quad G_{ba}^s \quad G_{bb}^s$$

the source cross spectrum matrix can now be formed by

$$p_s^{s'} = C p (C p)^\dagger = C p p^\dagger C^\dagger = \begin{bmatrix} G_{aa}^s & G_{ab}^s \\ G_{ba}^s & G_{bb}^s \end{bmatrix} = G^s$$

(87)

where the superscript $\dagger$ refers to a transposed and complex conjugated quantity, i.e. Hermitian transpose.

**Flow noise suppression techniques**

The flow induced noise will in the active measurements be more significant than in the passive measurements, and a suppression technique must be applied. The flow noise can be considered as uncorrelated over large distances, implying that an averaged cross spectrum would be free of this type of noise source. There are two methods of using cross spectra described in the literature. The first one, which is not applied in the present work, is based upon the use of one additional, noise free microphone placed on each side of the 2-port element. Then the auto spectra are formed by

$$G_{a_1a_1} = \frac{G_{a_1} G_{a_1}^*}{G_{a_1} G_{a_1}^*}, \quad \text{and} \quad G_{b_1b_1} = \frac{G_{b_1} G_{b_1}^*}{G_{b_1} G_{b_1}^*}$$

(88)
If the distance is long enough to allow the flow noise to be uncorrelated between microphone 1 and the noise free microphone 3, then the forming of cross spectra between these microphones will suppress the flow noise. There exists various microphone components which can be used in order to reduce the flow noise measured on microphone 3, e.g. wind screens. The problem with this method is that if the reference microphone is placed at a pressure node the signal to noise ratio can be very low. This can be treated by the use of several reference microphones, which if placed intellectually would at all frequencies provide at least one reference microphone with a large signal to noise ratio. Also, since there would be a pressure maximum near the end of the duct, to avoid a pressure node the reference microphone could be placed as close as possible to the end but outside of the acoustic near field. Then the reference microphone should be placed within a quarter wave length of the duct termination, and thus in this case only one reference microphone would be needed.

Here another method is applied, which was described by Lavrentjev, Åbom and Bodén [2]. This method also attempts to suppress the flow noise by correlation techniques, and again an additional microphone is used, but in this case it is not required to be noise free. In (88) the auto spectrum at microphone 1 is formed using the auto spectra at microphone 3. Since here neither microphone is noise free, the method only use cross spectra of the measured quantities.

The basic idea is based upon the transportation of a measured quantity from one cross section to another. Since it is possible to express what the sound pressure should be at microphone 1 using the sound pressure at another microphone and the theory of plane wave propagation, it is possible to express the auto spectra of the microphone 1 signal using the cross spectra between this signal and the signal which is transported from another microphone to the position of microphone 1. If the theory is exact then the noise free cross spectra between the microphone 1 signal and the transported signal is identical to the auto spectra of a noise free microphone 1 signal.

**Transportation of quantities between cross sections**

Choosing the origin of the x-axis at the first microphone and evaluate equations (66) and (67) will yield the pressure amplitudes at the first microphone. Since the 2-port scattering matrix is defined as the relative matrix between the amplitudes at both sides, the cross sections that define the 2-port will be the position at each side where the x-axis is put to zero. Hence regardless of microphone positions used during the measurements, the 2-port can be defined for any pair of cross sections during the calculations, even though significant errors can be induced by attenuation errors over long distances. The transportation equations in matrix form can be derived using the relations

\[ p_+'' = p_+ \exp(-ik_+x'') \]
\[ p_-'' = p_- \exp(ik_-x'') \]  

(89, 90)

where \( x'' \) refers to the new coordinate, defined in the old coordinate system where \( p_+ \) and \( p_- \) was at the origin of the x-axis. Writing (89, 90) on matrix form and rearranging them yields
\[
\begin{bmatrix}
\exp(ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0 \\
0 & \exp(ik_{a-} x_a^- - ik_{b-} x_b^-)
\end{bmatrix}
\begin{bmatrix}
p_{a+}^+ \\
p_{b+}^+
\end{bmatrix}
= \begin{bmatrix}
p_{a+} \\
p_{b+}
\end{bmatrix}
\]
(91, 92)

\[
\begin{bmatrix}
\exp(-ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0 \\
0 & \exp(-ik_{a-} x_a^- - ik_{b-} x_b^-)
\end{bmatrix}
\begin{bmatrix}
p_{a-}^+ \\
p_{b-}^+
\end{bmatrix}
= \begin{bmatrix}
p_{a-} \\
p_{b-}
\end{bmatrix}
\]

Putting (91) and (92) into (41) yields

\[
\begin{bmatrix}
\exp(ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0 \\
0 & \exp(ik_{a-} x_a^- - ik_{b-} x_b^-)
\end{bmatrix}
\begin{bmatrix}
p_{a+}^+ \\
p_{b+}^+
\end{bmatrix}
= \begin{bmatrix}
p_{a+} \\
p_{b+}
\end{bmatrix}
\begin{bmatrix}
\rho_a & \tau_b \\
\tau_a & \rho_b
\end{bmatrix}
\begin{bmatrix}
0 & \exp(-ik_{a-} x_a^- - ik_{b-} x_b^-) \\
\exp(-ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0
\end{bmatrix}
\begin{bmatrix}
p_{a-}^+ \\
p_{b-}^+
\end{bmatrix}
\]

which can be slightly rearranged into

\[
\begin{bmatrix}
\rho_a & \tau_b \\
\tau_a & \rho_b
\end{bmatrix}
\begin{bmatrix}
\exp(-ik_{a-} x_a^- - ik_{b-} x_b^-) & 0 \\
0 & \exp(-ik_{a+} x_a^+ - ik_{b+} x_b^+)
\end{bmatrix}
\begin{bmatrix}
p_{a-}^+ \\
p_{b-}^+
\end{bmatrix}
\]

(93)

What is sought is the relation between the new scattering matrix relating the new pressure vectors

\[
\begin{bmatrix}
p_{a+}^+ \\
p_{b+}^+
\end{bmatrix}
= \begin{bmatrix}
\rho_a & \tau_b \\
\tau_a & \rho_b
\end{bmatrix}
\begin{bmatrix}
p_{a-}^+ \\
p_{b-}^+
\end{bmatrix}
\]

(94)

Comparing this equation with (94) it is possible to identify the relationship:

\[
S^* = T_+ S T_-^{-1}
\]

(96)

where

\[
T_+ = \begin{bmatrix}
\exp(-ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0 \\
0 & \exp(-ik_{a-} x_a^- - ik_{b-} x_b^-)
\end{bmatrix}, T_- = \begin{bmatrix}
\exp(ik_{a-} x_a^- - ik_{b-} x_b^-) & 0 \\
0 & \exp(ik_{a+} x_a^+ - ik_{b+} x_b^+)
\end{bmatrix}
\]

(97,1,2)

For these special matrices it can be noted that the inversion of the matrices can be obtained by inverting the elements separately, since

\[
T_+^{-1} = \frac{1}{\exp(-i(k_{a+} x_a^+ + k_{b+} x_b^+))}\begin{bmatrix}
\exp(-ik_{a+} x_a^+ - ik_{b+} x_b^+) & 0 \\
0 & \exp(-ik_{a-} x_a^- - ik_{b-} x_b^-)
\end{bmatrix}
\begin{bmatrix}
(T_+^{11})^{-1} & 0 \\
0 & (T_+^{22})^{-1}
\end{bmatrix}
\]

(98)
The transportation for $R$ is easily derived using (91, 92) along with (79). The result is

$$R^r = T_+^{-1}RT_.$$  \hspace{1cm} (100)

The equation (86) can now be reformulated into

$$(E - S'R^r)(E + R^r)^{-1}p^r = C^r p^r = p^r_+$$  \hspace{1cm} (101)

This equation describes the source vector calculated at the cross section where the new microphone is mounted. Transporting it back to the 2-port is simply done by

$$p^r_+ = T_+^{-1}p^r = T_+^{-1}C^r p^r$$  \hspace{1cm} (102)

Using (102), i.e. a microphone some distance away from the 2-port, along with (86), which corresponds to a microphone at the cross section of the 2-port, it is now possible to calculate (87) with two different source vectors, i.e. four variables, which can only yield cross spectra. However, the diagonal elements of the source cross spectrum matrix will just as before represent the auto spectrum of the source generated sound wave on each side. Since the auto spectrum is a real quantity, any imaginary parts in the diagonal elements could be viewed as a measure of the error in this method.

**Over determination of the source cross spectrum matrix**

When many microphone signals are available it is desirable to combine them in order to further suppress errors. The main difficulty is to formulate the matrices in a proper manner. Obviously, if we are to use a measured $p$-vector twice as long, we will need to alter the dimensions of $C$ and $T_\pm$ accordingly in order to maintain the source vector as a two-element column vector. The following derivations will for simplicity be done for two new cross sections at each side, but can be extended to any number of cross sections.

Assigning them the indices $I$ and $II$, it is possible to reformulate equation (102) into

$$\begin{bmatrix} p^r_+ \\ p^r_+ \end{bmatrix} = \begin{bmatrix} T_+^{-1}_I & T_+^{-1}_{II} \\ \end{bmatrix} \begin{bmatrix} C^r_I & 0 \\ 0 & C^r_{II} \end{bmatrix} \begin{bmatrix} p^r_I \\ p^r_{II} \end{bmatrix} = T_+^{-1}_I C^r_I p^r_I + T_+^{-1}_{II} C^r_{II} p^r_{II}$$  \hspace{1cm} (103)

Again, this is a source vector at the cross sections of the 2-port calculated using data from microphones at other cross sections. The original cross section microphone data are treated in the normal manner to form the second source vector needed

$$\left(p^r\right) = (Cp)^\dagger = \left[p^*_d C^*_{11} + p^*_b C^*_{12} \quad p^*_a C^*_{21} + p^*_b C^*_{22}\right]$$  \hspace{1cm} (104)

From this the Source Cross Spectrum Matrix can be calculated using (87)

$$G^r_{I,II} = \left(T_+^{-1}_I C^r_I p^r_I + T_+^{-1}_{II} C^r_{II} p^r_{II}\right) (Cp)^\dagger$$  \hspace{1cm} (105)
We now recognize that if we use the transformed cross sections \( I \) and \( II \), and if the reference cross section remains the same, then

\[
G_{I,II}^s = G_I^s + G_{II}^s
\]  

(106)

The conclusions of this result are that what we have done is not an over determination but in fact an additive averaging, and also; since it is possible to transform a cross section to the reference cross section, then any two \( p^s \) vectors transformed in this way can be used to form a source cross spectrum matrix, as long as the same microphone is not found in both \( p^s \) vectors, in which case an auto spectra will be formed. This averaging technique is easily applied to an arbitrary number of different cross sections. However, in order to get the actual average one needs to divide the results by \((m-1)(n-1)\), where \( m \) and \( n \) are the number of microphones used for each \( p^s \) vector.

The other way of treating data from multiple cross sections is if (102) is reformulated into

\[
C^{-1}T_+p^s = p^*
\]

(107)

Using the cross sections \( I \) and \( II \) we now write (107) as

\[
\begin{bmatrix}
C_I^{-1}T_{+I} \\
C_{II}^{-1}T_{+II}
\end{bmatrix}p^s = \begin{bmatrix}
p_I^* \\
p_{II}^*
\end{bmatrix}
\]

(108)

It is now possible to calculate the over determined problem.

\[
p^s = \begin{bmatrix}
C_I^{-1}T_{+I} \\
C_{II}^{-1}T_{+II}
\end{bmatrix}^{-1} \begin{bmatrix}
p_I^* \\
p_{II}^*
\end{bmatrix}
\]

(109)

where the inversion of the matrices is the Moore-Penrose pseudo-inverse. Using for example 4 different cross sections, calculating two different source vectors using two cross sections for each, we can now obtain the source cross spectrum matrix by

\[
G_{I,IV}^s = p_{I,II}^s(p_{III,IV}^s)^\dagger = \begin{bmatrix}
C_{III}^{-1}T_{+III} \\
C_{IV}^{-1}T_{+IV}
\end{bmatrix}^{-1} \begin{bmatrix}
p_I^* \\
p_{II}^*
\end{bmatrix} \begin{bmatrix}
C_{III}^{-1}T_{+III} \\
C_{IV}^{-1}T_{+IV}
\end{bmatrix}^{-1} \begin{bmatrix}
p_{III}^* \\
p_{IV}^*
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_I^{-1}T_{+I} \\
C_{II}^{-1}T_{+II}
\end{bmatrix}^{-1}
\]

\[
\begin{bmatrix}
G_{p^a_1p^b_1a_{III}} & G_{p^a_1p^a_1b_{III}} & G_{p^a_1p^a_1a_{IV}} & G_{p^a_1p^b_1b_{IV}} \\
G_{p^b_1p^a_1a_{III}} & G_{p^b_1p^b_1b_{III}} & G_{p^b_1p^a_1a_{IV}} & G_{p^b_1p^b_1b_{IV}} \\
G_{p^a_1p^b_1a_{III}} & G_{p^b_1p^b_1a_{III}} & G_{p^b_1p^a_1a_{IV}} & G_{p^b_1p^b_1b_{IV}} \\
G_{p^b_1p^b_1b_{III}} & G_{p^b_1p^b_1b_{III}} & G_{p^b_1p^b_1a_{IV}} & G_{p^b_1p^b_1b_{IV}}
\end{bmatrix}
\]

36
The difference between equations (110) and (105) is that now an over determined system of equations has been solved, which is based upon a least squares method. If this is an improvement of the error suppression is however not obvious. In equation (110) two matrix inversions have been used, which means that computational errors have been induced. The sensitivity of errors in the input data to the Moore-Penrose pseudo-inverse has not been investigated in the present work, leaving some uncertainties concerning the applicability of this method. However, the direct influence on the results obtained during the measurements using this method are discussed. Also, it should be noted that the resulting over determined source cross spectrum matrix should not be divided by a factor to compensate for the number of spectra used, this is done by the matrix inversions.

Sources of error – Frequency limits

The sources of error associated with the two-microphone method were systematically investigated by Bodén and Åbom for the case without flow [6], and later with a mean flow present [7]. The later work did also focus on the error induced by neglecting the attenuation of a propagating sound wave, an error increasing with the Mach number. Their work treats numerous error types which in many circumstances might have a noticeable effect on the accuracy of the measurement. Apart from the fact that the matrix inverse must exist as already mentioned, they suggest that the frequency range should be restricted by

\[
0.1\pi < \frac{k\Delta x}{\left(1 - M^2\right)} < 0.8\pi
\]

(111)

where \(\Delta x\) is the microphone separation. Using (111) for one microphone separation will result in a frequency range of about three octave bands. The standard approach with the two-microphone method is therefore to use a third microphone which yields an additional usable frequency range. For convenience the usable frequency ranges when the Mach number is zero, applied for the microphone separations used during the passive measurements are listed in table 10.

<table>
<thead>
<tr>
<th>(\Delta x) [m]</th>
<th>Frequency range [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.061</td>
<td>281 &lt; f &lt; 2250</td>
</tr>
<tr>
<td>0.122</td>
<td>141 &lt; f &lt; 1125</td>
</tr>
<tr>
<td>0.183</td>
<td>94 &lt; f &lt; 750</td>
</tr>
</tbody>
</table>

Table 10 Usable frequency range during passive calculations as function of microphone separation.

If the test rig is to be used in the complete frequency region up to the first cut-on frequency of the duct, then the maximum microphone separation when effects of attenuations, imperfect microphones and flow are neglected is determined by
\[ \Delta x \leq \frac{c}{2f_c} \]  

(112)

where \( f_c \) is the cut-on frequency of the first propagating higher order mode. The corresponding maximum value for the smallest distance is thus \( \Delta x \approx 0.853D \), which for a duct with a diameter of 9 cm yields a \( \Delta x \) of 7.7 cm, half the wave length of a sound wave propagating with the frequency \( f_{c_01} \).

As already mentioned, the method of suppressing flow noise during the source spectra measurements is based upon the idea of the flow being uncorrelated at two different microphone positions. Therefore the microphone separation should be larger than the correlation length, which is approximated by [7]

\[ \Lambda \approx \frac{M c}{f} \]  

(113)

which implies

\[ k \Delta x > 2\pi M \]  

(114)

The relation above is yet another limit to the microphone separation. Comparing it with the equation (111), it is apparent that, depending on the Mach number, the same microphone separation may not be of use for both the passive and the active case. The lower the Mach number is the greater the frequency range will be in which both relations are fulfilled. For a maximum velocity of 89 m/s and a mean flow velocity factor of 0.82 the Mach number will be 0.213. This implies that the range during the passive calculations are

\[ \frac{16.4}{f} < \Delta x < \frac{131}{f} \]  

(115)

and for uncorrelated flow fields the range is

\[ \Delta x > \frac{79}{f} \]  

(116)

Thus if the same microphones are to be used both for passive and active measurements the usable frequency range is decreased into

\[ \frac{79}{\Delta x} < f < \frac{131}{\Delta x} \]  

(117)

For this reason the same microphone positions are not used for both the passive and the active measurements. The usable frequency ranges for each Mach number and
microphone separation possible to use in the active measurements are listed in the table below.

<table>
<thead>
<tr>
<th>Δx [m]</th>
<th>0.061</th>
<th>0.122</th>
<th>0.183</th>
<th>0.246</th>
<th>0.307</th>
<th>0.368</th>
<th>0.429</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.060</td>
<td>336</td>
<td>168</td>
<td>112</td>
<td>83</td>
<td>67</td>
<td>56</td>
<td>48</td>
</tr>
<tr>
<td>0.124</td>
<td>700</td>
<td>350</td>
<td>233</td>
<td>173</td>
<td>139</td>
<td>116</td>
<td>99</td>
</tr>
<tr>
<td>0.187</td>
<td>1049</td>
<td>524</td>
<td>350</td>
<td>260</td>
<td>208</td>
<td>174</td>
<td>149</td>
</tr>
<tr>
<td>0.213</td>
<td>1197</td>
<td>598</td>
<td>399</td>
<td>297</td>
<td>238</td>
<td>198</td>
<td>170</td>
</tr>
</tbody>
</table>

Table 11 The limitations in frequency range during active measurements as function of microphone separation.

Due to the fact that it is only possible to use 4 channels as references and that to avoid system crashes the total number of channels is 8 at maximum, the number of distances is narrowed into 3. The microphones chosen as reference channels during the active measurements are 1, 3, 8 and 10, and the other microphones used are 4, 5, 6 and 7, resulting in the three distances 0.429, 0.307 and 0.061 m. For calculations of the source cross spectrum matrix where only one distance is used, the microphones 1, 5, 6 and 10 are used, while for the over-determination all microphones are used in the usable frequency range given in the table above.

Choosing the 2-port element cross sections
The choice of the 2-port element cross sections depends on the application of the model and on the increase in error per length of transportation distance used. Due to the fact that the impact of the error in the wave number will increase with the length of the distance to a reflection, calculations based upon measured data in long test rigs will be more sensitive to this error. In the case of a test rig constructed out of different materials, the wave number will be different at different sections of the rig, adding to the difficulty in estimating this error. In the present work the influence on the error of the choice of cross section has not been studied, instead only the criterion of the application of the model is considered.

Since the mixer plate is placed upstream of a mixing section in an exhaust system, it may be proper to transport all quantities to the cross sections of this mixer section if their positions are known and if the mixer section is of the same pipe material and dimensions as the test rig. The reason for this is that any network analysis containing the 2-port element of the present work will then in the industrial application be using these cross sections. However, when this is not fulfilled, even if the materials and cross section dimension of the test rig and the real application are the same, the 2-port must be recalculated if any of the distances from the plate to the cross sections in the real mixer section are shorter than those already calculated. On the other hand, if the calculated distances to the cross sections are shorter than the real case, straight pipe 2-port elements
can be added in the network to make up for the difference. This implies that for the versatility of the 2-port, the shorter the length of the calculated 2-port the better.

In addition, since the plate has a length extension, questions arise whether to put the cross sections at a small duct length containing the plate, or to calculate all quantities to one single cross section. In the present work the 2-port is defined at one single cross section at the mounting of the plate. The transportation distances from each microphone to the plate are listed in Table 12.

<table>
<thead>
<tr>
<th>Upstream Mic Label</th>
<th>Distance to Plate [mm]</th>
<th>Downstream Mic Label</th>
<th>Distance to Plate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1562</td>
<td>6</td>
<td>1724</td>
</tr>
<tr>
<td>2</td>
<td>1501</td>
<td>7</td>
<td>1970</td>
</tr>
<tr>
<td>3</td>
<td>1439</td>
<td>8</td>
<td>2031</td>
</tr>
<tr>
<td>4</td>
<td>1378</td>
<td>9</td>
<td>2093</td>
</tr>
<tr>
<td>5</td>
<td>1132</td>
<td>10</td>
<td>2154</td>
</tr>
</tbody>
</table>

Table 12  Transportation distances used in the present work.

**Calibration**

The calibration in the passive acoustic measurements is made using a calibration tube, consisting of a straight steel pipe with a circular cross section containing a loudspeaker. Since the calibration tube has a smaller radius than the test rig, the sound field in the tube will only consist of plane waves in the plane wave frequency region of the test rig. Up to eight microphones are mounted at the same cross section, i.e. equally spaced throughout the circumference of the tube, which implies that if the amplitude and phase sensitivities and the complete measurement chain are identical for each microphone, the responses are also identical. Since the passive quantities, i.e. the reflection coefficients and the scattering matrix are relative quantities, only the relative differences between the microphones needs to be taken into account. In order to calibrate for the differences the transfer functions between one microphone chosen as a reference and the other microphones are formed

\[ p_r H_{p_r,p_i} = p_i \]  

(118)

where the transfer function estimate is formed by

\[ H_{p_r,p_i} = \frac{G_{p_r,p_i}}{G_{p_r,p_r}} \]  

(119)

Note that this is the transfer function of a system with \( p_r \) as input and \( p_i \) as output. Apparently the noise on the reference microphone will not be suppressed by a method of correlation since the auto spectra of the reference microphone is used for the transfer function estimate, however a few hundred averages will be enough to suppress the error.
below any significant level. This is not necessarily true however if there are constant disturbances present, e.g. electro-magnetic interference.

The microphone cross spectra used to form the transfer functions between the loudspeaker signal and the sound pressures during the measurements are then divided by the corresponding transfer function from the calibration

$$G_{lepl}^{\text{cal}} = \frac{G_{lepl}}{H_{p_r p_i}}$$

(120)

which implies that the calibrated transfer functions are formed by

$$H_{lepl}^{\text{cal}} = \frac{H_{lepl}}{H_{p_r p_i}}$$

(121)

For the active results the absolute value of the amplitude is of interest, implying that both a relative and an absolute calibration must be made. In the source measurements the cross spectra between two microphones are calculated and averaged over, and hence two calibration factors must be used for the relative calibration. Thus the relative calibration is made by

$$G_{p_i p_j}^{\text{cal}} = p_{i}^{\text{cal}}(p_{j}^{\text{cal}})^* = \frac{G_{p_i p_j}}{H_{p_r p_i}(H_{p_r p_j})^*}$$

(122)

For the absolute calibration information about the absolute sensitivity of the microphones are required. This information can be measured by the use of a calibration piston, constructed to generate a sine wave at a certain frequency and with such an energy content that when a microphone is mounted to the piston the sound pressure level in that frequency is known. For instance, the piston used in this work yields a sound pressure level of 94 dB (1 Pa) at 1 kHz. When a microphone is connected to the calibration piston, the read output value will depend on the sensitivity of the microphone, the attenuation of the signal in the system and on the amplification set in the signal conditioning system. The microphone sensitivity is also set in this system, and can thus be adjusted so that the read output displays a value corresponding to the set signal amplification. For instance, in this work the amplification was 100 mV/Pa during the calibration, and 3.16 mV/Pa during the measurements, implying that for a sound level of 94 dB during both situations, the read output should be 100 mV during the calibration and 3.16 mV during the measurements. To calibrate the results it is then simply a matter of defining the calibration factor

$$K = K_{\text{meas}}$$

(123)
which, again in this work is $3.16 \, mV$. The total calibration in phase and absolute sensitivity for the cross spectra measured is thus

\[
G_{total}^{cal} = G_{cal}^{cal} \Rightarrow G_{pp;pp_{total}}^{cal} = \frac{G_{pp_{pj}}}{K^2 H_{pr;pr_{pj}}}.
\]  

(124)

It should be noted that this method of using an absolute sensitivity factor which is constant in frequency is only applicable in the frequency range, where the sensitivity within an acceptable error is equal to that at $1 \, kHz$.

**Vibration measurement method**

For the purpose of analyzing the acoustic dependence of the stiffness of the mixer, it is of interest to measure the vibration level of the different plate thicknesses at different flow speeds. Here the choice of sensor is the strain gauge, since it will have the least effect on the fluid-structure interaction as well as the lowest weight of all non-optic sensors available. For simplicity, only one strain gauge is used and thus the vibration level will only be measured at one point on the plate. If there is a node in the mode shape for a given eigen-frequency at the strain gauge position this means that that mode will not show up in the results. However it is considered not likely that such a mode shape will exist in the frequency range of interest.

The main aspect of the vibration levels which is of interest is any presence of high vibration energy content which travels upwards in frequency with an increasing flow velocity, i.e. forced excitation from flow generated mechanisms. Also the relative vibration content per frequency between the different plates is of interest since it might yield important and possibly qualitative conclusions concerning the controllable effect on the fluid-structure interaction by varying the plate thickness.

Since it is the vibrations and not the static deflection of the plate which is of interest, it is the dynamic strain which is measured. Thus for each velocity step the static strain is set as the strain equilibrium level.

**Drag coefficient measurement method**

When comparing various measurements and simulations made with different dimensions and flow speeds, non-dimensional factors are of great importance. Such a factor is the Reynolds number, and another factor is the drag coefficient, $c_D$. For the possibility to compare the present work with future simulations the drag coefficient is determined using pressure loss data simultaneously obtained from the acoustic measurements. In a duct the drag coefficient over an object is given by
\[ c_D = \frac{\Delta p S_{\text{duct}}}{\frac{1}{2} \rho u^2 S_{\text{proj}}} \]  

(125)

where \( S_{\text{proj}} \) is the projected area of the mixer plate on the cross section area, \( S_{\text{duct}} \) is the cross section area of the duct, and \( \Delta p \) is the static pressure loss over the object. The static pressure is measured by connecting a pressure gauge to a hole in the duct wall on each side of the object. However even in a straight pipe there is a significant pressure loss, and thus the pressure loss is measured for the empty pipe in order to eliminate it from the pressure loss measured over the test object. This procedure only works if both sets of measurements are made at the same flow velocity. Since the pressure loss of the object will be different than that of an empty pipe, the flow velocity will not be the same for a given operating power of the fan, resulting in difficulties making measurements at exact flow velocities. For this reason, a function describing the pressure loss of the empty pipe is interpolated from the measurements at a few flow velocity increments. To start with the pressure loss is assumed to be due to shear stresses at the wall.

\[ \Delta p S_{\text{duct}} - \tau_w \pi D \Delta L = 0 \Rightarrow \tau_w = \frac{\Delta p D}{4\Delta L} \]  

(126)

where \( \tau_w \) is the wall shear stress and \( \Delta L \) is the distance in the x-direction between the pressure holes used for measuring \( \Delta p \). It is common to express the wall shear stress as a non-dimensional coefficient, e.g. the friction factor.

\[ c_f = \frac{\tau_w}{\frac{1}{2} \rho u^2} = \frac{\Delta p D}{2\rho u^2 \Delta L} \]  

(127)

There exists various empirical models for theoretical determination of the friction factor \( c_f \) which all agree well with experiments at, depending on the model, specific ranges of the Reynolds number. If there is an interest in a comparison between the measured friction factor and the one obtained using one of these models, the friction factor of the empty pipe can easily be obtained from the pressure loss data using the equation above. However, it should be noted that the test rig section over which the pressure loss has been measured consists of both aluminum and plastic pipes. The obtained friction factor is thus not truly representative for either of the pipes, leading to possible deviations from empiric models found in the literature.

In the present work, the mean value of the friction factors obtained from the pressure loss measurements at various velocities for the empty pipe is calculated, and then used in order to obtain the pressure loss exactly at the velocities used for the static pressure losses over the plates. The final drag coefficients of the various plates are then obtained by subtracting the calculated pressure loss of the empty pipe from the measured pressure losses of the plates.
Measurements and Results

Ideally the passive and active measurements for all plates and the empty pipe should be carried out at the same velocities and temperatures. Due to the fact that the controller of the frequency converter only allowed integer steps of the percentage of the maximum power, it was not possible to adjust the flow velocity to a precise value. Also, due to the turbulence, the velocity measured pended in a range of about 1 % of the total velocity. Since the pressure loss over each plate differed, the frequency converter had to be set at different percentages in order to achieve similar velocities for all plates. Due to this the flow velocities vary in a range of about $\pm 1 \text{ m/s}$ between the test objects at the highest velocity, and gradually less as the velocity is decreased.

First a measurement series were conducted aiming at generating all data needed for the calculations. However, due to poor active results, a new series of measurements were carried out in order to find and eliminate the error sources. After some modifications of the test rig, measurements of the reflection coefficients and the active part were finally conducted. The frequency resolution was set to 1.25 Hz for all acoustic and strain measurement data, and the excitation signal for all passive acoustic measurements was white noise. Unless differently stated, all two-port quantities are transported to the cross section at which the plate enters the duct.

Original test rig measurements and results

For each of the plates and also for the empty pipe three independent passive measurements and one active measurement were conducted at each of the axial velocities 25, 52, 78 and 89 m/s, corresponding to the Mach numbers 0.060, 0.124, 0.187 and 0.213, based on the mean flow velocity. Also, in order to investigate the influence of the number of microphones and independent measurements on the error suppression, 6 independent passive measurements were conducted for the empty pipe with a zero flow velocity. For all passive measurements the microphones 1-4 on the upstream side and 7-10 on the downstream side are used, see table 3. For the active measurements microphones 2 and 9 are replaced by microphones 5 and 6.

Zero velocity in an empty pipe

For the no flow case the number of averages was set to 800, which at zero flow velocity should be more than enough. However the coherence function between each microphone and the loudspeaker signal is still poor, see figure 14.
This coherence problem was not possible to overcome by any means without doing a thorough investigation of the complete system, for which there was no time. Instead effort was made trying to solve the problem by adjusting the gain of the amplifier and by using an equalizer, however only slight improvements were seen. Nevertheless the measurements were carried through with the hope that the new noise suppression techniques would to a great extent reduce the errors in the results. Poor coherence is thus the case for all measurements conducted in the present work.

In order to study the effect of over-determination of the scattering matrix from obtained pressure amplitudes, three scattering matrices were calculated using an over-determination of the pressure amplitudes with 4 microphones on each side and 2, 3 and 6 independent measurements respectively. The effect of over-determination of the pressure amplitudes is studied by comparing the results obtained using an over-determination and those obtained using the standard two-microphone method, both with data from 4 microphones available. The effect of increasing the number of independent measurements are shown for the magnitude of element (2,2) and the phase of element (2,1) in figures 15 and 16, while the comparison between the methods of determining the pressure amplitudes is shown for the magnitude of element (1,1), and the phase of element (1,2) in figures 17 and 18.

**Figure 14**  Coherence functions between the loudspeaker signal and the microphones for one of the measurements.
Figure 15  Error suppression in the magnitude of $S_{22}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.

Figure 16  Error suppression in the phase of $S_{21}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.
Figure 17  *Error suppression in the magnitude of $S_{11}$ using 4 microphones on each side and 6 independent measurements. Difference between the 2-microphone method and the method of over-determined pressure amplitudes.*

Figure 18  *Error suppression in the phase of $S_{12}$ using 4 microphones on each side and 6 independent measurements. Difference between the 2-microphone method and the method of over-determined pressure amplitudes.*
As can be seen huge improvements can be made by applying the over-determination of the pressure amplitudes and of the scattering matrix. Also, since the phase and magnitude are fairly constant in frequency, disregarding the unphysical ripple, it seems as though the appropriate lengths and wave numbers have been chosen.

**Passive results with flow**

Although it seems as if great improvements can be made by using 6 independent measurements over 3, the number for all passive 2-port measurements made with flow is set to the latter due to time shortage. The method of over-determining the pressure amplitudes is also used with 4 microphones on each side of the 2-port.

The main reason for measuring the empty pipe with flow is to obtain a reference result of the test rig and the methods in order to determine the actual effects of the plate. It is also possible to study the influence of errors in various parameters since the results in the empty pipe are known from theory. One parameter which is studied more closely is the ratio between the mean and maximum flow velocity inside the duct. The effect of slightly varying this parameter is seen by studying the magnitude of element $(2,2)$ and the phase of element $(2,1)$, see figures 19 and 20.

![Figure 19](image.png)

**Figure 19**  *The magnitude of $S_{22}$ for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.*
This result clearly shows that choosing the correct Mach number is very important when the scattering effects of the 2-port are considered. However, in the sources of error section it was stated that the influence of errors in the wave number on the results, due to e.g. errors in the Mach number increases with the transportation length. To visualize this the phase of element $(2,1)$ is shown in figure 21, where the quantity is calculated at the cross sections of the microphones which are closest to the plate on each side.
Figure 21  
*The phase of $S_{21}$ at the cross sections of the closest microphones, for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.*

From figure 119 it is seen that the curves representing the different mean velocity correction factors are almost identical. The periodic phase shifts are due to the wave propagation over the length of the two-port, which now is over 3 m. Unfortunately the application demands a two-port of a very short length – or preferably – no length at all. Consequently the quality of the scattering matrices will suffer from the long transportation, however the results are still interesting. The magnitude of element $(2,2)$ is shown at the Mach numbers 0.060 and 0.213 in figures 22 and 23, and the phase of element $(2,1)$ is shown at the same Mach numbers in figures 24 and 25.
Figure 22  The magnitude of $S_{22}$ at Mach 0.060 for each of the plate thicknesses and for the empty pipe.

Figure 23  The magnitude of $S_{22}$ at Mach 0.213 for each of the plate thicknesses and for the empty pipe.
Figure 24  The phase of $S_{21}$ at Mach 0.060 for each of the plate thicknesses and for the empty pipe.

Figure 25  The phase of $S_{21}$ at Mach 0.213 for each of the plate thicknesses and for the empty pipe.
It seems from these results that the influence of the plate is very small. The deviations in phase seen in figure 25 are assumed to be due to slight variations in the Mach number during the measurement. As can be seen in the results the data is very noise contaminated in general and especially below 250 Hz and above 1200 Hz for the highest Mach number, and below 50 Hz and above 1500 Hz for the lowest Mach number. These limits can also be seen when the coherence functions are considered, which are shown for Mach 0.213 in figure 26.

![The coherence function between the loudspeaker signal and the microphones at Mach 0.213, from the 3 mm plate measurements using all 6 loud speakers.](image)

Since the transmission loss of the plates might be of interest in the real application, it is shown in for the highest Mach number in figure 27.
Figure 27 The transmission loss over plate at Mach 0.213. For the empty pipe the transmission any deviation from zero is erroneous.

It is here very clear that the plate’s influence on the passive properties of a duct is very small.

**Erroneous active results**

The active data was measured for all four Mach numbers using 1000 averages, and using the passive data, i.e. the scattering matrix and the reflection coefficients the source cross spectrum matrix for each case was obtained. The difference between the 3 mm plate and the empty pipe for the highest Mach number is shown in figure 28.
It is obvious from this result that the dominating sound measured is present in the empty pipe, and thus not generated by the plate. This result is troublesome since it means that the influence of the plate is hard to study. The same characteristic source spectra are seen for all velocities and plates. However, the spectrum of the empty pipe should not look this way if it was completely smooth, implying that there are source mechanisms which are associated with an erroneous constructed test rig. The dominating peaks for instance cannot be generated by the wall turbulence alone. In order to see the influence of the Mach number on these peaks, some of the cross spectra measured in the empty pipe are shown for the four Mach numbers in the figures 29-32.

Figure 28  Differences in element (1,1) of the source cross spectrum matrix between the 3 mm plate and the empty pipe at Mach 0.213.
Figure 29  Cross spectra measured in the empty pipe at Mach 0.060.

Figure 30  Cross spectra measured in the empty pipe at Mach 0.124.
Figure 31  Cross spectra measured in the empty pipe at Mach 0.187.

Figure 32  Cross spectra measured in the empty pipe at Mach 0.213.

Studying these figures it is seen that the peaks are traveling upwards in frequency for increasing flow velocities.
**Error source identification**

Finding and eliminating the source of the peaks seen during the measurements are imperative since no conclusions can be drawn from the source spectra measured with the peaks present. A possible explanation for the peaks could be electromagnetic interference from the frequency converter supplying the fan in the wind tunnel with a direct current. This interference would indeed scale in frequency with the velocity, but it is regarded as unlikely that this is the source of the error since no problem of the kind has been experienced working with the wind tunnel in the past. This also suggests that the peaks are not multiples of the blade passing frequency of the fan.

Another possible source investigated was any flow induced noise due to a crude connection between the downstream termination of the duct and the muffler used to eliminate reflections. To test this measurements with the empty test section were carried out with and without the muffler connected at the axial velocity $46 \text{ m/s}$, see figure 33.

![Figure 33](image_url)  
*Cross spectra with and without the muffler connected to the downstream end termination of the loudspeaker, both measured at the axial flow velocity $46 \text{ m/s}$.*

As can be seen from the result the muffler does not cause these peaks. The influence of the side branches connecting the loudspeakers to the duct were then investigated. The probability of there being a flow separation process is very likely since such a mechanism would generate peaks which would indeed scale in frequency with the velocity. The introduction of an opening in the duct wall will cause the periodic shedding of vortices which propagate downstream until they encounter the other end of the opening, see figure 34.
When a vortex impacts on the downstream side of the wall opening a pressure pulse will be generated, which propagate upstream to hit the upstream side of the opening and a new vortex shedding will occur. The frequencies at which the feedback loop will be present can be roughly estimated by the use of the Rossiter formula [10]

\[
f_{St,n} = \left( \frac{n - \Phi_r/2\pi}{1/\eta + M} \right) \frac{u}{d}
\]

where \( n \) corresponds to a multiple of the resonance frequency, \( \Phi_r \) is a phase angle associated with the time delays, \( d \) the length of the wall opening and \( \eta \) is the ratio between the convection speed of the vortex and the free stream mean velocity. The inner diameter of the side branches connecting the loudspeakers is 90 mm. The uncertainty in the estimation lies in the values of the parameters \( \Phi_r \) and \( \eta \). For the case of a cavity in a wall the constants are:

\[
\eta \approx 0.4 \\
\Phi_r \approx \frac{\pi}{2}
\]

In order to draw some conclusions regarding the existence of this whistling phenomenon the resonance frequency as a function of the mean flow velocity for \( n = 1,2,3,4,5 \) has been plotted in figure 35.
Comparing figure 35 with figures 29-32, it is apparent that the 2nd and 3rd resonance frequencies might be the cause of the erroneous peaks. The deviations from the theory are assumed to be due to the uncertainty in the parameters. Further, the wall opening is in fact a perforation and not a true wall opening, see figures 36 and 37.

**Figure 35**  The resonance frequency as a function of the Mach number. The lines corresponds to the 1st resonance and its multiples.

**Figure 36**  The perforation at the side branches connecting the loudspeakers to the test rig.
The perforations were drilled out in an array of circular holes in the duct walls to minimize the flow disturbance.

The assumption of a wall opening could be worse in some frequency bands than in others, which might explain the absence of the 1st resonance in the measurements, present in the model. To test whether the side branches connecting the loudspeakers could be the source of the resonances, the loudspeakers were removed and all perforations were covered with duct tape, see figure 38.

The perforations covered with duct tape.

It was considered sufficient in this investigation to measure the cross spectra between different microphones without taking any kind of reflection or transmission into account. The cross spectra between microphones 1, 2, 3 and 4 were measured with all perforations taped for the reference pipe. The measurements were done with 1000 averages at the Mach numbers 0.124 and 0.213. The results are shown in figures 39 and 40,
Figure 39  Cross spectra measured with all perforations taped at Mach 0.124.

There are no visible resonances in the figures. The conclusion is thus that there is a resonant phenomena occurring at the perforations, which the tape cover seems to
suppress completely. If there are any resonant mechanisms still acting at the perforations, they are heavily damped and their contribution to the total sound level negligible.

**Test rig modifications**

Since a duct tape cover of the perforations at the loudspeaker side branches appeared to completely eliminate the erroneous peaks, an investigation on the possibility to use the reflection data from the original test rig for this crudely fixed test rig was made. As a quick check of the differences in the scattering properties of the modified rig, the reflection coefficient towards the upstream duct termination $R_a$ was measured and compared with that of the old rig. During the new measurements the tape was removed from the side branches connecting the loud speakers at side $b$, whilst on side $a$ the perforations were still taped. The measurements were made with the empty pipe at the Mach numbers $0.060$ and $0.124$, and the result was compared with the reflection coefficients calculated from the reference pipe measurements of the unmodified test rig. The result at the Mach number $0.124$ calculated at the cross sections closest microphones are shown in figure 41.

![Figure 41](image)

**Figure 41**  
Real part of $R_a$ for the original and the taped test rig at Mach 0.124.

It is evident from these results that the reflection coefficient changes dramatically when the covering tape is applied. The frequency interval of the peaks from the original test rig is at Mach $0.124$ approximately $140 \text{ Hz}$, whilst for the taped rig it is about $60 \text{ Hz}$. For an end reflection corresponding to a discontinuous infinite impedance change, the peaks at a certain cross section would appear at multiples of half the wave length. Writing the wave numbers for these frequencies and neglecting the effect of convection yields
\[
\lambda = \frac{c}{f} = \begin{cases} 
\frac{343}{60} & \Rightarrow \frac{\lambda}{2} \approx 2.86 \\
\frac{343}{140} & \approx 1.23 
\end{cases}
\]  

(130)

It is not possible to be precise about the reflection of the upstream end termination of the test rig, however the distance to the end of the steel pipe from the cross section at which the reflection coefficients are calculated is 2.8 m, which is fairly close to the value calculated above. The wave length calculated above for 140 Hz suggests that an area discontinuity appears at 1.23 m upstream of the cross section. The distance to the first loud speaker side branch is from that cross section 1.20 m. Therefore the conclusion is that the side branches were reflecting the sound waves in the original test rig, and covering them with tape removes this effect, implying that old reflection coefficients are not usable for active measurements in a taped rig.

Following this reasoning an alternative to the tape was investigated. A plastic foam was cut in circular discs of 5 mm and mounted to cover the perforations, see figure 42.

![Figure 42](image)

**Figure 42**  Plastic foam of 5 mm thickness attached on to the perforations inside the side branches.

As a first step, the influence of the foam on the whistling tones was investigated by covering the perforations on side \( b \) with foam, and those on side \( a \) with tape, and using microphone positions 1-4 to measure the cross spectra of the sound pressure. This was done at the Mach number 0.124, with 400 averages, and for foam thicknesses of 5 and 10 mm. The results is presented in figure 43.
The foam does indeed to a large extent reduce the level of the whistling tones. However, the tones are still appearing, which can be a problem if the generated sound level of the plate is of the same order. No significant improvement can be seen when the foam thickness is increased. Whether this modification of the rig is sufficient for this work is difficult to say, but a criterion which must be fulfilled is that the reflection coefficient has not changed significantly from the original test rig. To validate this, the perforations on both sides were covered with foam of 10 mm thickness and the reflection coefficient $R_a$ were measured at Mach 0.124 using 1000 averages. The difference between this reflection coefficient and the original one is shown in figure 44.

![Figure 43](image1.png)

**Figure 43**  *The effect on the whistling tones of applied foam thicknesses of 5 and 10 mm at Mach 0.124.*

![Figure 44](image2.png)

**Figure 44**  *Difference in real part of $R_a$ at Mach 0.124, with and without a cover of 10 mm thick foam attached on the perforations.*
Studying the differences in $R_a$ suggests that this modification might be applicable, at Mach 0.124 at least, providing that the generated sound pressure level of the plate is significantly higher than the level of the whistling tones.

As an alternative measurement process, it was suggested that the reflection coefficients from the original test rig were to be completely discarded, and the taped test rig were to be used for active measurements, with new reflection coefficients for this rig. To reduce the measurement time by days it was also suggested that the reflection coefficients could be considered independent on whether it was the reference pipe or a plate mounted at the rig. For the case with zero velocity this is completely true since the reflections depends on the properties of the test rig downstream or upstream of the 2-port element. However it could be argued that inserting an object inside the duct would severely effect the flow and thus changing the reflective properties towards both end terminations. To investigate this the reflection coefficients for the reference pipe and for the 1.5 mm object were plotted, both calculated from the measurements of the original test rig at Mach 0.213. The results are shown in figures 45.

![Figure 45](image.png)

**Figure 45**   *The differences of the real part of $R_a$ for measurements with the reference duct and the 1.5 mm plate at Mach 0.213.*

It is clearly seen in the figures that the agreement between the two sets of measurements is good in the frequency region 50-1200 Hz. Below and above this region measured data is noisy in general, and it cannot be expected that the reflection coefficients would be equal at those frequencies. In the light of these results, the applied approach is to assume that the reflection coefficients towards the end terminations are unaffected by placing a plate in the 2-port element. The old reflection coefficients are thus discarded and new
ones calculated from measurements of the taped rig with no plate inside are used for the final results.

**Final reflection coefficients and active results**

For the active measurements using the modified test rig, the number of averages are 4000, while for the reflection coefficients the number is 1000. The source cross spectrum matrix consists of 4 elements per frequency point, but in order to save space in this report only figures discussed are shown in the results section, for a complete set of figures see the appendix.

The reflection coefficients measured and transported to the plate mounting are for the Mach number 0.213 shown in figure 46.

![Figure 46](Image)

*Figure 46  The magnitude of the transported reflection coefficients at the Mach number 0.213.*

As can be seen in the figure the downstream end termination is much more reflective than the upstream end. This is to be expected since, apart from the mufflers, the downstream pipe ends in a large room while the upstream pipe ends in a series of continuously expanding pipes.

In order to investigate whether the over-determination of the source cross spectrum matrix improves the results, plots of results obtained using the over-determination versus the regular method have been studied. Since the differences are about the same for all cases, only two are presented here. Figure 47 shows the diagonal elements of the source cross spectrum matrix of the 3 mm plate at the Mach number 0.213, and figure 48 shows the element (2,2) of the 0.5 mm plate at the Mach number 0.060.
Figure 47  The effect of using over-determination of the source cross spectrum matrix of the 3 mm plate at the Mach number 0.213.

Figure 48  The effect of using over-determination of the source cross spectrum matrix of the 0.5 mm plate at the Mach number 0.060.

Figure 47 indicates that the over-determination yields no significant improvement, while figure 48 suggests that the over-determination smoothen the curve as one would expect.
using an averaging method. Studying comparisons made of other cases suggests that the effect of the over-determination is in fact a smoothening of the curve at low velocities, i.e. low sound levels, and that the over-determination has little or no effect at the highest velocity. For the Mach number 0.187 somewhat clearer results are seen. Figure 49 shows the element (2,2) for the 1.5 mm plate at this Mach number.

![Figure 49](image)

**Figure 49**  *The effect of using over-determination of the source cross spectrum matrix of the 1.5 mm plate at the Mach number 0.187.*

At this Mach number originally two peaks existed about 40 Hz apart with a shallow valley in between, and in addition a peak is seen at approximately 1150 Hz. After the over-determination however the results if very different. In light of this clear improvement of the results, the over-determination is used for active results presented in the remainder of the report.

The effect of transportation can be studied from the active results. Figure 50 shows the different elements (1,1) for the 3 mm plate at the Mach number 0.187 obtained with and without the transportation.
Figure 50  Differences in the element \((1,1)\) of the source cross spectrum matrix for the 3 mm plate at the Mach number 0.187 for the cases with and without transportation.

As suspected the results are almost identical. The only notable difference is that the transported element is about 0.2 dB higher, which is due to acoustic dissipation during the transportation length. For the remainder of the report all results presented are transported to the cross section at which the plate enters the duct. The differences in sound pressure level between the plates are in figures 51-54 shown for the four Mach numbers and the element \((1,1)\).
Figure 51  Differences in sound pressure level between the 4 configurations at the Mach number 0.060.

Figure 52  Differences in sound pressure level between the 4 configurations at the Mach number 0.124.
Figure 53  Differences in sound pressure level between the 4 configurations at the Mach number 0.187.

Figure 54  Differences in sound pressure level between the 4 configurations at the Mach number 0.213.
From these results it is clearly seen that the sound generated by the 0.5 mm plate is lower than that of the other plate thicknesses. It is also evident that at Mach 0.213 the plates generates a sound pressure level as much as 15 dB higher at some frequencies, than that of the empty pipe. Apart from the noisy characteristic of the sound spectrum of the empty pipe, the differences between the empty pipe and the plates seem to be a highly damped peak which for Mach 0.213 is found at approximately 440 Hz. Also, especially at high Mach numbers for the thicker plates, there exists smaller periodically occurring peaks, or ripple, which deserves special attention.

In order to investigate the character of the sound generated, the source cross spectrum elements (1,1) and (2,2) for the 3 mm plate are in figure 55 shown for all four Mach numbers and plotted versus the Strouhal number.

![Figure 55](image)

**Figure 55** Elements (1,1) and (2,2) of the 3 mm plate as function of the Strouhal number, for the different Mach numbers.

As can be seen in the figure the peak of the curves coalesce approximately at the Strouhal number 1.2. This strongly suggests that the sound is generated by mechanisms associated with the flow. In order to further analyze the characteristics of the mechanisms the element (1,1) above is plotted again but with the sound pressure divided by the velocity to the power of 4, where the results have been smoothed in order to better distinguish the magnitude.
The results now coalesce in magnitude as well for the three highest Mach numbers, while the lowest Mach number source spectrum is about 2 dB higher. The reason for this deviation might be that there are other mechanisms dominating at low Mach numbers, but it can also be due an erroneous flow velocity. Using a scaling law with the power of 4, 2dB corresponds to an error of about 12 % in the flow velocity, which for the axial flow velocity of 25 m/s, i.e. Mach 0.060 is about 2-3 m/s. It is possible that for low Mach numbers the differences in the flow velocity profile over a cross section downstream of the mixer plate is very different from a cross section upstream due to the efficiency of the mixer, while for large Mach numbers the mixer has less effect on the profile. However the reason, figure 56 strongly suggests that the sound source is of the dipole type.

In order to investigate the ripple, the element \((1,1)\) is plotted for the Mach number 0.213 for all three plates, where some frequency ranges have been cut out, see figure 57.
Figure 57  Element (1,1) of the source cross spectrum matrix of the three plates at Mach 0.213. Note that some of the frequency ranges are cut out.

There appears to be two periods of the peaks, one at approximately 45 Hz and one at 20 - 25 Hz. It is also seen that the lower frequency periodicity decreases at higher frequencies. One possible cause of the ripple is that some resonances in the system are excited. This idea is backed up by the fact that the ripple are seen at every magnitude level of the sound spectra. If the ripple was produced by an external sound source it seems very unlikely that the spectrum from that sound should at all frequencies be within 2 dB of the spectrum of the sound generated by the mixer plate. A quick approximation of the resonances of the air volume in a pipe can in the plane wave region be expressed as multiples of the first resonance, which has a wave length of twice the pipe length. This implies for the two frequencies

\[
\begin{align*}
\left\{ f = 22.5 \text{ Hz} \right\} & \Rightarrow \frac{\lambda}{2} = \frac{343}{2 \cdot 22.5} = 7.62 \text{ m} \\
\left\{ f = 45 \text{ Hz} \right\} & \Rightarrow \frac{\lambda}{2} = \frac{343}{2 \cdot 45} = 3.81 \text{ m}
\end{align*}
\]

The total length of the three pipes is 7.47 m, but then one muffler is connected to the pipe at each end termination, and on the upstream side of the upstream muffler there are various conically expanding elements which finally connects to the wind tunnel. It is as previously discussed in this report impossible to be very precise about the acoustic length of the system, however, it could be in the close vicinity of 7.62 m. The length of the pipe from the upstream muffler to the mounting of the plate is 3.87 m, which again is very close to 3.81 m. The fact that the lower frequency periodicity is not seen at low frequencies even though the plate is mounted could be explained by the fact that for low frequencies the wave length of the sound is much larger than the dimensions of the plate.

However, it has already been concluded that using the wrong flow velocity will result in errors in the phase, which may induce errors in the process of taking the reflections in the test rig and the scattering properties of the 2-port element into account. In figure 58 the element (1,1) of the source cross spectrum matrix is shown for various choices of flow velocities.
Figure 58  The element (1,1) of the source cross spectrum matrix for different choices of flow velocities, where the spectrum has been zoomed in order to magnify the differences.

As can be seen from this figure the ripple is still present even when a deviation of 20% corresponding to a difference of 17.8 m/s has been applied on the flow velocity, strongly suggesting that the ripple are not the result of errors in the Mach number. It should be noted that the different flow velocities have been used during the complete chain of calculations, i.e. for the determinations of the pressure amplitudes, the scattering matrix, the reflection coefficients and finally the source cross spectrum matrix.
**Vibration results**

During the measurements of the strain of the plates, high peaks at the frequency of the grid voltage and its multiples, i.e. 50 Hz, 100Hz, 150 Hz... were detected. Knowing what the peaks are no harm is done, and thus the measurements have been carried out without wasting time trying to find the source of the electric interference. The results presented are post processed using a crude algorithm to remove the peaks, still leaving some small remains of them. The measurements were carried out at the Mach numbers 0.060, 0.081, 0.124, 0.163, 0.187 and 0.225, see figures 59-61.

![Graph](image)

**Figure 59**  
*Dynamic strain of the 0.5 mm plate at 6 different Mach numbers.*

![Graph](image)

**Figure 60**  
*Dynamic strain of the 1.5 mm plate at 6 different Mach numbers.*
In order to get a feeling for the relative dynamic strain between the plates, they are shown in figure 62 for the highest Mach number.

As can be seen from these figures the vibrations are dominated by peaks which stay constant in frequency as the Mach number increases, unlike the Strouhal frequency. A strongly dominating peak at what seems to be the first elastic mode of the plates can be seen for all three plate thicknesses. In figure 63 the highest peak of the 0.5 mm plate has been zoomed in.
The frequency of the peak does indeed increase with frequency, something not seen when studying the strain auto spectra of the thicker plates. This increase in eigen frequency must be due to an increased stiffness. The cause of this is explained by an assumed increasing static deflection of the plate due to an increasing static force from the flow.

In order to investigate possible relations between the strain spectra and the sound spectra, they are plotted on top of each other, with an applied amplitude magnification of the strain. Figure 64 shows the auto spectrum of the dynamic strain and of element (1,1) of the source cross spectrum matrix for the 0.5 mm plate at the Mach number 0.187.

**Figure 63**  Dynamic strain of the 0.5 mm plate at 6 different Mach numbers, zoomed in at the highest peak.
Before any analysis can be made more such plots must be considered. Figure 65 shows the same comparison but for the 1.5 mm plate at the same Mach number.
Studying the figure 64 and 65 it might seem that only one conclusion can be drawn; that there is absolutely no information of the source generated sound to be gain by studying these strain measurements. However, this is not necessarily the case. A different interpretation is that the eigen-modes are very important if the vibration level is high enough. From the experiments it could bee seen by eye that the dynamic deflection of the tip of the 0.5 mm plate was more than 1 mm at Mach 0.123, however no visible vibrations were detected at any flow speed for the thickest plate. It can be seen from the figures above that the 1.5 mm plate is by orders of magnitude somewhere in between the other plates, and during the experiments vibrations were seen by eye only at the highest speeds for the 1.5 mm plate.

Also, it may be that the shape of the eigen-mode is of great significance. Since the dipole force on the plate stems from vortex shedding at the edges of the plate, the mobility at the edges at the mode shape may be of ultimate importance. For the 0.5 mm plate there seems to be one or possibly two additional peaks at the frequencies 380 Hz and 560 Hz, which both are fairly constant for increasing Mach numbers, again suggesting that they are associated with the eigen-modes of the plate. The fact that they appear only for the thinnest plate might be due to that a nodal line of that mode shape is in the close vicinity of the strain gauge. This is a notion which is backed up by the fact that the peaks are not present for all Mach numbers, i.e. for all static deflections, for which the nodal lines and the eigen-frequency differ a small amount due to the increased stiffness of the plate. To investigate which mode shapes exists below 2 kHz, a simple model of the plate was made in the finite element method program Abaqus. Putting the boundary condition to rigid at the duct wall, the results deviated some but not too much from the measurements for the first peak, see table 13.

<table>
<thead>
<tr>
<th>thickness</th>
<th>0.5 mm</th>
<th>1.5 mm</th>
<th>3 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode #</td>
<td>frequency [Hz]</td>
<td>frequency [Hz]</td>
<td>frequency [Hz]</td>
</tr>
<tr>
<td>1</td>
<td>78.852</td>
<td>235.240</td>
<td>467.050</td>
</tr>
<tr>
<td>2</td>
<td>336.17</td>
<td>1002.8</td>
<td>1986.2</td>
</tr>
<tr>
<td>3</td>
<td>475.54</td>
<td>1410.1</td>
<td>2761.1</td>
</tr>
<tr>
<td>4</td>
<td>789.75</td>
<td>2354.8</td>
<td>4648</td>
</tr>
<tr>
<td>5</td>
<td>1191.2</td>
<td>3526.5</td>
<td>6861.3</td>
</tr>
<tr>
<td>6</td>
<td>1323.7</td>
<td>3942</td>
<td>7321.4</td>
</tr>
<tr>
<td>7</td>
<td>1747.6</td>
<td>5174.6</td>
<td>7739.1</td>
</tr>
<tr>
<td>8</td>
<td>2068</td>
<td>6114.7</td>
<td>10050</td>
</tr>
<tr>
<td>9</td>
<td>2184.6</td>
<td>6492.3</td>
<td>11817</td>
</tr>
<tr>
<td>10</td>
<td>2834</td>
<td>7321.2</td>
<td>12650</td>
</tr>
</tbody>
</table>

Table 13  
*Eigen frequencies of the first 10 eigen-modes obtained using Abaqus.*

It is also clear that there indeed are modes which have not been detected in the strain measurements. Of the first 6 modes, the modes number 3 and 5 are missing in the strain figures. Studying the shape of these modes it is realized that they are torsional, see the appendix, and that the strain gauge was indeed put on a nodal line. From the sound
spectra a trough is constantly seen at 400 Hz for the 0.5 mm plate, but the first torsion mode is found at 475 Hz in the FEM simulation. In the case of the first bending mode shape, i.e. the first peak, the FEM simulation yielded an approximately 10% lower frequency than the strain gauge measurements. However it is not for certain that the same relation between simulations and the real situation holds for the torsional modes, implying that it still is possible that a torsional mode is the cause of the trough.
**Resulting drag and friction coefficients**

The measurements of the static pressure drop were conducted at 6 different axial velocities which varied slightly between the different objects and between the objects and the empty pipe, see table 14.

<table>
<thead>
<tr>
<th>Object</th>
<th>Axial velocity [m/s]</th>
<th>Pressure Loss Pa</th>
<th>Object</th>
<th>Axial velocity [m/s]</th>
<th>Pressure Loss Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,5 mm</td>
<td>25.5</td>
<td>290</td>
<td>3 mm</td>
<td>25.2</td>
<td>300</td>
</tr>
<tr>
<td>0,5 mm</td>
<td>34.1</td>
<td>510</td>
<td>3 mm</td>
<td>33.8</td>
<td>520</td>
</tr>
<tr>
<td>0,5 mm</td>
<td>52</td>
<td>1170</td>
<td>3 mm</td>
<td>51.9</td>
<td>1250</td>
</tr>
<tr>
<td>0,5 mm</td>
<td>68.7</td>
<td>1970</td>
<td>3 mm</td>
<td>68.4</td>
<td>2140</td>
</tr>
<tr>
<td>0,5 mm</td>
<td>78.4</td>
<td>2560</td>
<td>3 mm</td>
<td>78.6</td>
<td>2840</td>
</tr>
<tr>
<td>0,5 mm</td>
<td>95.5</td>
<td>3730</td>
<td>3 mm</td>
<td>93.4</td>
<td>4050</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>25.2</td>
<td>300</td>
<td>empty</td>
<td>29</td>
<td>210</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>33.7</td>
<td>530</td>
<td>empty</td>
<td>34.7</td>
<td>290</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>52.2</td>
<td>1310</td>
<td>empty</td>
<td>58</td>
<td>770</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>68.8</td>
<td>2220</td>
<td>empty</td>
<td>68</td>
<td>1060</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>78.8</td>
<td>2920</td>
<td>empty</td>
<td>84.5</td>
<td>1600</td>
</tr>
<tr>
<td>1,5 mm</td>
<td>92.8</td>
<td>4100</td>
<td>empty</td>
<td>102</td>
<td>2320</td>
</tr>
</tbody>
</table>

**Table 14**  
Measured static pressure loss.

From the pressure loss of the empty pipe the friction factor is calculated, and given in table 15.

<table>
<thead>
<tr>
<th>Axial velocity [m/s]</th>
<th>29</th>
<th>34.7</th>
<th>58</th>
<th>68</th>
<th>84.5</th>
<th>102</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f \cdot 10^4$</td>
<td>45</td>
<td>43</td>
<td>41</td>
<td>41</td>
<td>40</td>
<td>40</td>
<td>42</td>
</tr>
</tbody>
</table>

**Table 15**  
Calculated friction factor.

In order to check if the calculated pressure loss agrees well with the measured, they are both shown in figure 67.
Figure 67  Calculated and measured static pressure loss of a pipe length of 3081 mm.

The resulting drag force and drag coefficient are shown for the three plate thicknesses in figures 68 and 69.

Figure 68  The resulting drag force on the plates due to the air flow.
These results are very interesting, since it seems that the drag coefficient of the 0.5 mm plate decreases for increasing flow velocities. This is most likely due to increasing static deflection of the 0.5 mm plate as the flow velocity is increased, resulting in a smaller projected area in the normal direction of the flow, which in turn yields a lower pressure loss. However it is also likely that the yielding plate has a lower pressure loss due to less turbulence, which would also explain the decrease in the drag for increasing velocities.
Discussion and future work

The effects of the plate on the scattering matrix are evidently small, while on the source cross spectrum matrix the effect is tremendous. In the source spectra differences between the plates can also be observed, which is why this discussion will focus on the effect of the yielding plate.

As can be seen from the source results the peak of the 0.5 mm plate seems to be split into two. Whether this is an error or due to an actual mechanism is hard to say. It could be argued that for high velocities the static deflection of the 0.5 mm plate due to the flow is so severe that it changes the dynamic interaction of the plate and the flow, e.g. by introducing new eigen-modes. Another possible effect is a coupled system with the plate, the duct walls and the air in between, which has an anti-resonance at the trough seen at approximately 400 Hz. However there are also the high-level sound seen at all velocities just below the low frequency limit set by the correlation length. If this sound is more than errors due to the insufficient microphone separations, it could be that there are mechanisms in the duct which may disrupt the results seen above this limit. Since it has not been possible to identify the source of this sound, it is very hard to come to some sort of conclusion about the existence of the trough for the yielding plate. However, since the trough is only existent for the lowest plate stiffness, whatever interactive effect experienced by the plate and the surroundings, it can be said for certain that it is dependent on the stiffness of the plate. This more or less out-rules the possibility of there being another source present in the duct, and very strongly points towards the align of thinking that the trough is due to some resonance related phenomena.

Since the yielding plate shows such strange behavior it may be very interesting to study it further. In future work the velocity increments should be made much smaller, and using a laser vibro-meter a more detailed structural dynamic investigation of the plate should be made. However for this work to be of high quality, the problems with the poor coherence must be solved, and the ripple must be taken care of if the peaks and troughs seen in the results are to be trusted. Also, the coupling effects between the plate and the surroundings should be modeled in a computer in order to further investigate the phenomenon.

What is more certain however is that the 0.5 mm plate generates less sound than the thicker plates, implying that the 0.5 mm plate is the best from an acoustic point of view. However it should be noted that the purpose of the plate, i.e. to generate vortices might suffer from the static deflection and the dynamic behavior.

Also, it is instructive to comment about the influence of the errors in the Mach number. It is seen in the results that the error induced in the phase by using an erroneous Mach number is strongly dependent on the transportation length. This is due to the fact that the microphones in the current setup are very close to each other, implying that the intermediate distances over which the wave numbers are approximated are very short. Hence, the large errors induced by poor approximations of the wave numbers can be traced to the long transportation distance. From the results it is also seen that the magnitude is more or less unaffected. This is due to the fact that the magnitude is the
maximum amplitude of a sound wave, which for a wave inside a duct implies that the only spatial variation will be due to the losses, which are small. The phase on the other hand is extremely dependent on the spatial variation since the wave is harmonically oscillating as function of the axial coordinate. Following the same line of thinking, it is easily realized that an auto spectra formed from a pressure which has been transported over a significant distance, will not be significantly affected by errors in the wave numbers. Also, it can easily be realized that the error induced by transporting a quantity will be erased if the quantity is transported back to the original position. This finally leads to the conclusion, that the error induced by using erroneous Mach numbers or transportation lengths when transporting the results over long distances, are only of significance when the scattering matrix is of importance. However, this statement comes from observations of the experiments when the Mach number was varied but always equal for all calculation steps, e.g. the scattering matrix and the source cross spectrum matrix. In the real measurement situation in the present work the measurements needed for the scattering matrix, the reflection coefficients and the source cross spectrum matrix were conducted at different times. It is thus possible that the Mach numbers differs between these measurements. If this is the case the statement made above does not necessarily hold, and for instance the ripple may still be caused by the use of incorrect Mach numbers. This is something which should be evaluated in future work.

Summary

An investigation on the effects of having a yielding, triangular plate in a duct with a mean flow present have been carried out. The stiffness of the plate was modified by varying the plate thickness in between $0.5 - 3\ mm$, and measurements were conducted for Mach numbers in between $0.060 - 0.213$. The scattering properties as well as the source generated sound of the plate have been determined using a 2-port model, and the vibrations of the plate have been measured by a strain gauge. In addition the pressure loss of the plate for each stiffness and Mach number have been investigated. In the process of determining the acoustics, methods of over-determination have been applied in the calculations of the pressure amplitudes, the scattering matrix and the reflection coefficients. Also, a method for over-determining the source cross spectrum matrix has been suggested and the effects observed. The effects of over-determining the scattering properties of a 2-port element were studied by measurements without flow in an empty duct, and the result clearly shows that increasing the number of microphones and the number of independent measurements are very effective means of noise suppression. The over-determination of the source spectra was applied for the calculations with the plate inserted in a duct with a mean flow present, and also in this case improvements were seen.

The effect of errors in the wave numbers have been addressed, and it is observed in the results that the error induced on the results by for instance erroneous Mach numbers strongly depends on the distances over which calculations on the wave propagations are made.
The effect of inserting the plate is small when the scattering matrix is considered, however changes are clearly seen in the source cross spectrum matrix. The sound pressure level is overall in the order of 10 dB higher than the empty pipe, the exact deviation depending on the Mach number, and in addition a dipole source in the form of a Strouhal tone is observed, which at the Mach number 0.213 is approximately at 440 Hz. However in the case of the 0.5 mm plate the Strouhal tone is split into two smaller ones. The mechanism behind this is not determined, but it is discussed that it is related to resonance phenomena in a coupled system with the plate and it’s surroundings. The comparisons made with the vibration spectra and the sound spectra are inconclusive, however FEM calculations shows that there exists torsional modes not detected during the strain measurements, which may be more important since the mechanism behind the Strouhal tone is periodic flow separation at the plate edges, i.e. at the highest mobility of the torsional modes.

Overall in frequency having a yielding plate results in 2 dB less sound at the Mach number 0.213, clearly showing that the effect of having a yielding plate is a beneficial one from an acoustic point of view.
Acknowledgments

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References


Appendix

Test rig dimensions

The dimensions of the three pipes of the test rig are given in figures A1-A3.

Figure A1. The dimensions of the test object pipe.

Figure A2. The upstream aluminum pipe. 1: Loudspeakers, 2: Microphones, 3: Static pressure hole, 4: Prandtl pipe.
Plate dimensions

For the calculations of the drag coefficient the area of the plate projected in the normal direction of the axis is required. This is obtained as the sum of the areas $S_1$ and $S_2$, see figure A4.

Since the plate is placed in an angle of attack the curvature of $S_2$ is not circular, however when transposed in the normal direction of the axis it is, with the same radius at bend as the inner radius of the pipe. Then the projected area of $S_2$, referred to as $S_3$ is calculated according to the equation given in table A1, [11].
Table A1. Dimensions and equation needed for calculating $S_3$. Note that $w$ is the dashed border between $S_1$ and $S_2$ seen in figure A4. $\alpha$ is given by $r$ and $w$.

The resulting projected area of the plate used for the calculations of $c_D$ is $3620 \text{ mm}^2$.  

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<table>
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<tr>
<td>$w$</td>
<td>$64 \text{ mm}$</td>
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<tr>
<td>$r$</td>
<td>$45 \text{ mm}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$70,83^\circ$</td>
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</table>

$S_3 = \frac{r^2}{2}(\alpha - \sin \alpha)$

$= \frac{r^2}{2}(2\sin^{-1}\left(\frac{w}{2r}\right) - \frac{w}{r})$
Acoustic Results

Empty pipe zero flow

Figure A5  Error suppression in the magnitude of $S_{11}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.
Figure A6  Error suppression in the magnitude of $S_{22}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.

Figure A7  Error suppression in the phase of $S_{21}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.
Figure A8  Error suppression in the phase of $S_{12}$ as function of the number of measurements. The pressure amplitudes are over-determined using 4 microphones on each side.

Figure A9  Error suppression in the magnitude of $S_{11}$ using 4 microphones on each side and 6 independent measurements. Difference between the 2-microphone method and the method of over-determined pressure amplitudes.
Figure A10  Error suppression in the phase of $S_{12}$ using 4 microphones on each side and 6 independent measurements. Difference between the 2-microphone method and the method of over-determined pressure amplitudes.

Scattering matrices at Mach 0.060

Figure A11  The magnitude of $S_{11}$ at Mach 0.060.
Figure A12  The magnitude of $S_{22}$ at Mach 0.060.

Figure A13  The magnitude of $S_{21}$ at Mach 0.060.
Figure A14  The magnitude of $S_{12}$ at Mach 0.060.

Figure A15  The phase of $S_{21}$ at Mach 0.060.
Figure A16  The phase of $S_{12}$ at Mach 0.060.

Scattering matrices at Mach 0.124

Figure A17  The magnitude of $S_{11}$ at Mach 0.124.
Figure A18  The magnitude of $S_{22}$ at Mach 0.124.

Figure A19  The magnitude of $S_{21}$ at Mach 0.124.
Figure A20  The magnitude of $S_{12}$ at Mach 0.124.

Figure A21  The phase of $S_{21}$ at Mach 0.124.
Figure A22  The phase of $S_{12}$ at Mach 0.124.
Scattering matrices at Mach 0.187

Figure A23  The magnitude of $S_{11}$ at Mach 0.187.

Figure A24  The magnitude of $S_{22}$ at Mach 0.187.
Figure A25  The magnitude of $S_{21}$ at Mach 0.187.

Figure A26  The magnitude of $S_{12}$ at Mach 0.187.
Figure A27  The phase of $S_{21}$ at Mach 0.187.

Figure A28  The phase of $S_{12}$ at Mach 0.187.
Scattering matrices at Mach 0.213

**Figure A29**  The magnitude of $S_{11}$ at Mach 0.213.

**Figure A30**  The magnitude of $S_{22}$ at Mach 0.213.
Figure A31  The magnitude of $S_{21}$ at Mach 0.213.

Figure A32  The magnitude of $S_{12}$ at Mach 0.213.
Figure A33  The phase of $S_{21}$ at Mach 0.213.

Figure A34  The phase of $S_{12}$ at Mach 0.213.
Errors due to erroneous Mach number

Figure A35  The magnitude of $S_{11}$ for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.

Figure A36  The magnitude of $S_{22}$ for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.
Figure A37  The phase of $S_{21}$ for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.

Figure A38  The phase of $S_{12}$ for the empty pipe at the axial velocity 89 m/s, for the different mean/max flow velocity ratios 0.81, 0.82 and 0.83.
Source cross spectrum matrices

Figure A39  $G_{s11}$ at Mach 0.060.

Figure A40  $G_{s22}$ at Mach 0.060.
Figure A41  \(G_{s_{11}}\) at Mach 0.124.

Figure A42  \(G_{s_{22}}\) at Mach 0.124.
Figure A43 \( G_{s1} \) at Mach 0.187.

Figure A44 \( G_{s2} \) at Mach 0.187.
Figure A45  \( Gs_{11} \) at Mach 0.213.

Figure A46  \( Gs_{22} \) at Mach 0.213.
Vibration Results

Since all strain plots are presented in the results section of the report, they are not repeated here. The table containing the calculated modes using Abaqus is however, since the indices of the modes shapes shown in figures A47-A53 are listed in it.

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</tr>
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</table>

Table A2  Eigen frequencies of the first 10 eigen-modes obtained using
Figure A47  Mode no 1.

Figure A49  Mode no 2.

Figure A50  Mode no 3.

Figure A51  Mode no 4.

Figure A52  Mode no 5.

Figure A53  Mode no 6.