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Friction Coefficients and Grasp Synthesis

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***Abstract*— We propose a new concept called *friction sensitivity* which measures how susceptible a specific grasp is to changes in the underlying friction coefficients. We develop algorithms for the synthesis of stable grasps with low friction sensitivity and for the synthesis of stable grasps in the case of small friction coefficients. We describe how grasps with low friction sensitivity can be used when a robot has an uncertain belief about friction coefficients and study the statistics of grasp quality under changes in those coefficients. We also provide a parametric estimate for the distribution of grasp qualities and friction sensitivities for a uniformly sampled set of grasps.**

I. Introduction

Friction coefficients are important for determining the quality of a specific grasp and for understanding whether a grasp is force-closed or not. Most state of the art grasp synthesis approaches typically assume fixed friction coefficients and evaluate an associated grasp quality measure such as the L^1 grasp quality Q_μ, [1]. In reality, friction coefficients may vary depending on temperature, humidity and the presence of dirt on an object. Also, a robot will rarely have knowledge of precise friction coefficients to start with. Instead, we may only be able to estimate a confidence interval of friction coefficients. In this work, we address the following related issues:

a) We systematically study the impact of changes in friction coefficients on the stability of grasps in the context of a popular L^1 grasp quality measure Q_μ.
b) We propose the concept of *friction sensitivity* $S^n_{a,b}(g)$ of a grasp g with respect to Q_μ and fit a Dirichlet distribution to the distribution of $(Q_\mu(g), S^n_{a,b}(g))$ for uniformly sampled grasp configurations with three contact points.
c) We propose and evaluate algorithms for synthesizing stable grasps with low friction sensitivity and for small friction coefficients.

The paper is structured as follows: In Section II, we discuss related work and introduce preliminaries. In Section III, we define friction sensitivity and describe our algorithms for grasp synthesis. We discuss our experiments in Section IV. Finally, we conclude our work and discuss future directions in Section V.

II. Background and Related Work

In the following, we review the grasp quality function Q_μ and the basics of friction coefficients and grasp synthesis.

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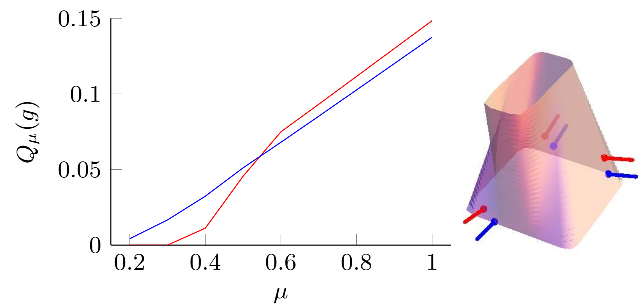


Fig. 1: For the red and blue contact point configurations depicted on the object, we consider how the grasp's stability, measured in terms of a popular grasp quality function, varies with changing friction coefficients. The vertical axis depicts grasp quality, while the assumed friction coefficient μ is varied from 0.2 to 1.0. The blue grasp remains more stable under changes in friction, while the red grasp yields more stable grasps for higher friction values.

A. Grasp synthesis and L^1 grasp quality

Similarly to the work reported in [2], we focus on determining contact point configurations on a surface S which result in a force-closed grasp g. We consider grasps

$$g = (c_1, \dots, c_m, n_1, \dots, n_m, z) \in \mathbb{R}^{3m} \times (\mathbb{S}^2)^m \times \mathbb{R}^3$$

consisting of contact points $c_i \in S$ on some surface $S \subset \mathbb{R}^3$ and with corresponding inward-pointing unit normal vectors $n_i \in \mathbb{S}^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ such that S has centre of mass $z \in \mathbb{R}^3$. To determine if such a grasp g can withstand external forces, we need to estimate if g is a force-closed grasp [3]. Ferrari and Canny [1] introduced an L^1 grasp quality measure Q_μ which can be used for this purpose. For a fixed friction coefficient $\mu > 0$, the Coulomb friction model states that – under the assumption that no slippage occurs – forces applied at a contact $c_i \in \mathbb{R}^3$ on some surface S and with corresponding inward pointing unit normal vector $n_i \in \mathbb{R}^3$ satisfy $\|f_i^t\| \leqslant \mu f_i^\perp$, where $f_i^t \in \mathbb{R}^3$ denotes the component of f_i tangent to S at c_i, $f_i^\perp \in \mathbb{R}$, and $f_i^\perp n_i$ denotes the component along the normal direction n_i - *i.e.* these forces have to lie within the friction cone $C_i = \{f_i \in \mathbb{R}^3 : \|f_i^t\| \leqslant \mu f_i^\perp\}$. For a particular grasp g as above, these friction cones C_i can then be approximated by $C_i \approx \{\sum_{i=1}^l \alpha_i f_{ij} : \alpha_i \geqslant 0\}$ for l uniformly spaced vectors $f_{i1}, \dots, f_{il} \in C_i$ satisfying $\langle f_{ij}, n_i \rangle = 1$.

In this paper, we use $l = 8$ such uniformly spaced vectors. To define this L^1 quality measure, one then approximates the set of wrenches satisfying $\sum_{i=1}^m |f_i^\perp| \leqslant 1$ by the convex hull $\mathrm{Conv}(\{0\} \cup S(g))$, where

$$S(g) = \mathrm{Conv}\left(\{w_{ij} : i \in \{1, \dots, m\}, j \in \{1, \dots, l\}\}\right),$$

and where $w_{ij} = (f_{ij}, (c_i - z) \times f_{ij})$. $Q_\mu(g)$ is then defined to be the radius of the largest ball inside $\mathrm{Conv}(\{0\} \cup S(g))$ and centred at the origin. To compute $Q_\mu(g)$, $S(g)$ is represented as an intersection of affine half-spaces $S(g) = \cap_{j=1}^{s} \{x \in \mathbb{R}^6 : \langle x, v_j \rangle \leqslant \lambda_j\}$ for some $\lambda_j \in \mathbb{R}$, $v_j \in \mathbb{R}^6$, $\|v_j\| = 1$, which can be obtained using the Quickhull algorithm [4]. Then $Q_\mu(g) = \max(0, \min_j \lambda_j)$. If a grasp g satisfies $Q_\mu(g) > 0$, it is force-closed and can withstand external wrenches in arbitrary direction. Furthermore, grasps are considered more stable the larger $Q_\mu(g)$ is.

B. *Friction coefficients and grasp synthesis*

For the purpose of robotic grasping, friction is commonly modelled using Coulomb's friction laws [3] for some friction coefficient μ as above. Friction coefficients depend on various parameters: [5] discusses in particular the influence of humidity on friction, while [6] study the dependence of friction on temperature. Further factors influencing friction include surface properties such as dusty or oily vs. dry and clean surfaces [7].

Since robots are to work in extreme conditions such as in search and rescue operations and in manufacturing applications, the impact of environmental factors on friction coefficients should be considered an important component in the analysis of grasp hypotheses. Even in less-extreme scenarios, such as that of a service robot in a home environment, friction can be influenced by dusty or dirty surfaces and can vary even during a manipulation task, *e.g.* when a robot is washing dishes. Clearly, none of these environmental factors can be determined exactly, and the robot hence needs to operate with an *expected friction value*. In current grasp synthesis work, such friction coefficients are often set to a fixed value according to friction tables for various material combinations [3], [2].

To the best of our knowledge, the problem of assessing the goodness of a force-closed grasp with respect to robustness under changes in friction has so far not been studied in depth. One work which mentions the problem of uncertainty in friction coefficients is [7] where the impact of uncertainties in friction and contact positions on grasp synthesis is discussed. In order to deal with uncertainty in friction coefficients, the authors suggest to work instead with 'effective friction coefficients' which are obtained by multiplying the coefficients of friction by some fixed reduction rate $\frac{1}{\kappa} \leqslant 1$ which is assumed to be known. In the work of [8], independent contact regions are computed on discretized objects taking into account uncertainties in friction coefficients. There, these uncertainties are also modeled using a reduction rate. Based on the same concept, [9] developed an algorithm to compute minimal required friction coefficients and contact forces.

III. METHODOLOGY

Fig. 1 displays two examples showing how the grasp quality measure Q_μ changes with respect to changes in the assumed friction coefficient μ for the depicted contact configurations and for $\mu \in [0.2, 1.0]$. This figure highlights several important features of the function $\mu \mapsto Q_\mu(g)$. Observe, for example, that the graphs are monotonically increasing with increasing friction and that they have a distinct almost piecewise-linear looking shape which seems to be a generic property we encountered also for other object shapes. Furthermore, we observe that, for $\mu = 1$, a ranking of these two grasps based on grasp quality alone would return the red grasp as a preferable grasp hypothesis, while this grasp is unstable for $\mu = 0.2$ where the blue grasp remains stable. A natural question arises: which grasp should we choose if we only have knowledge of a confidence interval $\mu \in [0.2, 1]$?

A friction coefficient of 0.2 corresponds to the friction of a polythene (plastic) surface in contact with a steel surface, while a friction of 1.0 corresponds to *e.g.* copper against copper. In this section, we introduce a simple sensitivity measure $S^n_{a,b}(g)$ which we will use to assess a grasp's stability under variations in friction. Furthermore, we devise a parametric approach for studying the sensitivity of generic grasps as well as grasps on specific objects. Finally, we describe a gradient based approach for synthesizing grasps robust under changes in friction coefficients and develop a new algorithm that can be used to determine force-closed grasps even for small friction coefficients.

A. *Quantifying a grasp's sensitivity to friction*

To provide a computationally tractable first-order approximation of the average slope of the graph $\mu \mapsto Q_\mu(g)$, for $\mu \in [a, b]$ and for a fixed grasp g, we make the following definition:

Definition 3.1. *Consider a grasp configuration g and a friction interval $[a, b] \subset \mathbb{R}_{\geqslant 0}$. Fix $n \in \mathbb{N}$ and consider $\delta = \frac{1}{n}(b - a)$, $\{x_0, \ldots, x_n\} \in [a, b]$, $x_i = a + i\delta$ for $i \in \{0, \ldots, n\}$. We define the sensitivity $S^n_{a,b}(g)$ of g with respect to the parameters a, b, n to be:*

$$S^n_{a,b}(g) = \frac{1}{n-m} \sum_{i=m}^{n-1} k_i,$$

where $k_i = \frac{1}{\delta}(Q_{x_{i+1}}(g) - Q_{x_i}(g))$ and m is the smallest integer $i \in \{0, \ldots, n-1\}$ such that $Q(x_i)$ is not zero. If no such m exists, we define $S^n_{a,b}(g) = 0$.

We then consider grasps with large $S^n_{a,b}(g)$ to be sensitive to changes in friction, while grasps with small $S^n_{a,b}(g)$ are considered to be insensitive to such variations.

Suppose a robot has computed a set of grasp hypotheses $H_\mu = \{g_1, \ldots, g_m\}$ of grasps g_i with underlying friction coefficient $\mu > 0$ and such that $Q_\mu(g_i) > 0$. While traditional ranking based approaches would select a grasp with largest grasp quality Q_μ, our definition of grasp sensitivity allows us to react to uncertainty in the friction coefficients. Returning to Fig. 1, we can compute that $S^{20}_{0.2,1}(g_{blue}) \approx 0.1640$ for the blue grasp, while $S^{20}_{0.2,1}(g_{red}) \approx 0.2287$ for the red grasp. If we are working under the assumption that $\mu \approx 0.6$, a ranking by $Q_{0.6}$ now favours g_{red}, while a ranking by $S^{20}_{0.2,1}$ for grasps with $Q_{0.6}(g) > 0$ returns g_{blue}, which indeed stays stable over the whole friction interval $[0.2, 1]$. To provide a simple measure balancing the benefits of large grasp quality with low sensitivity, we define

$$\Phi^n_{a,b,\mu}(g) = \frac{Q_\mu(g)}{S^n_{a,b}(g)},$$

which provides a simple scoring function for grasps. We propose that grasps with large $\Phi^n_{a,b,\mu}(g)$ are desirable since they arise from a comparatively large grasp quality and low friction sensitivity.

B. Statistical properties of grasps and friction

Let us now describe how we shall study some of the basic statistical properties of grasp quality and friction sensitivity.

Generic random sampling: To study grasps with m contact points generically, that is without a notion of an object, we consider the set $\mathcal{D}(r) = \mathbb{B}(r)^m \times (\mathbb{S}^2)^m \times \{0\}$, where $\mathbb{B}(r) = \{x \in \mathbb{R}^3 : \|x\| \leqslant r\}$ and $\mathbb{S}^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ and 0 denotes the origin in $\mathbb{R}^3$. An element $g = (c_1, \dots, c_m, n_1, \dots, n_m, z) \in \mathcal{D}(r)$ then represents a grasp with contacts c_i, inward pointing unit contact-normals n_i and with centre of mass z at the origin, and where the contacts are constrained to lie in the ball $\mathbb{B}(r)$ around the origin. Using the uniform probability distribution on $\mathcal{D}(r)$, we can now produce an arbitrary number of random grasps in this set. We employed a similar approach of grasp sampling in our work [10]. The grasp quality Q_μ and our sensitivity measure $S^n_{a,b}$ can in this context be considered as random variables on this space whose properties we can study statistically. In our experiments, we will in particular show that a Dirichlet distribution provides a good fit to $(Q_\mu(g), S^n_{a,b}(g))$.

Random sampling and surfaces: To study grasps on an arbitrary surface S, we shall employ uniform random sampling on S (as in [10]) to obtain a set $C = \{c_1, \dots, c_l\}$ of contact points. We can then study the set of $\binom{l}{m}$ tuples of distinct configurations of m such contact points as grasp candidates. Using this procedure, we shall then obtain information about the distribution of Q_μ and $S^n_{a,b}$ for a specific surface.

C. Synthesizing stable grasps with small friction coefficient

As we shall show, stable grasps are difficult to synthesize with sampling based approaches such as the ones used by GraspIT [11] if the friction coefficients are small (*e.g.* $\mu \leqslant 0.5$). We hence propose a new procedure using 'virtual' friction coefficients. Suppose we have a parametric form for our graspable surface S, so that points and unit normals on S are given by coordinates $(x, y) \in \mathbb{R}^2$ as $c(x, y)$ and $n(x, y)$ respectively. Since we would like to execute a gradient based method using Q_μ, we shall use a modified definition, where $\hat{Q}_\mu(g) = \min_j \lambda_j$, rather than $Q_\mu(g) = \max(0, \min_j \lambda_j)$, where λ_j are the offsets of the hyperplanes defining the wrench space $S(g)$ which we mentioned in Section II. The advantage of $\hat{Q}$ here is that we can obtain numerical gradients even when $\hat{Q} < 0$, while Q just takes on the value zero in those regions. Note that $\hat{Q}_\mu(g) = Q_\mu(g)$ when $Q_\mu(g) > 0$.

For grasps with m contacts, we then obtain a function $F_\mu : \mathbb{R}^{2m} \to \mathbb{R}$ mapping m contact point coordinates to the grasp quality $\hat{Q}_\mu(g)$ of the corresponding grasp at those contact points. We then proceed by iterating M steps of a standard gradient ascent of F_μ with a small decrease in μ until a desired target friction value μ_{end} is reached. Alg. 1 provides details, where $GradientAscent(F_\mu, g, \delta, M)$ executes M gradient ascent steps with step size δ with respect to F_μ and starting configuration g. Here, gradients are approximated using finite differences. $SampleGrasp(S)$ returns a uniformly sampled grasp configuration on the surface S.

Algorithm 1 Search for a stable grasp for a small friction coefficient $\mu_{end} > 0$ on a parametric surface S.

Require: S, $0 < \mu_{end} < \mu_{start}$, $\delta > 0$, $N, M \in \mathbb{N}$
  $g \leftarrow SampleGrasp(S)$
  for $i \in \{0, \dots, N-1\}$ **do**
    $\mu \leftarrow \mu_{start} - \frac{i}{N-1}(\mu_{start} - \mu_{end})$
    $g \leftarrow GradientAscent(F_\mu, g, \delta, M)$
  end for
  return g

IV. Evaluation

In the following, we describe an evaluation of our proposed methodology.

A. The impact of friction on grasp stability

Let us now study the impact of changes in friction coefficients on Q_μ. For this purpose, we sampled 10 sets of 10000 uniform samples, $U_1, \dots, U_{10}$, from the uniform distribution on $\mathcal{D}(2)$. Fig. 2 displays the mean percentage of stable grasps (*i.e.* $Q_\mu(g) > 0$) among the grasps in these sets U_i against friction coefficients ranging from 0 to 5 in increments of 0.1.

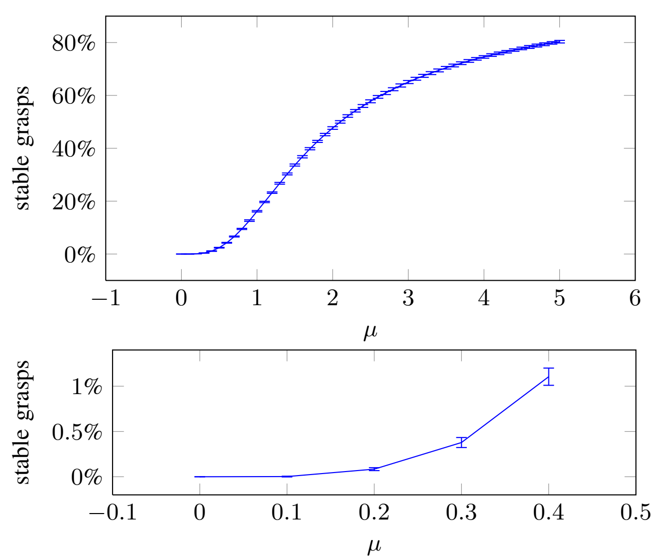


Fig. 2: Percentage of stable grasps for uniformly sampled grasps from $\mathcal{D}(2)$.

In this and all the following experiments, we used $l = 8$ edges to approximate the fiction cones used in the calculation of Q_μ. Fig. 2 additionally indicates the standard deviation for our 10 sets of grasp samples $U_1, \ldots, U_{10}$. Observe that the percentage of stable grasps increases substantially as the friction coefficient is increased and that this percentage decays rapidly as μ tends to zero as can be seen in the second plot. A friction coefficient of $\mu = 0.2$ corresponds to the friction of polythene plastic against steel, while a friction of 1.0 corresponds to copper against copper. For $\mu = 0.2$, only about 0.084% of the grasps were stable, while for $\mu = 0.5$, 2.4% were stable and, for $\mu = 1.0$, about 16.2% of the grasps were stable.

Friction coefficients hence significantly influence the success of sampling based grasp synthesis algorithms such as [2]. While previous work has certainly been aware of this phenomenon, the above simple 'generic' sampling based approach provides us with a first quantitative analysis of this phenomenon which, to the best of our knowledge, has not previously been formalized in this way. Fig. 2 provides clear evidence that 'straight-forward' sampling approaches for grasp synthesis used *e.g.* by the popular simulation environment GraspIT [11], are inappropriate for low friction coefficients.

B. Friction sensitivity for generic grasps

Recall that, when $Q_\mu(g)$ of a grasp g is relatively small for the expected friction coefficient μ, a big $S^n_{\mu-\epsilon,\mu+\epsilon}(g)$, for $\epsilon > 0$, indicates that it may be inappropriate to use the grasp g when we are uncertain about the exact value of μ. Let us now investigate the relationship between friction sensitivity and grasp quality Q_μ for a generic set of grasps.

For this purpose, we sampled a set W of 1 million grasps with three contact points uniformly from the set $\mathcal{D}(2)$. We assume that the true underlying friction coefficient μ lies in the interval $[0.2, 1.0]$ with an expected value of 0.6, and we hence compute $Q_{0.6}(g)$ for all grasps $g \in W$. Let us consider the set of grasps $W' = \{g \in W : Q_{0.6}(g) > 0.001\}$. W' contained 29236 stable grasps. For each $g \in W'$, we now compute an associated fiction sensitivity $S^{20}_{0.2,1.0}(g)$, using a partition of the interval $[0.2, 1.0]$ into 20 equally spaced sub-intervals.

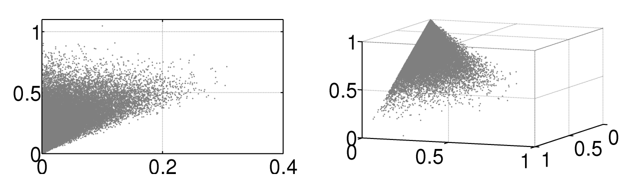


Fig. 3: The distribution of grasp quality $Q_{0.6}$ (horizontal axis) against sensitivity $S^{20}_{0.2,1}$ (vertical axis) is displayed on the left and the mapping of this data onto the standard simplex in $\mathbb{R}^3$ is shown on the right.

Fig. 3 displays the distribution of $(Q_{0.6}, S^{20}_{0.2,1.0})$ for our set of stable grasps W'. We will now study this distribution in more detail.

A parametric density estimate: Observe that the data in the left part of Fig. 3 is located in a cone with apex at the origin. We can see that grasps with low sensitivity and high grasp quality are sparse in this data-set. In order to be able to quantify statements about the likelihood of encountering grasps with prescribed grasp quality and friction sensitivity, we propose a parametric density fit as follows: as a first step, we determined edges e_1, e_2 of the smallest triangle in $\mathbb{R}^2$ enveloping all the samples and with apex at the origin. Both e_1, e_2 have one end-point at the origin and satisfy $\langle e_1 - e_2, d\rangle = 0$, where d is the vector which equally divides $\angle(e_1, e_2)$. Moreover, the length of e_1, e_2 is chosen as small as possible, and such that the triangle still contains all the samples. In our case, $e_1 = (0.5756, 0.9094)$ and $e_2 = (0.0017, 1.0762)$. The triangle containing the edges e_1, e_2 is mapped to the standard 2-simplex $\Delta = \{(x_1, x_2, x_3) : x_i \geqslant 0, x_1 + x_2 + x_3 = 1\}$ by an affine map mapping the origin to the vertex $(0, 0, 1) \in \Delta$. The right part of Fig. 3 displays the image of our data-points on Δ. Given our transformed data points in Δ, we determined a Dirichlet distribution fit to the data. Recall that a Dirichlet distribution $Dir(\alpha_1, \alpha_2, \alpha_3)$ on Δ is determined by three concentration parameters $\alpha_i > 0$. We performed a maximum likelihood fit of the parameters to the data using the *fastfit* Matlab toolbox [12]. The estimated parameters were $(\alpha_1, \alpha_2, \alpha_3) = (1.0001, 2.2273, 9.8739)$.

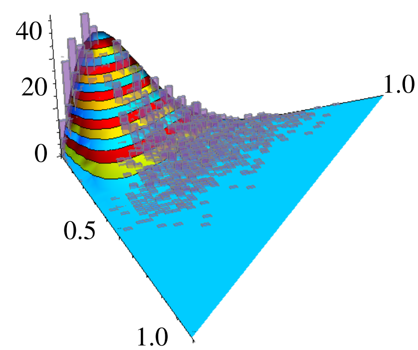


Fig. 4: Comparison between the fitted Dirichlet distribution and the observed data.

The surface plot in Fig. 4 shows the resulting density function of $Dir(1.0001, 2.2273, 9.8739)$ over the projection of Δ onto the xy plane together with a standard histogram density estimator. As we can see in that figure, the chosen Dirichlet density provides a visually satisfying fit to the data following the histogram density estimate closely.

To further quantify the quality of the fit, we ran a Pearson χ^2 test [13] to test the difference between the observed and expected frequencies. For this purpose, we used Mathematica's Monte-Carlo-based χ^2 testing function to evaluate the goodness of our fit and used a significance level of $\alpha = 0.05$. After repeating the test 10 times, the resulting average p-value was 0.833, indicating that our fit is of high quality. The usefulness of our parametric fit lies in the fact that it provides a summary of the data enabling us to compute probabilities for the occurrence of samples in different regions of the quality/sensitivity parameter space.

Table I provides examples of computed probabilities for encountering grasps in selected parameter regions based on our Dirichlet distribution fit. The bracketed expressions in

TABLE I: Probabilities for encountering a grasp in the selected parameter regions in $(S^{20}_{0.2,1.0}, Q_{0.6})$ space for uniform samples in $\{g \in \mathcal{D}(2) : Q_{0.6}(g) > 0.001\}$ determined using our Dirichlet distribution fit. The corresponding relative observed frequencies from our data-set are displayed in brackets below each such value.

$Q_{0.6}$	$S^{20}_{0.2,1.0}(g) \leqslant 0.2$	$S^{20}_{0.2,1.0}(g) \leqslant 0.4$	$S^{20}_{0.2,1.0}(g) \leqslant 0.6$
$\geqslant 0.02$	0.1003 (0.1127)	0.4317 (0.4576)	0.6002 (0.6109)
$\geqslant 0.05$	0.0196 (0.0189)	0.2097 (0.1994)	0.2913 (0.3062)
$\geqslant 0.10$	0.0001 (0.00007)	0.0352 (0.0379)	0.0923 (0.0902)
$\geqslant 0.15$	0 (0)	0.0042 (0.0046)	0.0203 (0.0236)

the table indicate the number of samples lying in those regions divided by the total number of samples. Since these are very close to the probabilities predicted by our Dirichlet distribution fit, this provides further assurance that the parametric representation can be used to calculate these probabilities without the knowledge of the full sample set.

C. Friction sensitivity for example objects

Having studied properties of grasp quality and friction sensitivity in a generic setting, we now concentrate on grasps on the four surfaces displayed in Fig. 5. For the purpose of this experiment, we assume a parametric representation of these surfaces allowing us to compute normals at each surface point p. We used four of the surfaces studied in [10] and followed the same uniform contact point sampling procedure as outlined in that paper.

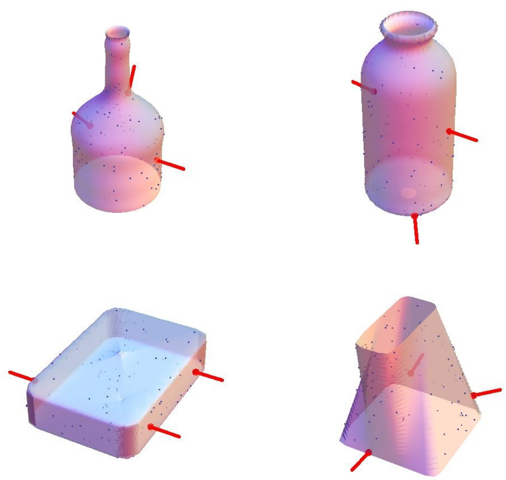


Fig. 5: Example surfaces and grasps on them

In particular, to study the surface-specific distributions of grasp quality and friction sensitivity, we sampled 100 contact points C uniformly from each of these surfaces and computed the resulting $\binom{100}{3} = 161700$ distinct grasps with three contact points chosen from C. Fig. 6 displays the resulting distributions for each of the surfaces depicted in Fig. 5 analogously to the generic case in Fig. 3. Observe that, while the general concentration of the grasp qualities and sensitivities towards the origin remains a dominant feature, we can observe object-specific properties in these distributions such as the sparse fringes for the box object and a stronger concentration towards the origin for the top left and bottom right object. Fig. 5 additionally displays sample grasps which corresponds to the respective red points in Fig. 6. These initial investigations provide evidence that such distributions in terms of grasp quality and friction sensitivity could be used also for the classification of the graspability of various objects under varying friction assumptions.

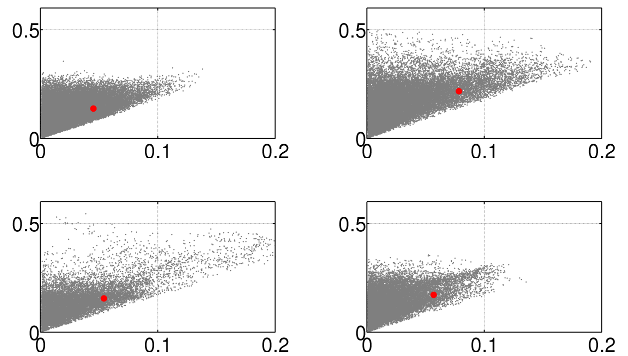
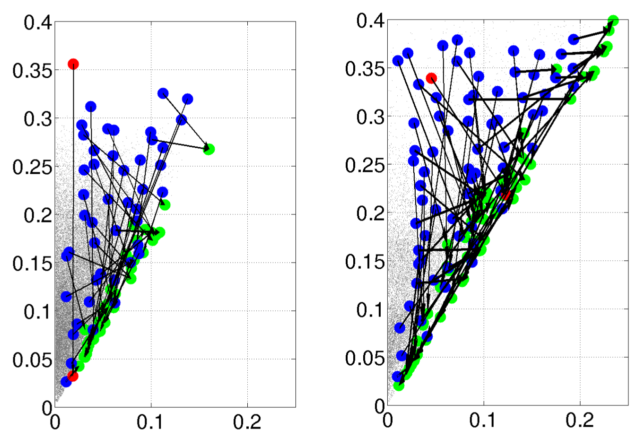


Fig. 6: Distributions of $Q_{0.6}$ (horizontal axis) and $S^{20}_{0.2,1}$ (vertical axis) for random grasps on the 4 surfaces displayed in Fig. 5 are shown in the same order as in that figure. The red dots correspond to the example grasps in Fig. 5 respectively.

D. Gradient ascent on $\Phi^n_{a,b,\mu}(g)$



(a) Bottle surface evaluation (b) Box surface evaluation

Fig. 7: Results of gradient ascent on $\Phi^n_{a,b,\mu}(g)$ represented in (Q_μ, $S^n_{a,b}$) parameters. Original grasps (blue points) are improved by gradient ascent resulting in the green points. The pairs of red dots correspond to initial and final grasps displayed in Fig. 9.

We now experimentally verify that, for any grasp g with $Q_\mu(g) > 0$, a simple fixed step-size gradient ascent can dramatically improve the value of $\Phi^n_{a,b,\mu}(g)$ and hence result in a more desirable grasp. In the following, unless specified elsewhere, we set $a = 0.2$, $b = 1.0$, $n = 20$ and $\mu = 0.6$. Analogously to the proposed algorithm Alg. 1, we consider a parametric surface representation $\varphi : \mathbb{R}^2 \to S$ of our object S and perform gradient ascent of the function $H^n_{a,b,\mu}$ sending a grasp g specified by the centre of mass z of the surface and a triple of surface contact point coordinates to the resulting value of $\Phi^n_{a,b,\mu}(g)$.

We consider the bottle and the box surface depicted in the left column of Fig. 5. The grey points in Fig. 7 display the distribution of grasp quality and sensitivity values for the bottle and the box surface which we computed previously and which are also displayed in Fig. 6. We divide the parameter region $[0.01, 0.285] \times [0.01, 0.385]$ into uniformly spaced boxes of size 0.025×0.025 and picked a grasp from the grey sample points for each non-empty box. This results in a set of grasps G for each of the two surfaces. G is depicted by blue dots in Fig. 7. We then apply 200 steps of a standard fixed step-size gradient ascent with respect to $H^n_{a,b,\mu}$ for every grasp in G and compute gradients numerically using small finite differences.

Fig. 7 shows the result of this gradient ascent on both the bottle and the box surface by green points. We can see that almost all the blue dots in Fig. 7 have been moved towards the right edge of the distribution cone, indicating an improvement in $\Phi^n_{a,b,\mu}(g)$. Fig. 8 illustrates the performance of the gradient ascent using bar-plots, with black and blue bars showing $\Phi^n_{a,b,\mu}(g)$ values before and after gradient ascent respectively. It is worth mentioning that, looking at Fig. 8, the final value of $\Phi^n_{a,b,\mu}$ seems to bounded by similar upper bounds, both for the bottle and the box surface. Fig. 9 displays two examples of gradient ascent on both surfaces and corresponding to the red dots in Fig. 7. The trajectory on the object surface represents the location of contacts in each iteration of the gradient ascent. Note that, if we imagine the bottle to be wet or slippery, the red grasp is intuitively less stable than the blue grasp, which is confirmed by the graph of the grasp quality depicted next to the bottle.

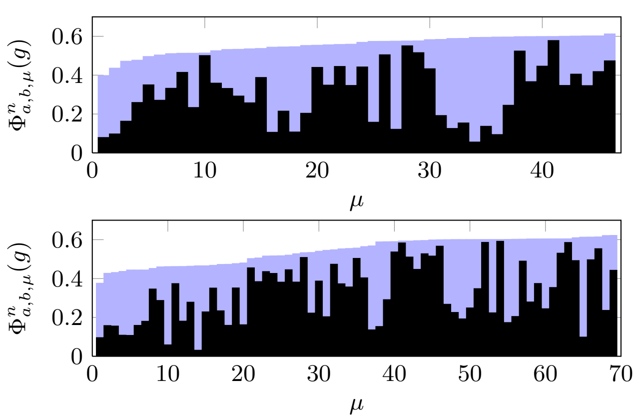


Fig. 8: Results of the gradient ascent on $H^n_{a,b,\mu}$ on the bottle (top) and the box (bottom) surface. Each bar represents a grasp sample shown in Fig. 7. Bars are sorted in ascending order of the final $\Phi^n_{a,b,\mu}(g)$ values which is depicted along the vertical axis. Black bars depict $\Phi^n_{a,b,\mu}$ values of the original grasp samples and blue bars are values after gradient ascent.

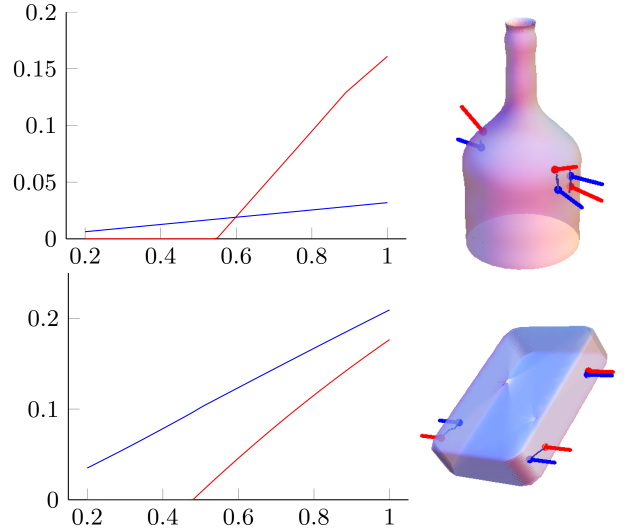


Fig. 9: An example of gradient ascent for $\Phi^n_{a,b,\mu}$ corresponding to the red points in Fig. 7. The red grasps converge to the blue ones under our gradient ascent. The trajectories are depicted as faint lines.

E. An evaluation of Algorithm 1

We now come to an evaluation of Alg. 1 which we proposed in order to synthesize force-closed contact configurations on surfaces with low friction coefficients. Again, we consider the bottle and box surfaces displayed in Fig. 5.

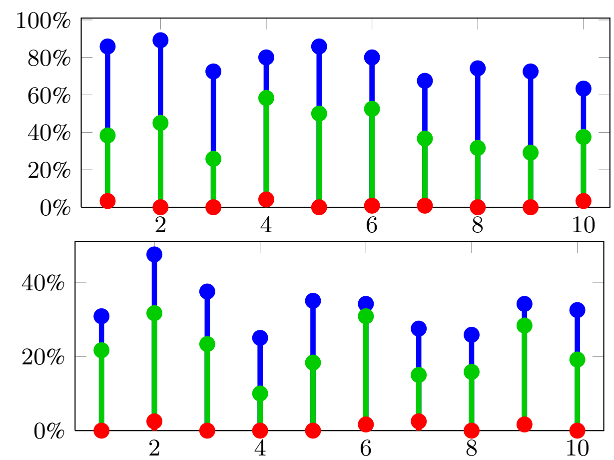


Fig. 10: Percentages of stable grasps for each of the 10 runs of our experiment with 120 grasps per experiment for the bottle (top) and box (bottom) surfaces and for $\mu_{start} = 1$ and $\mu_{end} = 0.2$. In red, we display the percentages of stable grasps for the original random grasps, in green, the percentages of stable grasps after a simple gradient ascent of $F_{\mu_{end}}$ and, in blue, the percentage of stable grasps synthesized using Alg. 1.

We sampled 10 contact points uniformly on these surfaces and computed all $\binom{10}{3} = 120$ distinct 3-contact grasp configurations for these contacts, resulting in a grasp set G for each surface. Next, we studied the effectiveness of Alg. 1 for these grasps, setting $\mu_{start} = 1.0$, $\mu_{end} = 0.2$ and descending from μ_{start} to μ_{end} in $N = 50$ steps and using a gradient ascent with step size $\delta = 0.05$ and $M = 40$ steps per iteration according to Alg. 1. We repeated this experiment 10 times for each surface, resulting in the percentages of stable grasps ($Q_{\mu_{end}}(g) > 0$) depicted by blue dots in Fig. 10.

To compare our result to a more straightforward approach, we tested the alternative approach of simply performing a gradient ascent of $F_{\mu_{end}}$ for each grasp, and with step-size $\delta = 0.05$ and for $M = 200$ iterations which resulted in the much lower percentages of stable grasps depicted in green. If we simply use a sampling based approach and evaluate the grasp quality for each grasp in our set with friction μ_{end}, almost none of the sampled grasps had positive grasp quality as indicated by red dots. Our results hence indicate that Alg. 1 can be used to successfully synthesize stable grasp configurations on objects with low friction coefficients by repeating the algorithm a few times with different random starting grasps until a stable grasp is found.

V. Conclusion

Studying grasping under uncertainty is an important area in robotics [14], [15], [16], [17], [18]. While most current state of the art approaches concentrate on aspects of imperfect object or robot models, we studied another fundamental problem in grasp synthesis in this work: the dependence of grasp stability on friction coefficients. We believe that this is an important problem when robots are to operate in open-ended environments with changing conditions.

We have in particular studied the statistics of stable grasps under changes in friction coefficients and have introduced the notion of friction sensitivity measuring the susceptibility of a grasp's quality to changes in friction. Furthermore, we have proposed and evaluated two gradient ascent algorithms for synthesizing force-closed contact configurations on parametric surfaces with potentially low friction and for the synthesis of stable grasps with low friction sensitivity.

In our future work, we would like to evaluate our approach with a real robot and study changes in friction coefficients in a real application such as a household robot cleaning dishes which might be dirty or wet, impacting heavily on the resulting friction properties. Further directions might also include the study of alternative friction models and grasp quality scoring functions.

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