Algorithms and Complexity. Exercise session 8

Approximation Algorithms

Approximation of Independent Set Let INDEPENDENT SET$-B$ be the problem of finding a maximum number of independent vertices in a graph whose degree (for each vertex) is limited by the constant B. Show that this problem is in APX, ie. it can be approximated with a constant in polynomial time.

Probabilistic not-all-equal-satisfying The NP-hard problem MAX NOT-ALL-EQUAL 3-CNF SAT is defined as follows.

INPUT: A CNF-formula consisting of clauses $c_1, c_2, \ldots, c_m$ where each clause is a disjunction of exactly three literals (variables or negated variables). The variables are $x_1, x_2, \ldots, x_n$.
SOLUTION: A variable assignment.
OBJECTIVE FUNCTION: The number of clauses that contain at least one true literal and at least one false literal.
PROBLEM: Maximize the objective function.

Since this problem is NP-hard, we want an algorithm that approximates the input within a constant in polynomial time. Constructing a probabilistic approximation algorithms for the problem to give an expected approximation factor of 4/3. Analyze your algorithm time complexity and expected approximation guarantee. You will need a randomized algorithm.

Approximation of linear inequalities MAX SAT LR$^{\geq}$ (maximum satisfiable linear subsystem) is the problem that, given a set of linear inequalities of type $\geq$, find a variable assignment that satisfies as many inequalities as possible. Construct an approximation algorithms that approximates MAX SAT LR$^{\geq}$ in factor 2.

Upper bound for approximation of homogeneous bipolar inequalities
MAX HOM BIPOLAR SAT LR$^{\geq}$ (maximum homogeneous bipolar satisfiable linear subsystem) is the same problem as MAX SAT LR$^{\geq}$ but the variables may only assume the values 1 and -1, and all inequalities are homogeneous, ie no constant terms.

Show that MAX HOM BIPOLAR SAT LR$^{\geq}$ can be approximated with a factor 2 and that MAX HOM BIPOLAR SAT LR$^{>}$ can be approximated with a factor 4.

Lower bound for the approximation of binary inequalities MAX BINARY SAT LR$^{\geq}$ (maximum binary satisfiable linear subsystem) is the same problem as MAX SAT LR$^{\geq}$ where the variables may only assume the values 0 and 1.

Show that MAX BINARY SAT LR$^{\geq} \notin$APX.
