Notation Syllabus

Vector and Matrices

A sequence of variables $x_i$ for $i$ going from 1 to $n$ can be interpreted as a column vector $x$ or a row vector $x^T$ in the corresponding standard basis. The inner product $\sum_i x_i y_i$ is denoted as $x^T y$. The length of a vector is $\sqrt{x^T x}$ and is denoted as $|x|$.

A double-indexed set of variables $a_{ij}$ is interpreted as a matrix $A$. We usually denote vectors in small caps and matrices in big caps, but this convention may change to adapt to the context.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

The product between a column vector $x$ with $m$ values and a row vector $y^T$ with $n$, is denoted by $xy^T$ and is a matrix of rank at most 1 with $m$ rows and $n$ columns. The product between two matrices $A$ and $B$ with $m$ rows and $n$ columns is denoted as $A \cdot B$.

$$xy^T = \begin{bmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{bmatrix}, \quad A \cdot B = \begin{bmatrix} a_{1,1} b_{1,1} & a_{1,2} b_{1,2} & \cdots & a_{1,n} b_{1,n} \\ a_{2,1} b_{2,1} & a_{2,2} b_{2,2} & \cdots & a_{2,n} b_{2,n} \\ \vdots \\ a_{m,1} b_{m,1} & a_{m,2} b_{m,2} & \cdots & a_{m,n} b_{m,n} \end{bmatrix}$$

The notations $x \geq y, x \leq y, x < y, x > y$ for vectors mean that the corresponding inequalities hold pointwise (i.e., if $x \leq y$ then $x_i \leq y_i$ for every $i$). We assign a similar meaning to notations $A \geq B, A \leq B, A < B, A > B$. The notation $A \succeq 0$ means that $A$ is positive semidefinite, the notation $A \succ 0$ that $A$ is positive definite. The notation $A \succ B$ and $A \succeq B$ are shortcuts for, respectively, $A - B \succ 0$ and $A$

Linear programs

We denote linear programs in one of this fashion.

$$\text{maximize } c^T x \quad \text{subject to } Ax \leq b$$

$$\text{maximize } c^T x \quad \text{subject to } Ax = b \quad x \geq 0$$

For a linear program such that we denote as $P$ the set of feasible solution for that program, and as $P_I$ the convex hull of set of integer feasible solutions of the same program, i.e., $P_I = \text{convexhull}(\mathbb{Z}^n \cap P)$. 

\[1\]
Semidefinite programs

We denote semidefinite programs (SDP) in one of this way.

\[
\begin{align*}
\text{maximize} & \quad c^T x \\
\text{subject to} & \quad A_1 \cdot X \leq b_1 \\
& \quad A_2 \cdot X \leq b_2 \\
& \quad \vdots \\
& \quad A_\ell \cdot X \leq b_\ell \\
& \quad X \succeq 0
\end{align*}
\]

\[
\begin{align*}
\text{maximize} & \quad \sum_i c_i (v_0^T v_i) \\
\text{subject to} & \quad \sum_{i,j} a_{ij} (v_i^T v_j) \leq b_1 \\
& \quad \sum_{i,j} a_{ij}^2 (v_i^T v_j) \leq b_2 \\
& \quad \vdots \\
& \quad \sum_{i,j} a_{ij}^\ell (v_i^T v_j) \leq b_\ell \\
& \quad |v_0| = 1
\end{align*}
\]

There is no much difference if the constraints are given in form of equations of in form of inequalities. For semidefinite programs such that we denote as \( \mathcal{P} \) the set of feasible solution for that program, and as \( \mathcal{P}_I \) the convex hull of set of integer feasible solutions of the same program, i.e., \( \mathcal{P}_I = \text{convexhull}(\mathbb{Z}^n \cap \mathcal{P}) \).

Other notation

For \( n \leq 0 \) and \( k \leq n \),
\[
\binom{n}{\leq k} = \sum_{i=0}^{k} \binom{n}{i}
\]

For a set \( S \) and integer \( k \),
\[
\binom{S}{k} = \{ T \subseteq S, |T| = k \} \quad (\leq k) = \bigcup_{i=0}^{k} \binom{S}{i}.
\]