Designing Digital Filters by Model Order Reduction Techniques

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***Abstract*—In this study, we compare properties of the reduced IIR filters obtaining from a linear-phase FIR filter by applying three different model order reduction techniques; Balanced Model Order Reduction (BMR), Singular Perturbations Method (SPR) and Optimal Hankel-Norm Approximation (OHA). As a prototype example, we use a low pass FIR filter which can be used in complex audio power amplifiers and use that to obtain a low order, stable IIR filter. Magnitude specification was satisfied by all three techniques. However, deviation could be seen in phase linearity. We found that the SPR method is more applicable in reducing order of FIR filters with smaller pass-band.**

I. INTRODUCTION

Linear-phase digital filters is being used in many industrial engineering applications, such as digital class-D audio power amplifiers (ZePoC). The linear phase filters allow distortion free transmission of signals. It is well known that adequate selection of the impulse response of a Finite Impulse Response (FIR) filter yields a linear phase characteristic, i. e. a constant group delay. However, the main drawback of FIR filters is their high order with respect to IIR filters, which have an equivalent magnitude response characteristic. A long signal delay and high power consumption are two disadvantages of higher order filters. Model order reduction techniques [3, 2] offer a solution for designing low order IIR filters preserving amplitude and phase characteristics in the pass band of the original FIR filter [5, 6]. In this study, we compare three such techniques, which we are going to discuss, in obtaining IIR filters from designed FIR filter.

II. MATERIALS AND METHODS

In this section, we describe the prototype application on which the model order reduction techniques were applied and follows a brief discription about the three techniques used.

A. Problem Formulation

We select to design a prototype low-pass FIR filter which can be used in ZePoC systems, with the following specifications: a cut-off frequency of $48kHz$, a stop band suppression of $-80dB$ and a sampling rate of $400kHz$. The FIR filter was designed using Matlab signal processing toolbox (fdatool) and it is an equiripple LP filter with pass-band ripple of 0.0575 (see Figure 1).

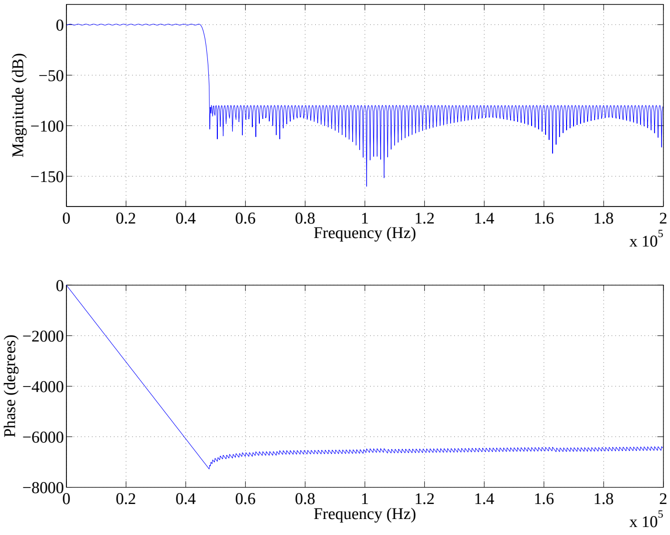

Fig. 1. Low pass FIR filter characteristics.

B. Model Order Reduction techniques

Various techniques of model reduction have been proposed in technical literature [6]. In this investigation, we use three such techniques; Balanced Model Order Reduction (BMR), Singular Perturbations Method (SPR) and Optimal Hankel-Norm Approximation (OHA). All these reduction methods are based upon obtaining a balanced realization of the original system, and then removing the weakly controllable and observable states [2, 4]. All can transform high order FIR into low order IIR filters, while maintaining original magnitude and phase characteristics. In theory, SPR gives better approximation at low frequencies, whereas BMR approximates the FIR better at high frequencies. However, for the design of highly selective filters, it is important that

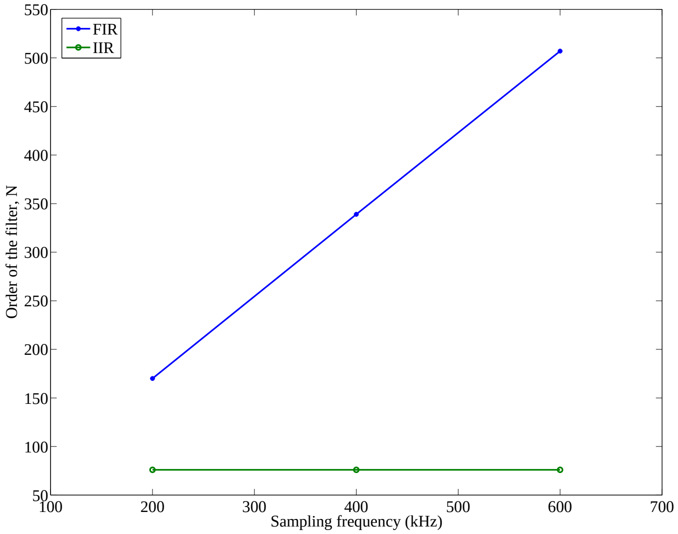

Fig. 2. Relation between order of the FIR and the reduced IIR filter and sampling frequency.

the error of approximation is small for all the frequencies especially in the pass band.

For the comparison, we use magnitude response error and the group delay, which is related to the phase linearity. For all these model order reduction techniques, it is common that stability can be guaranteed if the original filter/model is stable, and which is true in this case.

C. *Effect of Sampling Frequency*

The order of the FIR filter increases with the sampling frequency. Thus the number of *Hankel Singular Values (HSV)* is also increasing with the sampling frequency. However, the pass band is constant, the stop band edges are steady but only the stop band increases. The pass band and previous stop band are described by existing HSVs, so that new HSVs describe broadened stop band and do not change previous HSVs. New HSvs are smaller and can be removed during reduction [5]. Therefore reduced IIR filters with different sampling frequencies have the same order if all other properties are equal (see Figure 2).

III. RESULTS AND DISCUSSION

In a FIR filter, the order, number of states and the number of HSVs are the same. Therefore magnitude of a HSV tells the importance of the state [1]. The states that are recognized as weak can be removed from the filter due to their negligible influence. The order of the designed FIR filter with the sampling frequency of 400kHz is 339 and the sudden drop can be seen in the HSVs at the order of 76 (see Figure 3). Thus 76 was chosen as the common truncation point for all the methods. Eventhough we tested HSVs for different sampling frequencies (200 and 600 kHz), comparison was done only for the FIR filter designed with 400kHz sampling frequency.

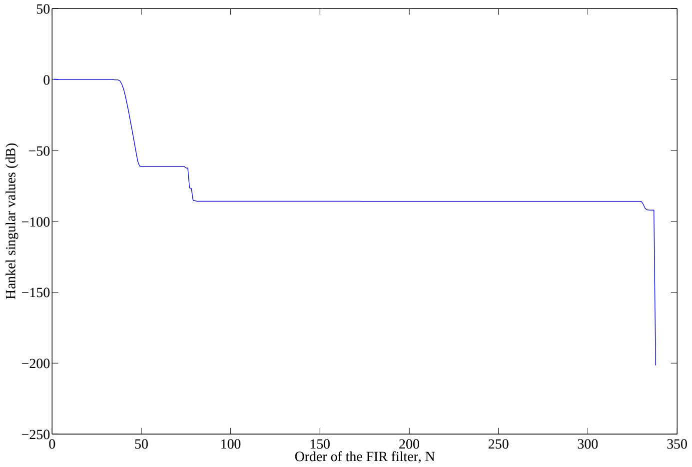

Fig. 3. Hankel singular values of FIR filter, N = 339.

The IIR filter, obtained using SPR method, has the smallest magnitude response error and is maintaining almost constant group delay in the pass-band region (see Figure 6). The magnitude response error ranges from -120dB to -80dB with an avarage around -100dB within the pass-band. The results achieved by the other two methods are also significant. As figure 4 shows, the error magnitude of the OHA is oscillating with the gain around -80dB. Furthermore, the group delay is decreasing with the increasing frequncy within the pass-band. A better result is achieved by the BMR method, because the average magnitude response error is around -85dB and it is almost flat except towards the cut-off region. However, phase linearity is deteriorated and it is somewhat constant towards the high frequency range of the pass band (see Figure 5). Note that the abrupt change in the group delay towards the transition point (from pass band to stop band).

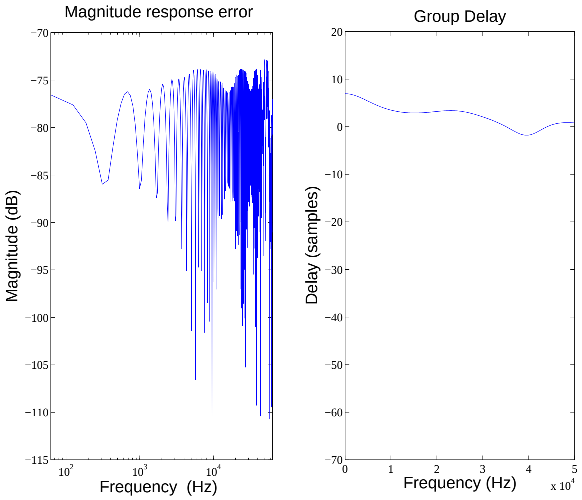

Fig. 4. Magnitude response error and group delay error for the OHA method.

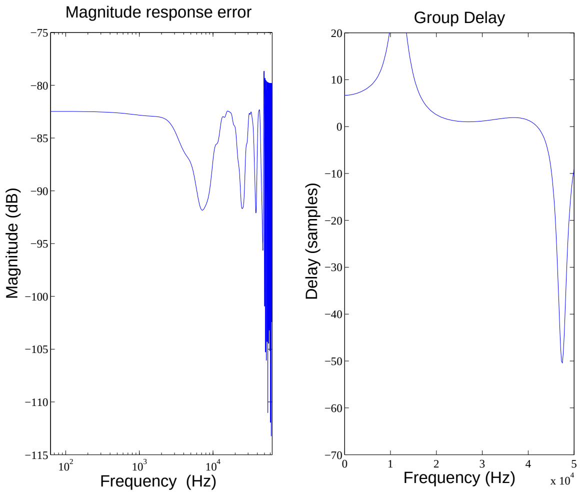

Fig. 5. Magnitude response error and group delay error for the BMR method.

The common opinion is that IIR filters cannot be stable and have a linear phase at the same time. But it is not accurate, especially when the filter is obtained from an FIR filter using model order reduction method. All the IIR filters, that were obtained from the original stable FIR, were stable.

IV. CONCLUSIONS

All three model order reduction techniques were able to transform the higher order FIR into low order IIR, while maintaining the magnitude specification of the original FIR filter. However the phase linearity was different from each other. The order reduction with matching DC gain or singular perturbation method produces the smallest error in phase linearity specially in low frequency range. Hence we conclude that this method is more applicable for filters with smaller pass-band. Furthermore, the model order of the FIR increases with the sampling frequency, but the order of the reduced model is independent of the sampling frequency.

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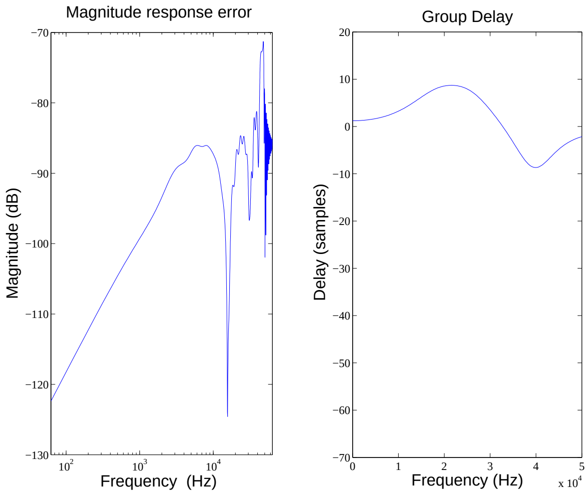

Fig. 6. Magnitude response error and group delay error for the SPR method.

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