KTH Computer Science
and Communication

Homework IV, Approximation Algorithms 2010

Due on Tuesday November 23 at 10.15 (hand in at start of lecture or send an email to osven@kth.se). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Ola Svensson. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually. On this problem set the first problem is about finding information and here you should feel free to use any source.

1 (30 p) Prove the Strong duality theorem (Theorem 12.1 in Vazirani's book). You are allowed to use any source for this exercise but the proof should be written using you own words. (Many proofs use Farkas' Lemma which you should explain if you use it but not necessarily prove.)

2 (30 p) Consider the standard IP formulation of the Set Cover problem, and its LP-relaxation. Consider the algorithm that picks all sets associated with non-zero values in the optimal fractional solution to the LP-relaxation. Show that this algorithm achieves an approximation guarantee of f if each element can be covered by at most f sets.

Hint: Use the complementary slackness conditions to prove this.

3 (40 p) This exercise is about generalizations and modifications to Jain & Vazirani's 3-approximation algorithm for the metric uncapacited facility location problem that was explained in class and that can also be found in Chapter 24 of Vazirani's book.

3a (20 p) Consider the following modification. Define the cost of connecting city j to facility i to be c_{ij}^2. The c_{ij}'s will still satisfy the triangle inequality (but the new connection costs, of c_{ij}^2, do not). Show that their approximation algorithm achieves an approximation guarantee of 9 in this case.

(This is exercise 24.6 in Vazirani's book.)

3b (10 p) It is reasonable to assume that as more cities connect to a facility, it's opening cost increases. Hence, assume that the opening cost of a facility is now $f_i + l \cdot s_i$ when there are l cities connected to facility i. How can we use Jain-Vazirani's algorithm to solve this variant?

3c (10 p, bonus exercise) Consider the variant in which each facility can be opened an unbounded number of times and if facility i is opened y_i times, it can serve at most $u_i y_i$ cities. Give a 6-approximation algorithm for this variant.

Hint: reduce the problem to that considered in 3*b so that you only lose a factor of* 2 *in the approximation guarantee.*