KTH Computer Science
and Communication

# Homework V, Approximation Algorithms 2010

Due on Tuesday, December 7, at 10.15 (hand in at start of lecture or send an email to moemke@kth.se). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Tobias Mömke. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually.

**1** (25 p) Let $Ax \geq b$ be the constraints of an LP, where $x$ is a vector containing an assignment of values to the variables, i. e., $x$ is a point in the polytope defined by the constraints. As defined in the lecture, $A^=$ is $A$ restricted to the rows corresponding to tight constraints with respect to $x$ and $A^=_x$ is $A^=$ restricted to the columns that correspond to nonzero values in $x$.

Show that $x$ is an extreme point solution if and only if $A^=_x$ has full column rank.

You may use information from the internet including the proof stated in the book draft, but you have to and use your own words and provide references to your sources.

**2** (45 p) This exercise is about the iterative linear programming approach for bipartite matchings.

**2a** (10 p) Show that the LP for the bipartite matching problem is not integral if we allow graphs that are not bipartite. (This is exercise 3.2 in the book draft from Lau, Ravi, and Singh.)

**2b** (35 p) Consider a modified version of the bipartite matching problem: we want to *minimize* the weight of a *perfect* bipartite matching (a matching, where each vertex is incident to exactly one edge, i. e., there are no isolated vertices left). You may assume that only those bipartite graphs are given as input that have a perfect matching.

State a linear programming relaxation for the modified problem, design an iterative algorithm analogous to that for the maximum bipartite matching problem, and show that the algorithm finds at least one assignment $x_e = 0$ or $x_e = 1$ in each iteration. (This is exercise 3.1 in the book draft from Lau, Ravi, and Singh.)

**3** (30 p) Within the iterative algorithm of the *generalized assignment problem*, constraints are removed if a machine $i$ has two jobs and $\sum_{j \in J} x_{ij} \geq 1$.

Consider instead $\sum_{j \in J} x_{ij} \geq \alpha$ for some positive constant $\alpha$. Analyze the results of the lecture (Section 3.2 of the book draft) depending on $\alpha$. (You do not have to state the parts of the proofs that are identical to the lecture; you may refer to the lecture instead.)