Minimizing Mission Risk in Fuel Constrained UAV Path Planning*

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I. Introduction

One of the main reasons for using a UAV instead of a manned aircraft is that the mission that need to be carried out is very dangerous. In such cases, we believe that the risk of losing the vehicle should be made an explicit and important part of the mission planning. The mentioned risks can often be estimated on a small scale, using detailed simulation models, but the tools to propagate such estimates to the overall mission planning level has been missing. In this paper we propose one way to take such local risk estimates into account in the mission planning, and at the same time make the tradeoffs against other path properties, such as fuel- or time constraints, explicit. First we show how to rewrite the accumulated risk estimate into a form that matches the formulation of the so-called weight constrained shortest path problem. Then we approximately solve it using a series of regular shortest path problems. These are in turn easily solved by standard algorithms such as Dijkstras[2] or A*.[3]

UAV path planning has been extensively studied in the literature.[4–11] However, to the best of our knowledge, only Chaundry et al.[10] give the operator an explicit estimate of the risk associated with the proposed mission path. Furthermore, only Zabarankin et al.[12] enable the user to clearly state constraints in terms of available fuel or time. In contrast to both Chaundry et al.[10] and Zabarankin et al.[12] who focus on the interactions of the UAV with a single SAM battery, the proposed approach enables planning of missions dealing with multiple threats, in the same way as the papers by Beard and others.[4,5,8,11]

The organization of this paper is as follows. In Section II, we state the proposed risk vs fuel path planning problem as well as two other path planning problems from the litterature. We then show how the proposed problem can be rewritten and approximately solved in Section III. A detailed example illustrating the approach is presented in Section IV, and finally, the paper is concluded in Section V.

II. Problem Formulations

In this section we will define three problems: the shortest path problem (SPP), the weight constrained shortest path problem (WCSPP) and the risk vs fuel path problem (RFPP). These three problems, and the corresponding acronyms, will be used throughout the paper. Before defining the three problems, we briefly review some graph notation.

Definition 1 *A graph $G = (V, E)$ is a collection of vertices $V = \{1 \dots n\}$ and edges $E \subset V \times V = \{(i, j),\ i, j \in V\}$. A path P is a sequence of vertices $P = (v_1, v_2, \dots, v_m)$ such that the set of consecutive pairs, $E(P) = \{(v_i, v_{i+1}),\ i = 1 \dots m-1\}$ is a subset of E. Each edge in a graph can furthermore be assigned weights $w_{ij} \in \mathbb{R}_+$ and costs $c_{ij} \in \mathbb{R}_+$.*

We are now ready to state a general SPP.

*Parts of these results were previously published in an AIAA conference paper.[1]

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Problem II.1 (SPP) *Given a graph $G = (V, E)$, costs $c_{ij} \in \mathbb{R}_+$ and start and destination vertices s and d. The Shortest Path Problem (SPP), is defined as follows:*

$$\begin{aligned} \min_{P} \quad & \Sigma_{(i,j)\in E(P)} c_{ij}, \\ s.t. \quad & s, d \in P. \end{aligned} \tag{1}$$

That is, find the path P from s to d such that the sum of costs c_{ij} is minimized.

The weight constrained version of the SPP is similarly defined,[12] as

Problem II.2 (WCSPP) *Given a graph $G = (V, E)$, weights $w_{ij} \in \mathbb{R}_+$, costs $c_{ij} \in \mathbb{R}_+$ and start and destination vertices s and d. The Weight Constrained Shortest Path Problem (WCSPP), is defined as follows:*

$$\begin{aligned} \min_{P} \quad & \Sigma_{(i,j)\in E(P)} c_{ij}, \\ s.t. \quad & \Sigma_{(i,j)\in E(P)} w_{ij} \leq W, \\ & s, d \in P. \end{aligned} \tag{2}$$

That is, find the path P from s to d such that the sum of costs c_{ij} is minimized while the sum of weights w_{ij} is kept below the bound W.

At this point we note that the WCSPP is in fact an NP-hard problem,[13] which means that large problem instances are in practice very hard to solve to optimality in reasonable time. Having reviewed the SPP and the WCSPP we now define the RFPP and discuss its properties in two remarks.

Problem II.3 (RFPP) *Given a graph $G = (V, E)$, with weights $w_{ij} \in \mathbb{R}_+$, corresponding to the amount of fuel needed to traverse the edge between node i and j, and costs $R_{ij} \in [0, 1]$ corresponding to the risk of losing the vehicle when traversing the edge between note i and j, and start and destination vertices s and d. The Risk vs Fuel Path Problem (RFPP) is defined as*

$$\begin{aligned} \max_{P} \quad & \Pi_{(i,j)\in E(P)} (1 - R_{ij}) \\ s.t. \quad & \Sigma_{(i,j)\in E(P)} w_{ij} \leq W \\ & s, d \in P, \end{aligned} \tag{3}$$

i.e. finding the path that minimizes the accumulated risk of losing the vehicle, while the required amount of fuel is less than W

Remark 1 *Note that the UAV survival probability for the whole mission only equals $\Pi_{(i,j)\in E(P)}(1 - R_{ij})$ if the path segment risks, R_{ij} are uncorrelated. This is obviously a quite strong assumption. However, we believe that it still makes more sense than the options of summing threat distances, radar exposure, or other measures of UAV risk. Furthermore, the choice of graph, G and corresponding path segments, can be made to increase the validity of this assumption by e.g. using a Voronoi graph where segment lengths are on the same order of magnitude as the threat regions.*

Remark 2 *Throughout this paper we are using fuel as the main constraint, W. However, other path properties such as distance or time of traversal can just as easily be used instead of fuel.*

III. Proposed Solution

In this section we will see how to solve the RFPP by rewriting it as a WCSPP which in turn can be solved approximately using a series of SPPs.

A. Writing the RFPP as a WCSPP

Note that the fuel constraint and path criteria of the RFPP maps nicely into a WCSPP. The objective function is however a product in the RFPP, reflecting the combined probability of surviving all the path segments, and not a sum of costs as in the WCSPP. In many papers, such as Beard et al.[4] and Zabarankin et al.[12] overall risk is not minimized. Instead, the objective functions captures a sum of kill probabilities, a sum of inverse squared threat distances, or a sum of some other measure of badness. Here however, we can find the solution that minimizes the actual risk estimate, by an elaborate choice of edge costs described in the following Lemma.

Lemma 1 *Let* $c_{ij} = log\left(\frac{1}{1-R_{ij}}\right)$*. Then the RFPP (Problem II.3), has the same solution as the WCSPP, (Problem II.2).*

Proof Since the constraint are the same in Problems II.3 and II.2, we only need to show that the objective function of Problem II.3, i.e. $\max \Pi_{e_i \in Path}(1 - R_{ij})$ can be manipulated into something of the form $\min \Sigma_{e_i \in Path} c_{ij}$ without changing the corresponding optimal solution. Now, using "$\Leftrightarrow$" to denote that two optimization problems have the same solution we have

$$\max\ \Pi_{e_i \in Path}(1 - R_{ij}) \Leftrightarrow \max log(\Pi_{e_i \in Path}(1 - R_{ij})),$$

since log is a strictly increasing function and can therefore be applied to any objective function in an optimization problem without changing the corresponding optimal solution. Using $log\Pi = \Sigma log$ we get

$$\max log(\Pi_{e_i \in Path}(1 - R_{ij})) \Leftrightarrow \max \Sigma_{e_i \in Path} log(1 - R_{ij}).$$

Furthermore, since a maximization problem can be turned into a minimization problem by multiplying the objective function by -1, we have

$$\max \Sigma_{e_i \in Path} log(1 - R_{ij}) \Leftrightarrow \min \Sigma_{e_i \in Path} - log(1 - R_{ij}),$$

and finally applying $-logx = log(1/x)$ we get

$$\min \Sigma_{e_i \in Path} - log(1 - R_{ij}) \Leftrightarrow \min \Sigma_{e_i \in Path} log(1 - R_{ij})^{-1},$$

which is exactly the proposed c_{ij}. ■

B. Solving the WCSPP

In Zabarankin et al.,[12] a modified version of the Label Setting Algorithm[13] was used. In this paper we use bisection search and a standard shortest path algorithm, such as Dijkstra,[2] to get a good feasible solution and an upper bound on the gap to optimum. The details of this approximation scheme can be found in our previous work.[1] Knowing how to transform the RFPP into a WCSPP which can be approximately solved by a series of SPP or a Label Setting Algorithm,[13] we go on to illustrate the approach with an example.

IV. Example Problem

In this section, we will find minimum risk paths for different fuel constraint levels in the example scenario depicted in Figure 1.

The task of the planner is to find a path from the start position at (250,500) to the target position at (450,400). As can be seen in Figure 1, the target is located within a threat area. This is fairly reasonable, since the target is supposed to be worth flying to and thus probably worth protecting. It should also be noted that this case immediately disqualifies approaches where all threat areas must be avoided. In order to formulate a RFPP we need to estimate the risks of flying in different parts of the mission theatre. Finding a risk estimate for a given flight path segment is a task that in itself can be made arbitrarily complex, as noted in the Remark below.

Remark 3 *We acknowledge that risks are very hard to estimate. However, using intelligence information and high fidelity simulation models of threat systems and electronic warfare components, it is possible to make coarse estimates, similar to the ones found in the paper by Chaudhry et al.[10] Furthermore, incorporating such results in a UAV path planner is one way to make sure that the knowledge of the subject matter experts is indeed put to use by the men and women making tactical decisions in the field.*

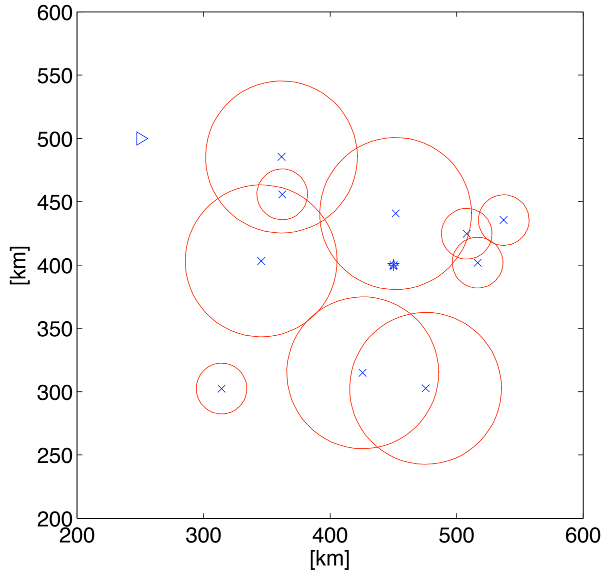

Figure 1. The setup of a path planning problem. Threat regions are illustrated by circles.

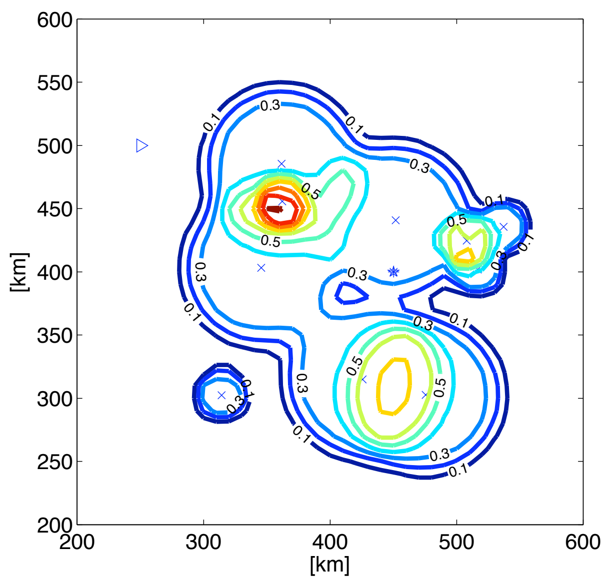

Figure 2. The estimated probability of loosing the UAV when flying a path segment.

We do not go into details about risk estimation here, but note that a threat level map, indicating the risk of flying a path segment of given length, could look like the one depicted in Figure 2.

The next step is to choose the form of the Graph G in which to plan the path. There are many options, including Voronoi grids,[4] visibility graphs,[14] or a classical grid. This choice should be made depending on the environment and nature of each type of UAV mission, but since graph choices is not the focus of our work we choose the simple grid depicted in Figure 3. Note that this choice is made to illustrate the main concepts of this paper, not to give the best possible path. Obviously, paths restricted to eight main directions of travel are not tactically optimal.

To illustrate the standard way of UAV path planning we first form a SPP where the cost of each path segment is calculated as a linear combination of path length and the risks of Figure 2. Applying a standard

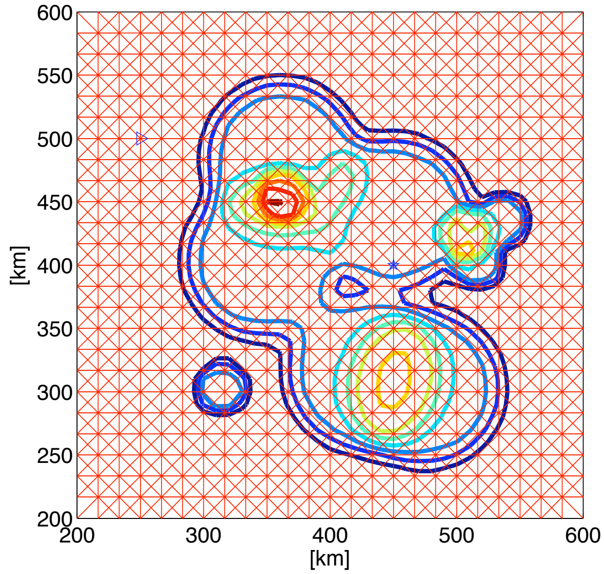

Figure 3. One possible choice of the graph G, drawn on top of the risk estimates.

Dijkstra type of algorithm,[2] we get the results in Figure 4.

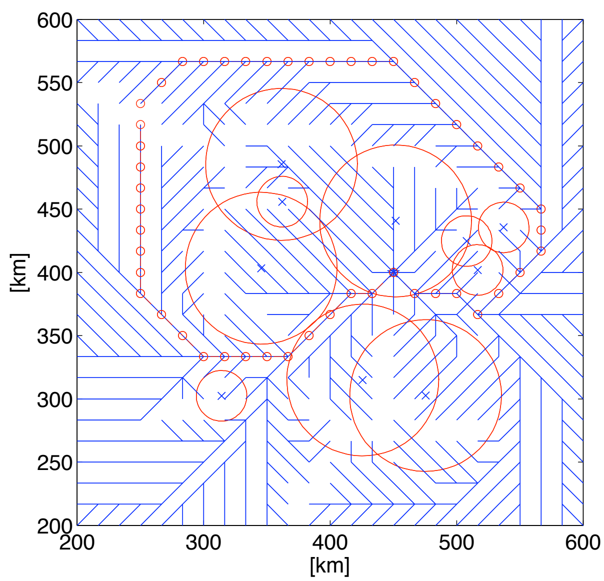

Figure 4. The solution paths to a Shortest Path Problem using a weighted sum of distance and threat exposure.

Figure 4 illustrates the lack of transparency offered to the operator. The difference between the paths starting at (250,520) and (250,530) is significant. From the former position a south-bound route is recommended that is short, but fairly risky. From the later position, a north-bound route is suggested that is longer but safer. These differences should be controlled by operator preferences, not by small changes in starting position.

The objective of the path planning is to find the least risky path respecting the given fuel (or time, see Remark 2) constraint. If there is a shortage of fuel, and the UAV is at (250,530) the suggested northern route is not an option. And similarly, if there is a lot of fuel available and the UAV is at (250,520), the southern route is unnecessarily dangerous, and the northern route is the preferred option. Using the WCSPP instead of the SPP enables the operator to make this distinction directly, instead of having to tune the coefficients in the combination of path length and risk. Furthermore, using the RFPP formulation the operator gets clear risk estimates for each path, that enable him or her to make better tactical decisions.

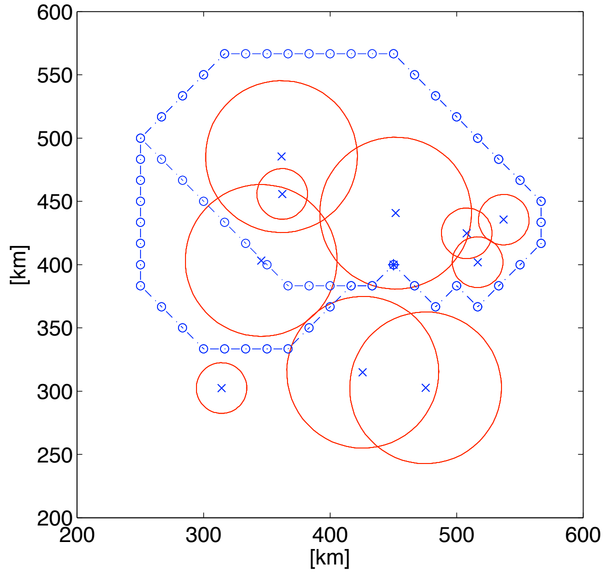

Figure 5. Three flight paths with different fuel constraints.

The results of the RFPP is shown in Figure 5. Paths for three different fuel bounds, corresponding to three different maximal path lengths are shown. The path corresponding to a range of 300km is forced to go straight through a threat area, barely avoiding the overlapping threat region, in order to reach the target. The corresponding overall mission risk is estimated to a survival probability of only 6%. A range of 400km corresponds to the southmost path with a survival probability of 22%, and finally a range of 600km gives the northmost path and a survival probability of 58%. These numbers are likely to make the operator abort the mission if there is only fuel for 300km. Note again that the numbers are very coarse estimates, and when making the decision, the operators can often take information into account that is not available to an algorithm. However, a survival probability is much easier to interpret than e.g. a number representing overall mission radar exposure, see Remark 3. Finally, please note again that the purpose of the example is to illustrate the idea of risk vs fuel path planning, not to serve as an example of a good UAV mission path. For example, the zig-zagging near (500,370) is tactically insane. A more complex choice of G must be used to plan real missions, perhaps in combination with some post processing of the path.

V. Conclusions

In this paper we have shown that it is possible to improve the transparency of UAV path planning in two ways. The first way is to include fuel- or time constraints as actual optimization constraints, instead of as parts of the objective function. The second way is to use risk estimates of individual path segments in the planning. The combination of both these suggestions enables the algorithm to maximize an estimate of the overall mission success rate, given the constraints imposed by the available amount of fuel, and present this estimated rate to the UAV operator making tactical decisions.

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