Vehicle State Estimation Using GPS/IMU Integration

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***Abstract*—New driver support systems require knowledge of the vehicle position with great accuracy and reliability. Satellite navigation (GNSS) is generally insufficiently accurate for positioning and as an alternative to using a ground station, combinations with high quality motion sensors are used in so-called Inertial Navigation Systems. However the system specifications and related cost are not suitable for Automotive applications. In this article a Vehicle model based concept is presented in a state estimator setup that will use signals that are available on modern vehicles. An extension of a commonly used Bicycle representation of the vehicle is applied with an automated tuning for signal disturbances. For coping with different update frequencies from GNSS and motion sensors a Bezier extrapolation is used. The resulting Adaptive Kalman Filter approach is compared to recorded signals from driving tests with an instrumented vehicle. The comparison shows that with the new setup a clear improvement is achieved for the vehicle motions compared to more commonly used Kalman filtering. This verifies that sensor disturbances can better be compensated with the presented concept, and also better results for positioning can be expected.**

I. Introduction

As a part of the High Tech Automotive System (HTAS) Driving Guidance program, the Connected Cruise Control (CCC) system [1], supports the driver with speed, heading and lane advice to anticipate to and eventually prevent congestion. As a first step, a robust method is needed to provide accurate assessment of the vehicle position at the road and direction of its movement (e.g. the vehicle speed, acceleration and heading). This study focuses on the concept of using vehicle state estimation in combination with multirate sensor integration.

State estimation is a model based method where a combination is made of a prediction model and sensor signals. Using a kinematic prediction model may be generic and easy to implement, however it lacks information about physical dynamics of the vehicle. This leads to more dependency on external aiding sensors [6], [13] and noisy estimates [6]. High fidelity vehicle models, e.g, those proposed in [9], capture the most significant dynamics accurately only when all parameters are known, but they can be too computational demanding for a real-time application and may be difficult to implement and tune. For the ease of implementation and computation, 2DOF linear Bicycle_model is widely used. However, some parameters, e.g, tyre stiffness, vehicle inertia etc, are still required for this model. Different on-line parameter estimation methods, e.g. those declared in [1], [5] and [4], are proposed to compensate the parameter uncertainties. These methods require knowledge of relative position between the sensor and the vehicle center of gravity (COG), which itself may be changing and is difficult to estimate. Therefore, an alternative is adopted to solve this difficulty [8], which applies innovation-based adaptive estimation.

Global Positioning System (GPS) is the most popular navigation sensor, despite its significant noise sources, e.g, multipath effect, shifts in the satellite orbits and clock errors, it has typically a sampling rate lower than 10 Hz. In [3], [12] and [6], different methods have been proposed in order to integrate GPS with a higher sampling rate Inertial Measurement Unit (IMU). However, in the Extended Kalman Filter (EKF) scheme, different sampling rates generally leads to a downgrade of the performance and may even cause stability problems. According to [2], primitive functions could be implemented in multi-rate hold functions and provide better performance in terms of smoothness than simple (i.e. zero, first and second order) hold functions.

In the current work, a new vehicle model is proposed to achieve a better accuracy for positioning using an innovation based technique in an adaptive estimation of signal disturbances. The innovation_based adaptive estimation technique is also incorporated in a general (EKF) scheme. The sideslip angle is chosen as the case study for assessing the performance of the new approach because it is typically difficult and challenging to estimate [7], but at the same time highly relevant for vehicle control applications. Moreover, sideslip angle is dependent on other states and therefore its accuracy is only guaranteed when the other states are accurately estimated [13].

The remainder of the paper is organized as follows. The new vehicle model, GPS measurement extrapolation and adaptive filtering are introduced in Section II, III and IV respectively. The experimental setup is described in Section V-C. Corre-

[1]This work is done in the framework of HTAS CCC project, which involves a Dutch consortium: Technical Universities from Delft (project leader), Eindhoven and Twente, Research Organization TNO, Industry Partners, Navteq, NXP, Technolution(SME) and Clifford(SME)

sponding experimental results are shown in Section V-B1, V-B2 and V-D accordingly. An overall view of the proposed vehicle state estimator performance can be found in Section V-C. The final conclusion is drawn in Section VI.

II. New Vehicle Model

The popular linear Bicycle_model representation of the vehicle is characterized by the linearity of tyre characteristics and generally captures the vehicle dynamics up to about 40% of the friction limit. It will be pointed out in Section V-B1 that *Fourier analysis* of the innovation sequences indicates bias error in the state estimates obtained using the Bicycle_model. Therefore, modifications of this model are proposed. In the light of [3], bias state could be combined with kinematic relation to improve the filter performance:

$$\begin{aligned} a_x &= \ddot{x} + b_x + \omega_x, \\ \dot{b}_x &= -\frac{1}{T} b_x + \frac{1}{T}\omega_{bias}, \end{aligned} \tag{1}$$

where ω_x and ω_{bias} are white noise for longitudinal velocity $\dot{x}$ and its bias state b_x, respectively. Time constant T in (1) determines how fast the bias state b_x evolves. Notice that $\ddot{x}$ represents the true longitudinal acceleration and a_x represents the estimated one. Because the true longitudinal acceleration is not known, measurements from accelerometer will be used instead. Based on this information, we add bias states to the yaw rate and lateral velocity states of the Bicycle_model [2] described in [9] as shown in (2).

$$\begin{aligned} a_{x_b} &= \ddot{x}_b + b_{x_b}, \\ \dot{b}_{x_b} &= -\frac{1}{T} b_{x_b}, \\ a_{y_b} &= \frac{C_1 + C_2}{mu}(v_{y_b} - b_{y_b}) \\ &\quad + (\dot{x}_b + \frac{aC_1 - bC_2}{mu})(\dot{\psi} - b_{\dot{\psi}}) + \frac{C_1}{m}\delta, \\ \ddot{\psi} &= \frac{aC_1 - bC_2}{I\dot{x}_b}(v_{y_b} - b_{y_b}) \\ &\quad + \frac{a^2C_1 + b^2C_2}{Iu}(\dot{\psi} - b_{\dot{\psi}}) + \frac{aC_1}{I}\delta, \\ \dot{b}_{y_b} &= -\frac{1}{T} b_{y_b}, \\ \dot{b}_{\dot{\psi}} &= -\frac{1}{T} b_{\dot{\psi}}, \\ \dot{\psi} &= \dot{\psi}, \\ \dot{x}_e &= x_b \cos\phi + y_b \sin\phi, \\ \dot{y}_e &= -x_b \sin\phi + y_b \cos\phi. \end{aligned} \tag{2}$$

Here, the remaining noise is assumed to be zero mean Gaussian. a_{x_b} and b_{x_b} describing the longitudinal kinematic relationship similar to (1). Bias states b_{y_b} and $b_{\dot{\psi}}$ are added to lateral velocity state v_{y_b} and yaw rate state $\dot{\psi}$ respectively. Using yaw angle ψ, the vehicle body frame variables indicated by subscript b, are related to the navigation frame variables x_e and y_e rather than using sideslip angle as in [3] and [7]. Comparison between the new model given by (2) and the Bicycle_model is drawn in Section V-B1.

[2]Subscripts b and e of the states indicate vehicle body frame and global frame respectively

III. GPS extrapolation using Bezier curve

In this work, *Bezier curve* given by (3) is used to predict the missing GPS measurement $y(t)$ in between $t_j \le t \le t_{j+1}$, using the last $n+1$ GPS measurements $u(t)$:

$$\begin{aligned} y(t) &= \sum_{l=0}^{n} f_{n,l}(t - t_j)u(t_{j-l}), \\ &where \\ f_{n,l}(t - t_j) &= \frac{n!}{l!(n-l)!}(\frac{t - t_{j-n}}{t_j - t_{j-n}})^{n-l}(-\frac{t - t_j}{t_j - t_{j-n}})^l. \end{aligned} \tag{3}$$

GPS measurements $u(t)$ serve as *control points* for the *Bezier Curve* and their sampled time t works as the corresponding weights. Using the predicted measurements $y(t)$, the multirate integration regresses back to a normal integration problem. Example of GPS measurements extrapolation will be shown in Section V-B2.

IV. Adaptive Kalman Filtering

Insufficiently known prediction model accuracy, noise statistics and a changing estimation environment (different driving maneuvers) usually reduce precision of the vehicle state estimation or introduce biases.

According to [8], based on the whiteness of the filter innovation sequence the process noise covariance Q could be adapted based on the maximum likelihood criterion as follows:

$$\hat{Q}_k = K_k \hat{C}_{vk} K_k^T, \tag{4}$$

where K_k is the calculated Kalman gain and $\hat{C}_{vk}$ is the innovation covariance matrix as shown in (5). It is calculated through averaging inside a moving estimation window of length N:

$$\hat{C}_{vk} = \frac{1}{N}\sum_{j=j_0}^{k} v_j v_j^T, \tag{5}$$

where $j_0 = k-N+1$ is the first epoch inside the estimation. In order to be merged into the general EKF scheme, $\hat{Q}_k$ could be simply calculated/updated at each time step after the filtering is performed. Corresponding experimental result given in Section V-D shows that when adapting the filter weight (and hence the filter gain), the adaptive Kalman filter outperforms the conventional Kalman filter.

V. Experimental Results

The experimental work has been conducted by Netherlands Organization of Applied Scientific Research TNO at the Ford Lommel Proving Ground in Belgium.

A. Experimental Setup

The sensor systems on the test vehicle (Toyota Prius 3^{rd} generation) are:

- Trimble Real-Time Kinematic (RTK) system.
- CORRSYS Non-Contact Optical ground speed Sensor.
- OxTS RT3100: GNSS supported IMU.
- Xsens MTi-G: GNSS supported IMU.

Besides, the ESC system sensors of the test vehicle are also logged (i.e. steering angle, wheel speeds, yaw rate, longitudinal and lateral acceleration). The experimental setup is shown in Fig. 1.

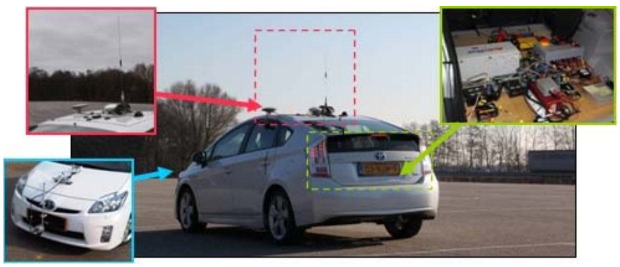

Fig. 1. Experimental sensors setup

B. Performance Analysis

In this section, the performance is analyzed separately for the new model concept and the GPS measurement extrapolation method.

1) Model verification: The Kalman filter concept involves a state correction of the prediction model based on the comparison with measurement signals. This is done at each time step and the correction amount is further referred to as the innovation of the Kalman filter. According to [10], the innovation $\nu = z - H\hat{x}$ is a white noise process with zero mean and a covariance of $S = (HPH^T + R)$, where H is the observation matrix, P is the state covariance matrix and R represents the model uncertainty. Therefore *Fourier analysis* of the innovation sequence provides a measure of the filter performance. Persistent peaks in the frequency spectrum indicate characteristic frequencies of unmodeled effects. In Fig. 2, contents at zero frequency implies bias error. The concept of using bias states in the new model in (2) is verified below for the innovation on the yaw rate state. More explicitly, in Fig. 2, one can find that the histogram of the yaw rate innovation sequence from the bicycle model is not exactly Gaussian compared with Gaussian approximation indicated by the red line. On the contrary, the histogram of the yaw rate innovation sequence from the new model reveals its whiteness as shown in Fig. 3.

2) GPS Measurement Extrapolation: The Bezier curve extrapolation concept presented in (3) is demonstrated on position data from the RTK system to show the improvements for multi-rate application. RTK measurements of 10 Hz are extrapolated to 100 Hz and compared to recorded measurement data as shown in Fig. 4. The recorded measurement data is

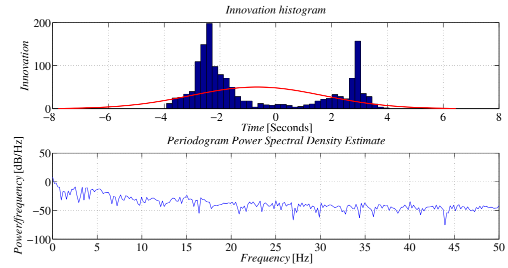

Fig. 2. Analysis of yaw rate from the bicycle model

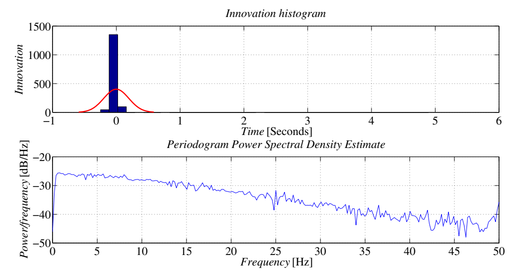

Fig. 3. Analysis of yaw rate from the new model

constant until an update from the sensor is received, resulting in a zero-order-hold (ZOH) signal.

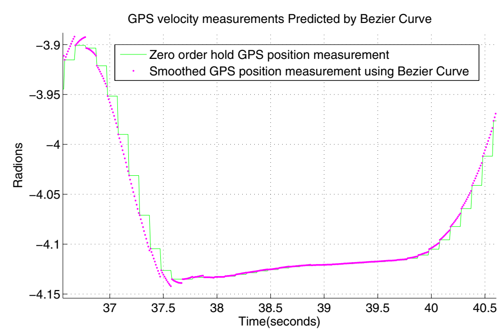

Fig. 4. Comparison of ZOH and Bezier curves

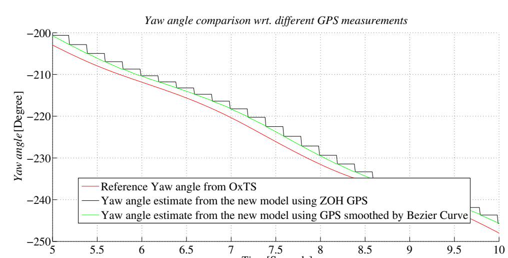

Fig. 5. Simulation results

Another comparison shown in Fig. 5 contains the heading angle estimates of the Kalman Filtering using ZOH measurements and measurements from the Bezier curve extrapolation. Both results are compared to the heading angle provided by the OxTS system. Clearly the benefit of using the Bezier curve extrapolation is observed. However close inspection of Fig. 4 reveals overshoots in the extrapolated GPS measurements. Therefore, outlier rejection through probability test/survival analysis may be required, especially for consumer grade GPS. As described in Section V-B1, the innovation sequence is a white noise process such that, chi-squared distribution could be used to test for outliers:

$$\nu^T S^{-1} \nu \leq \Gamma, \tag{6}$$

where value of Γ is determined prior to the estimation process and represents the percentage probability that a particular observation is an outlier or not [11].

C. Sideslip Estimation

As mentioned in Section I, the sideslip angle estimate is chosen as a case-study. In general, there are two methods of sideslip angle estimation using GPS as given by (7) and (8). The method given by (7) requires accurate estimates of the states and the method given by (8) relies on a typically slow measurement $\gamma|_{slow}$ [3] such that this method only captures low frequency content.

$$\beta|_{fast} = \arctan \frac{v_{y_b}|_{fast}}{v_{x_b}|_{fast}} \tag{7}$$

$$\beta|_{fast} = \gamma|_{slow} - \Psi|_{fast} \tag{8}$$

The method described by (7) is used because it captures more dynamics and relies less on GPS measurements than method (8). One illustrative sideslip angle estimation example for a sine wave driving test is shown in Fig. 6 where 5 Hz GPS measurements are used.

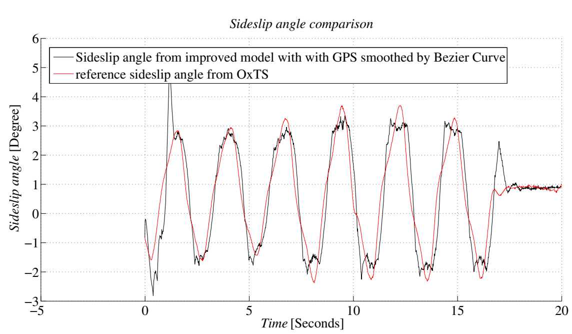

Fig. 6. Sideslip angle estimation

D. Noise Covariance Matrix Adaption

To demonstrate the capability of *Adaptive Kalman Filtering* described in (4) a comparison is made with standard Kalman Filtering as shown in Fig. 7. For this trial, 100Hz GPS measurements from OxTS is used and the front tyre stiffness of the prediction model is set at 50% of the value approximated [4] from vehicle testing. As can be seen from the right hand side of Fig. 7, with *Adaptive Kalman Filtering*, the estimated sideslip angle is able to converge to the reference.

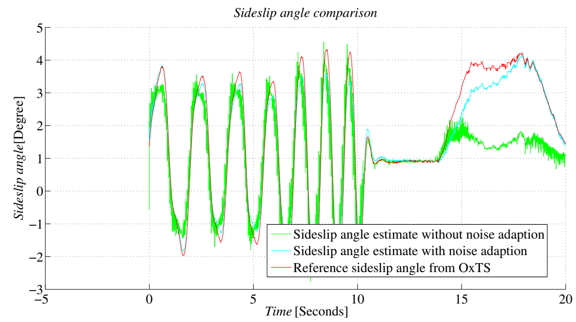

Fig. 7. Comparison between Kalman filtering and Adaptive Kalman filtering

VI. Conclusions

This paper introduces a modified vehicle state estimator from three aspects: new vehicle model, GPS measurement extrapolation and model uncertainty compensation. A good sideslip angle estimate is achieved using low frequency (5Hz) GPS measurements. Future research aims at applications on consumer grade GPS with an even lower update frequency and an evaluation of positioning accuracy under different driving conditions.

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[3]The subscript 'slow'/'fast'indicates sampling rate coincident to GPS and IMU respectively.

[4]This is estimated using TNO Tyre Estimator