Docent Lecture: Compositional Verification of Interaction Behaviour

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Interaction Behaviour

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- **Computation**: Data Transformation + *Interaction*

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  - teller machine (bankomat)
  - server accepting requests and sending responses
  - applications on a mobile device interacting via method calls

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- **Examples**:
  - concurrent server spawns off component to handle request
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- **Problem**:
  - how can we reason formally about the interaction behaviour of such systems?
  - *compositional* reasoning needed!

Concurrent Server

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Server gets request

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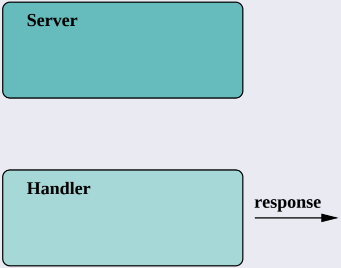
Server spawns off Handler



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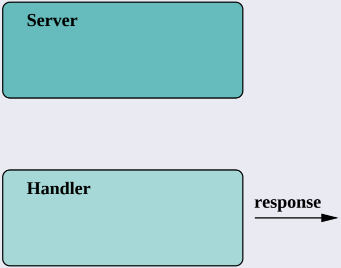


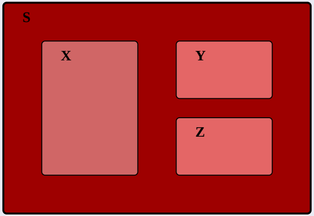
Property of Interaction Behaviour

Concurrent server always stabilizes (STAB)

Compositional Reasoning

Compositional Reasoning

Pure approach: Composing properties



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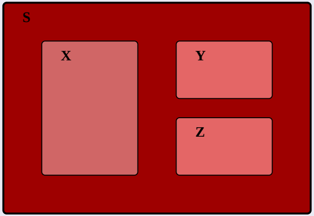
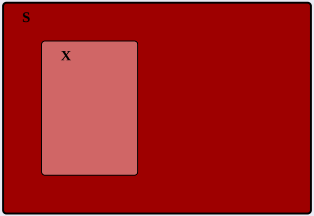


**More general approach:
Cut on component**



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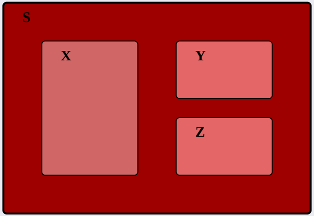

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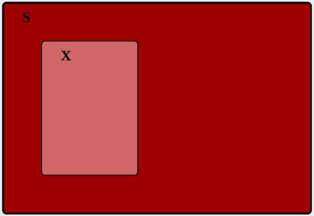

Concurrent Server

How does compositional reasoning help?

Proving Stabilization of Concurrent Server

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Original goal:

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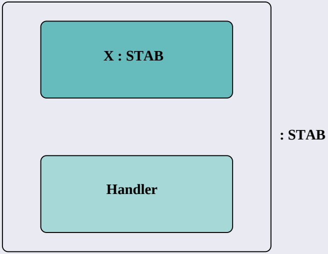
Reduces to:



Overview

Framework for Formal Reasoning: Ingredients

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Semantic Domains for Interaction Behaviour

- function from initial to final states: not suitable
- rather: sequences, or even trees, of interactions

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Defining Interaction Behaviour

- semantic domain too low–level and unstructured
- composing behaviours
- meaning of behavioural definition given in semantic domain

Framework for Formal Reasoning: Ingredients

Specification and Verification

- *specification* captures desired behaviour
- *verification* establishes whether model/implementation meets specification

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Compositional Verification

- inferring system properties from component properties

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- Traces (or runs, executions, paths)

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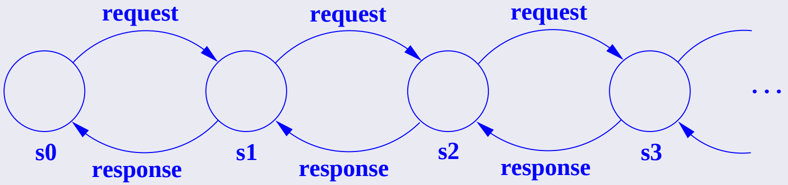
- Modal Transition Systems

LTS Example: Concurrent Server

Interaction Behaviour of Concurrent Server



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- Process Algebra: Calculus of Communicating Systems

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LTS Semantics

Induced by *transition rules*

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CCS Syntax

$$E \ ::= \ \mathbf{0} \ | \ A \ | \ \alpha.E \ | \ E + E \ | \ E|E$$

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CCS Semantics: Transition Rules (induce LTS)

$$\textsc{Prefix} \ \frac{-}{\alpha.E \xrightarrow{\alpha} E} \qquad \textsc{Def} \ \frac{E \xrightarrow{\alpha} F}{A \xrightarrow{\alpha} F} \ A \triangleq E$$

$$\textsc{Choice} \ \frac{E \xrightarrow{\alpha} E'}{E+F \xrightarrow{\alpha} E'} \qquad \textsc{Comm} \ \frac{E \xrightarrow{\alpha} E'}{E|F \xrightarrow{\alpha} E'|F}$$

CCS Example: Concurrent Server

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Defining Concurrent Server

$$CServer \triangleq request.(CServer \mid response.\mathbf{0})$$

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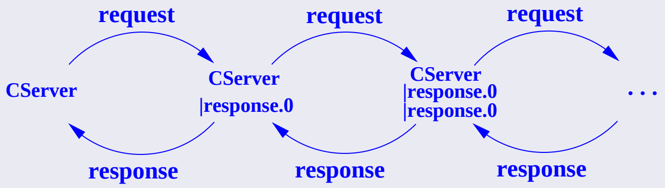
Induced LTS



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Example: Formalizing STAB

- CTL: AG (AF stab)

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- CTL: AG (AF stab)
- μK: $\nu X.\ \mu Y.\ [\text{request}]\, X \wedge [-\text{request}]\ Y$

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HML Syntax

$$\Phi \ ::= \ \mathbf{tt} \ | \ \mathbf{ff} \ | \ \Phi \vee \Phi \ | \ \Phi \wedge \Phi \ | \ \langle\alpha\rangle\,\Phi \ | \ [\alpha]\,\Phi$$

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HML Semantics: Satisfaction Relation $s \models^{\mathcal{T}} \Phi$

$$s \models^{\mathcal{T}} \langle\alpha\rangle\,\Phi \;\stackrel{\text{def}}{\Leftrightarrow}\; \exists s' \in \mathcal{S}.\,(s \xrightarrow{\alpha} s' \wedge s' \models^{\mathcal{T}} \Phi)$$
$$s \models^{\mathcal{T}} [\alpha]\,\Phi \;\stackrel{\text{def}}{\Leftrightarrow}\; \forall s' \in \mathcal{S}.\,(s \xrightarrow{\alpha} s' \Rightarrow s' \models^{\mathcal{T}} \Phi)$$

Verifying Behavioural Properties: Interactive

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Proof System Based: Judgements $s \vdash^{\mathcal{T}} \Phi$

$$\textsc{True}\ \frac{-}{s \vdash^{\mathcal{T}} \mathbf{tt}} \qquad \textsc{OrL}\ \frac{s \vdash^{\mathcal{T}} \Phi}{s \vdash^{\mathcal{T}} \Phi \vee \Psi} \qquad \textsc{OrR}\ \frac{s \vdash^{\mathcal{T}} \Psi}{s \vdash^{\mathcal{T}} \Phi \vee \Psi}$$

$$\textsc{And}\ \frac{s \vdash^{\mathcal{T}} \Phi \qquad s \vdash^{\mathcal{T}} \Psi}{s \vdash^{\mathcal{T}} \Phi \wedge \Psi} \qquad \textsc{Dia}\ \frac{s' \vdash^{\mathcal{T}} \Phi}{s \vdash^{\mathcal{T}} \langle \alpha \rangle \Phi}\ s' \in \partial_\alpha(s)$$

$$\textsc{Box}\ \frac{s_1 \vdash^{\mathcal{T}} \Phi \ldots s_n \vdash^{\mathcal{T}} \Phi}{s \vdash^{\mathcal{T}} [\alpha]\,\Phi}\ \partial_\alpha(s) = \{s_1, \ldots, s_n\}$$

Verifying Behavioural Properties: Algorithmic

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Complexity of Model Checking

- For Finite–State Systems:
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- For Pushdown Automata:
  exponential in number of non–terminals and in size of formula

Compositional Verification

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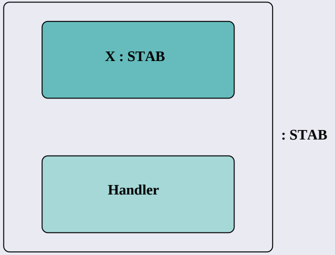
Task to prove:



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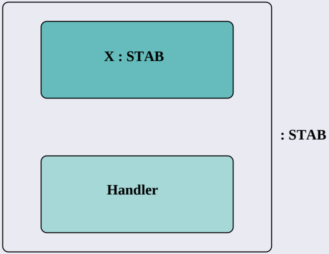

Notation:

$X : \mathrm{STAB} \models X | \mathrm{Handler} : \mathrm{STAB}$

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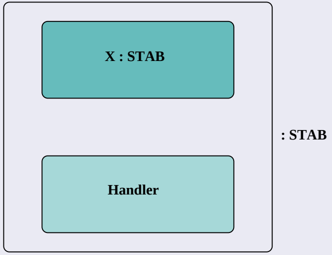

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Approaches:

- Interactive: proof systems
- Algorithmic: maximal models

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Global Discharge Rule

- explicit ordinal approximation
- proof tree embodies a valid proof by well–founded induction
- powerful mechanism for inductive and co–inductive proofs

Proving Stabilization of Concurrent Server

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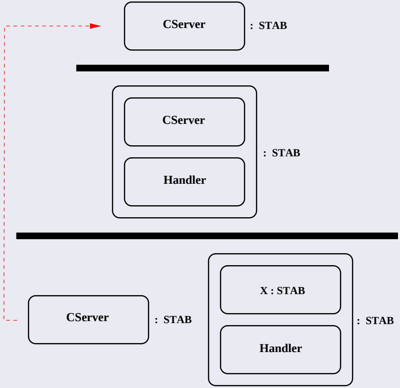
Proof Outline



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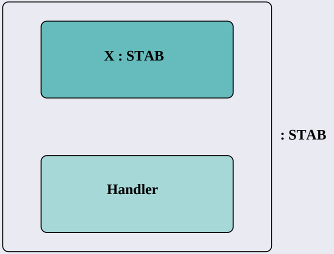

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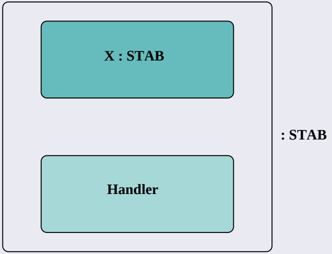
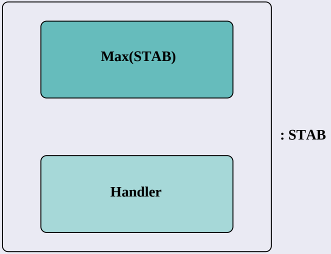


...reduces to model checking:



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Maximal Model Principle

$$\textsc{MaxMod} \quad \frac{\models Max(\Phi)|E : \Psi}{X : \Phi \models X|E : \Psi}$$

Maximal Models for Compositional Verification

Derived Compositional Verification Principle

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Applies to:

1. ACTL (Kripke models)
2. Simulation Logic (Control Flow Graphs)
3. modal μ–calculus (EMTS)

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How to reason about *complex phenomena* such as:

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Algorithmic Verification

How to achieve *scalability* of verification?

- separating concerns
- abstraction mechanisms