

Persistence & Topological Data Analysis for Robotics

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M. Laskey

Recent Collaborators on Topological Methods



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S. Ramamoorthy



L. E. Kavraki



J. Carvalho

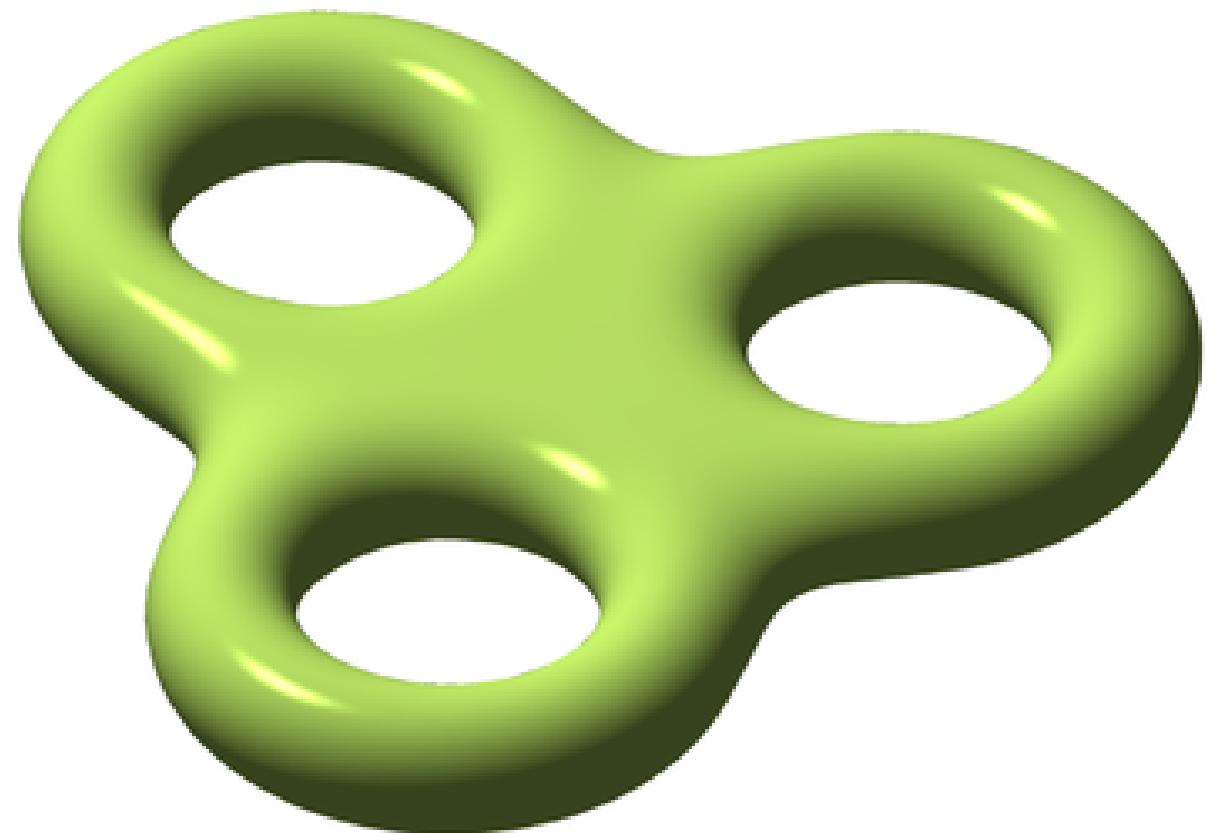


A. Varava

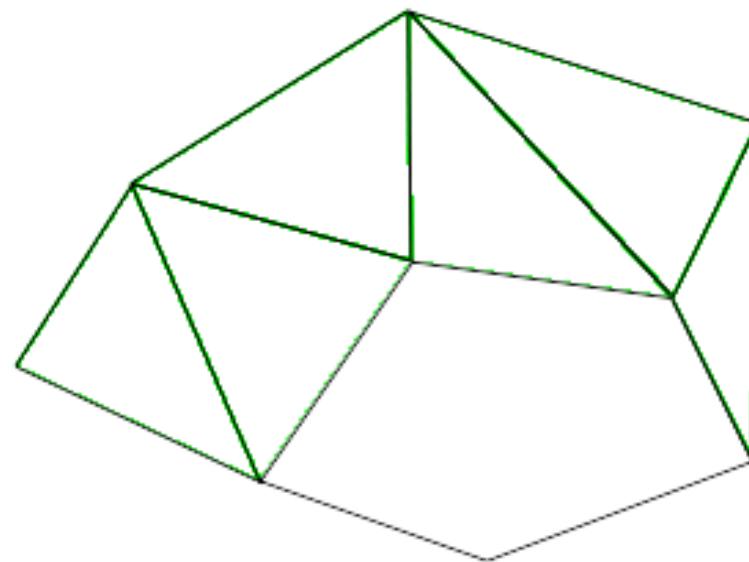
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S. Stazak, W. Y. Hsieh, K. Hang, J.A. Stock, M. Li, Y. Bekiroglu

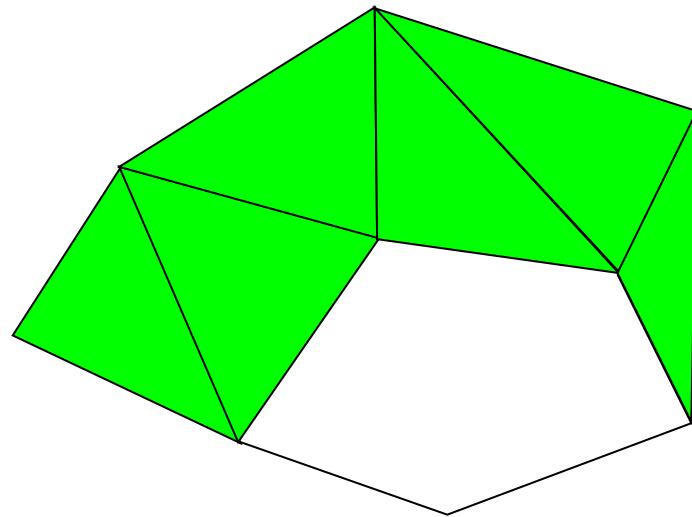
Topology



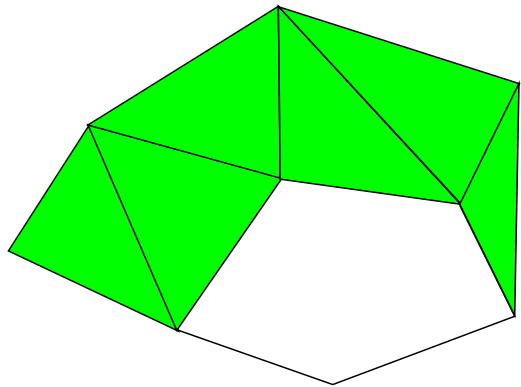
Graphs



Simplicial Complexes



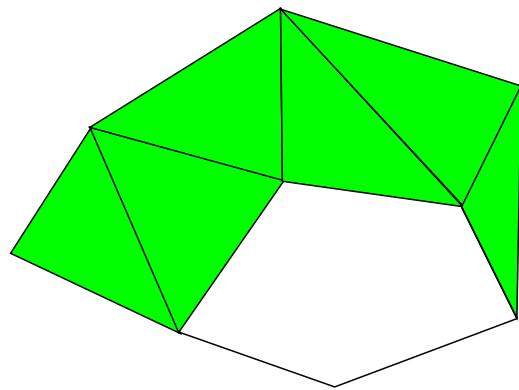
Simplicial Homology over $\mathbb{Z}_2 = \{0, 1\}$



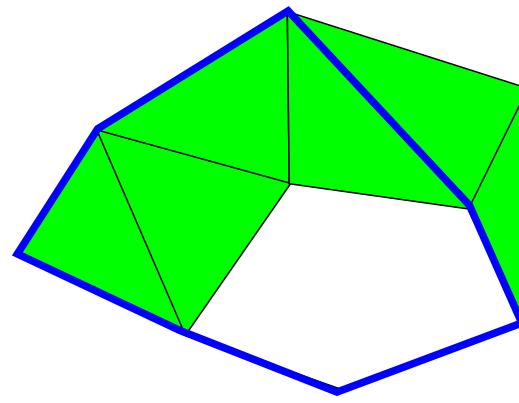
p-chains and boundary operator

$$\partial : C_p(\mathcal{K}) \rightarrow C_{p-1}(\mathcal{K})$$

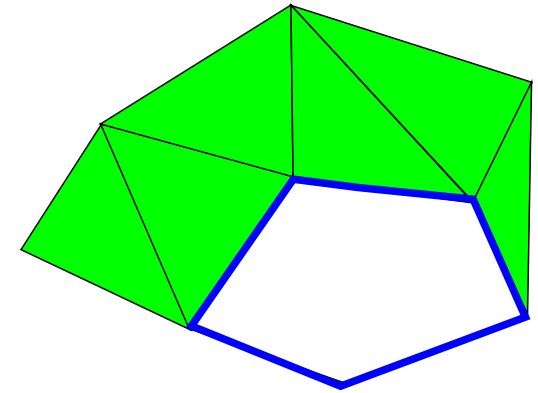
1-cycles over $\mathbb{Z}_2 = \{0, 1\}$



complex

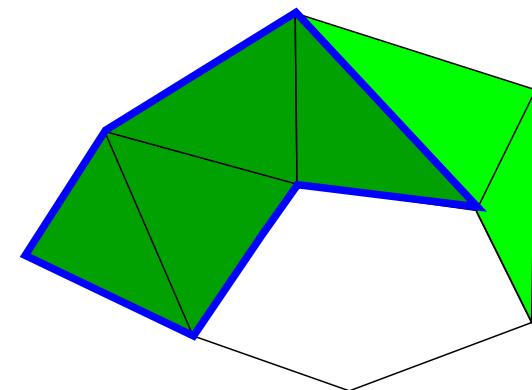
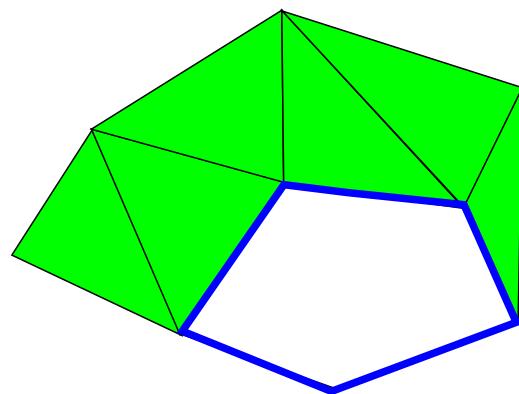
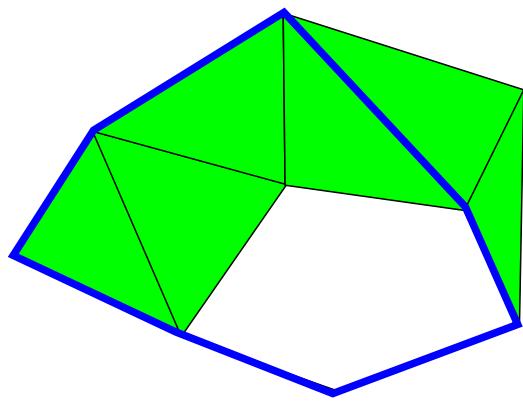


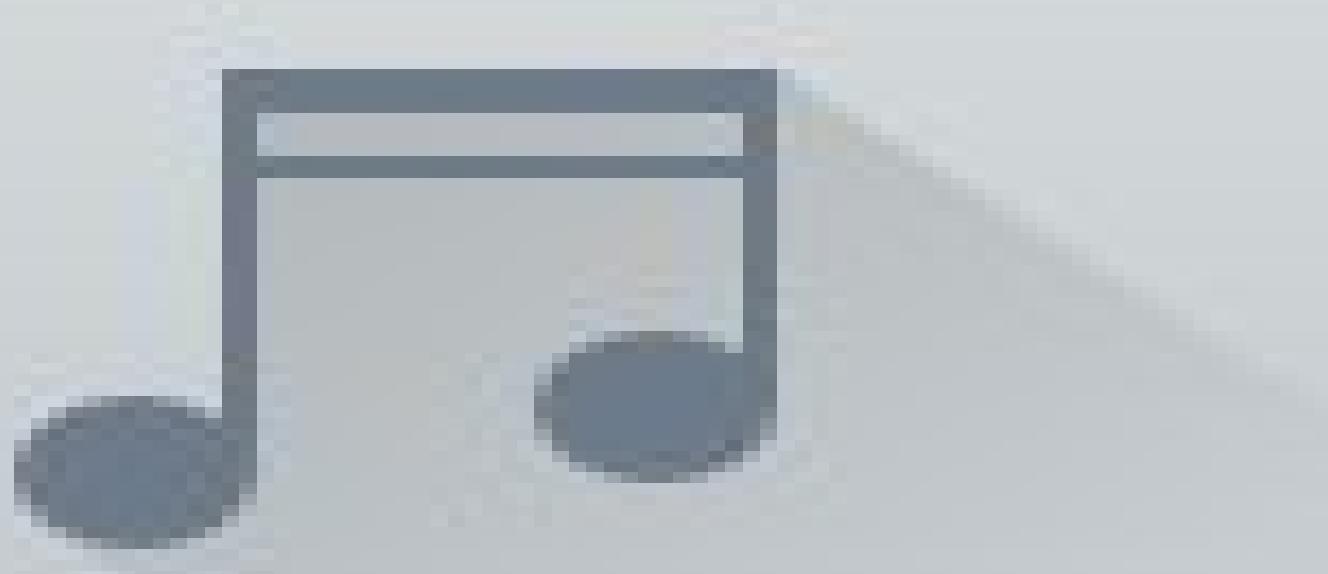
a 1-cycle

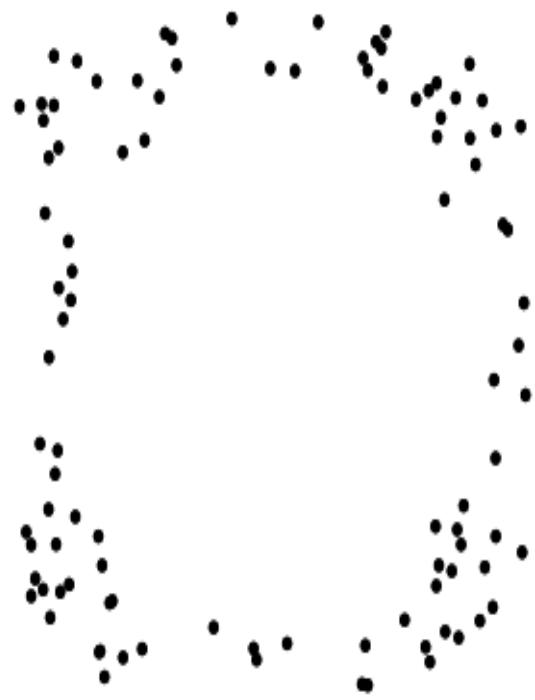


another 1-cycle

1-st homology group over $\mathbb{Z}_2 = \{0, 1\}$

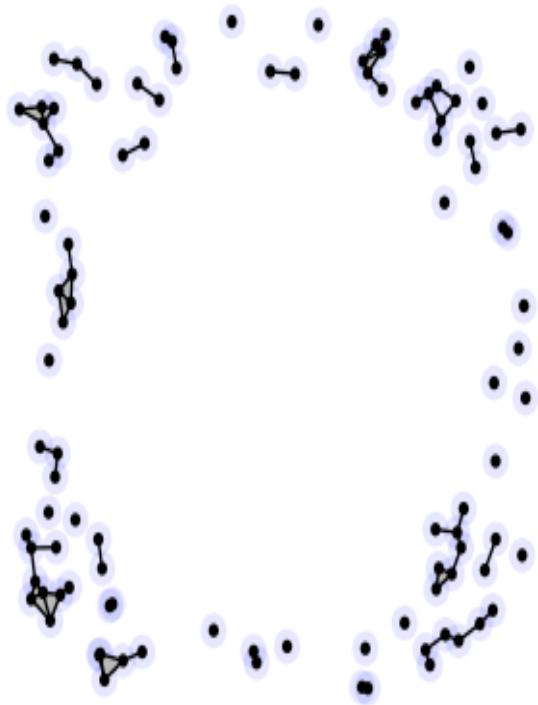


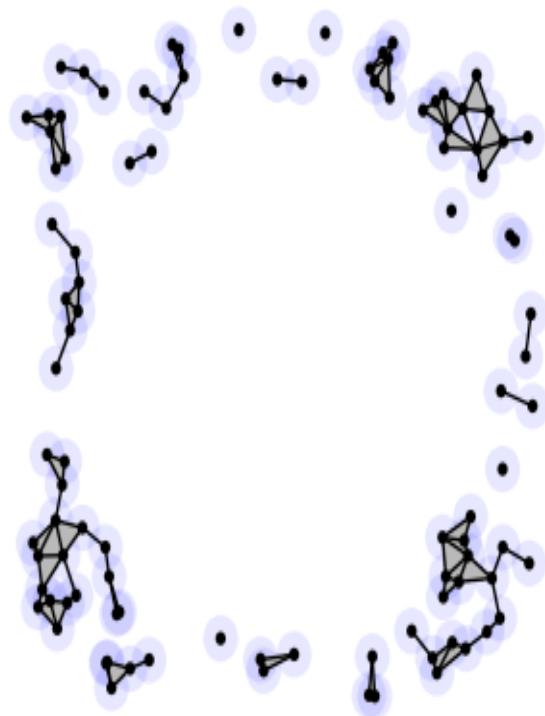


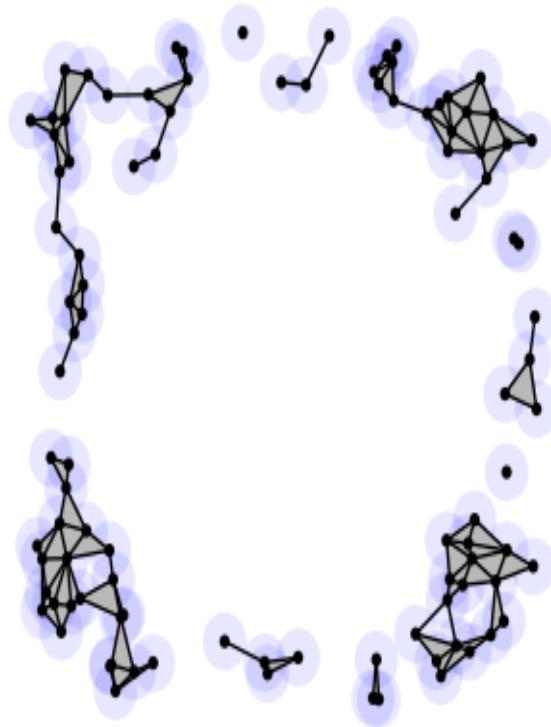


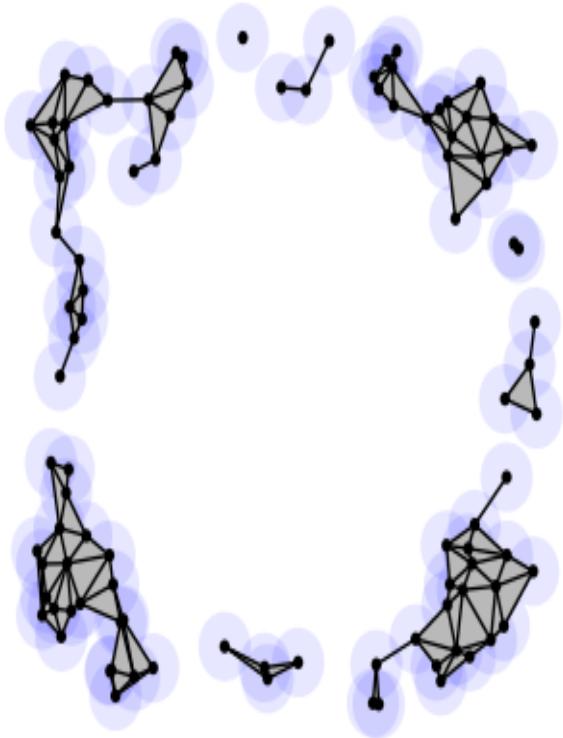


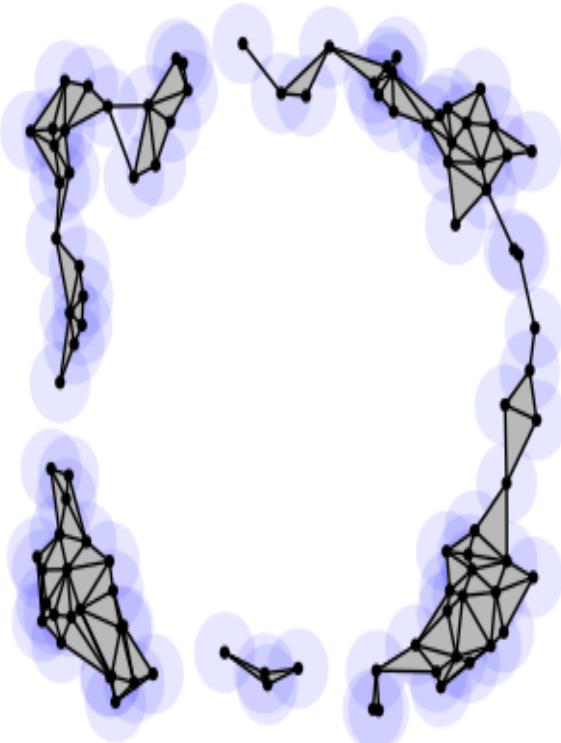


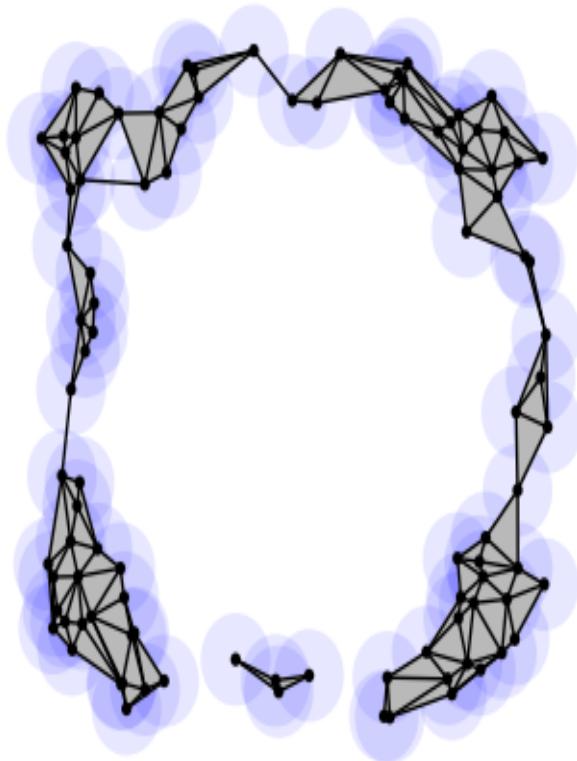


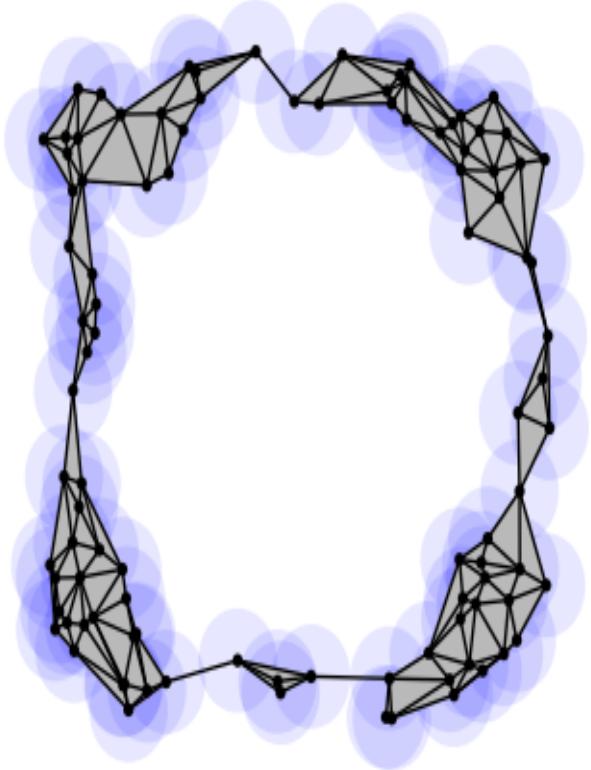


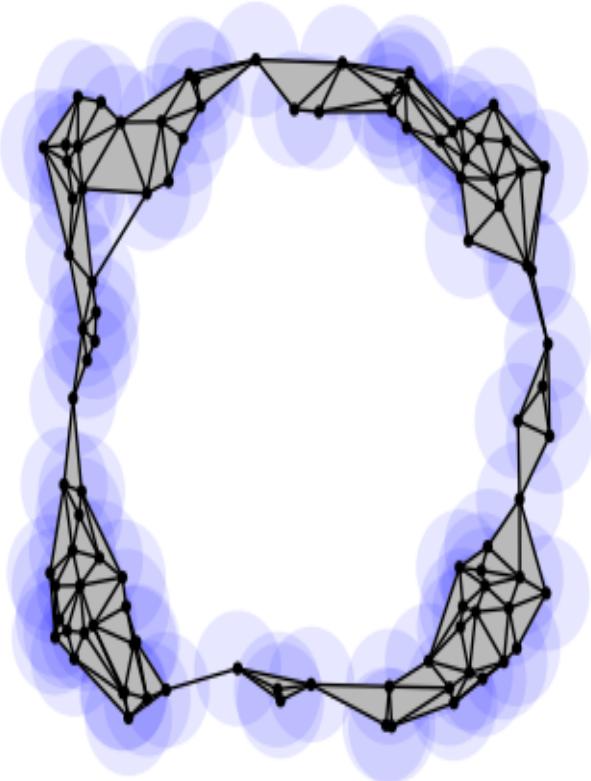


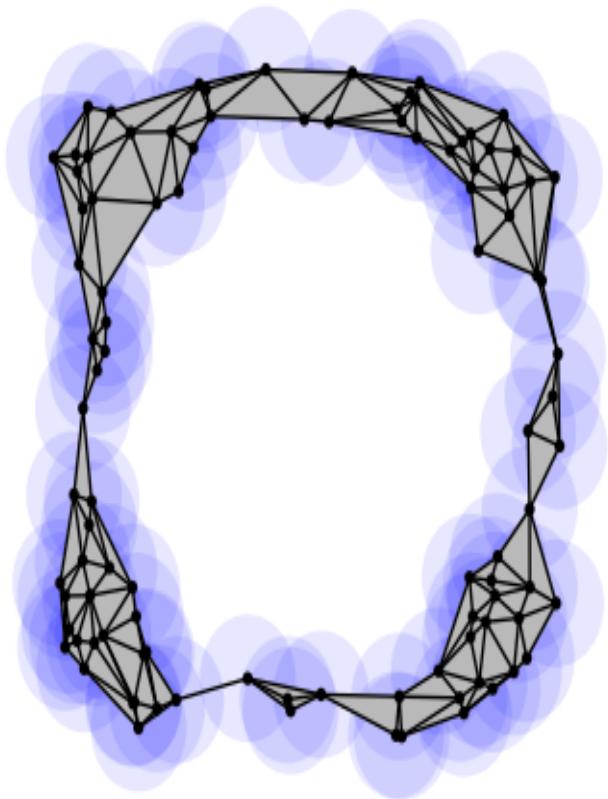


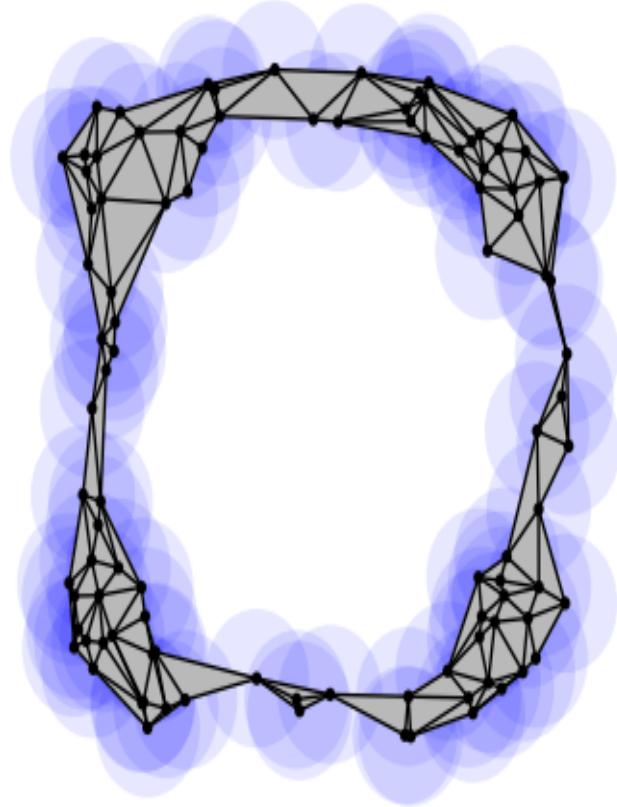


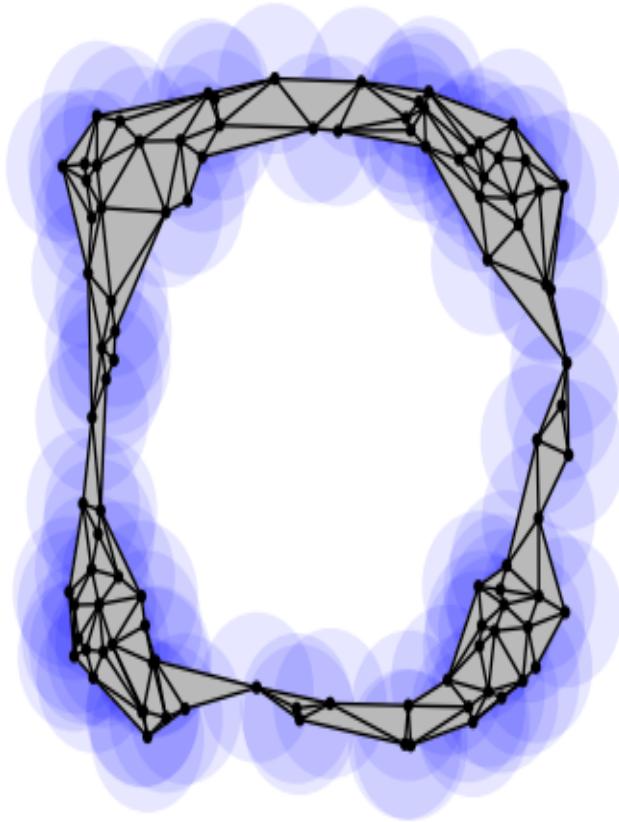


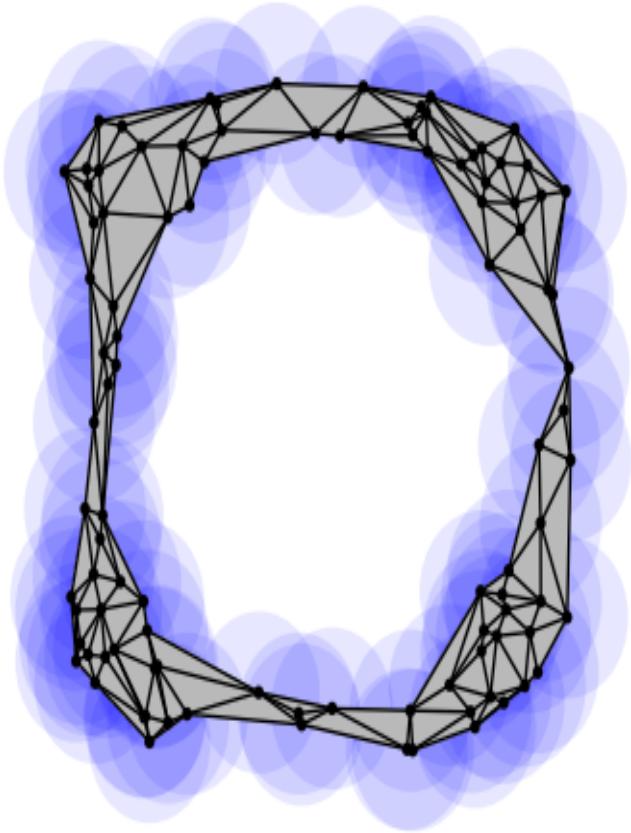


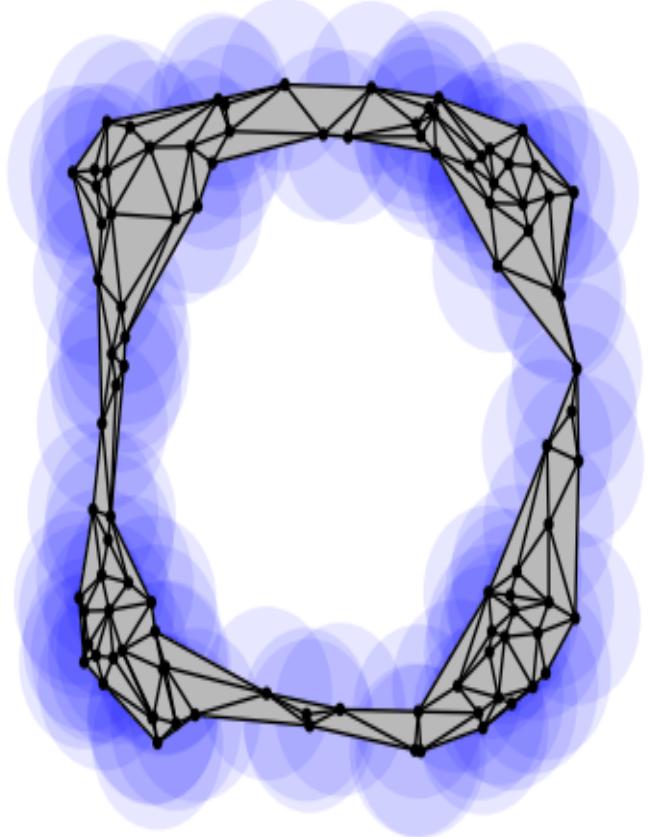


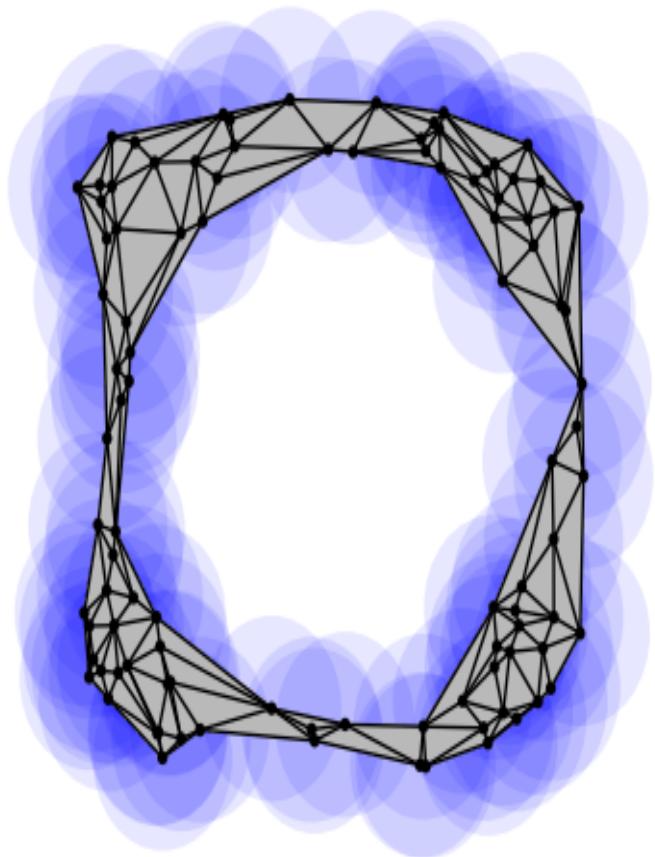


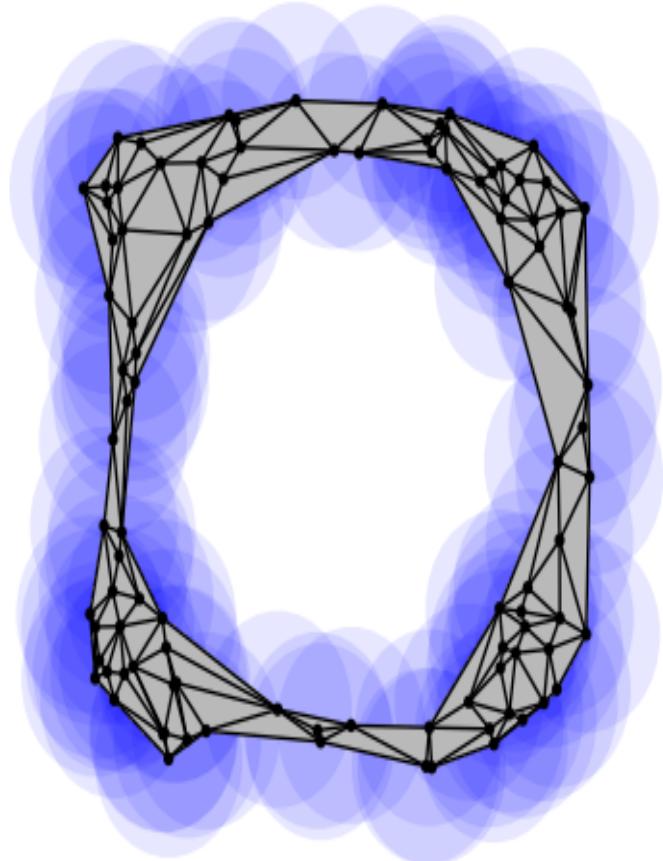


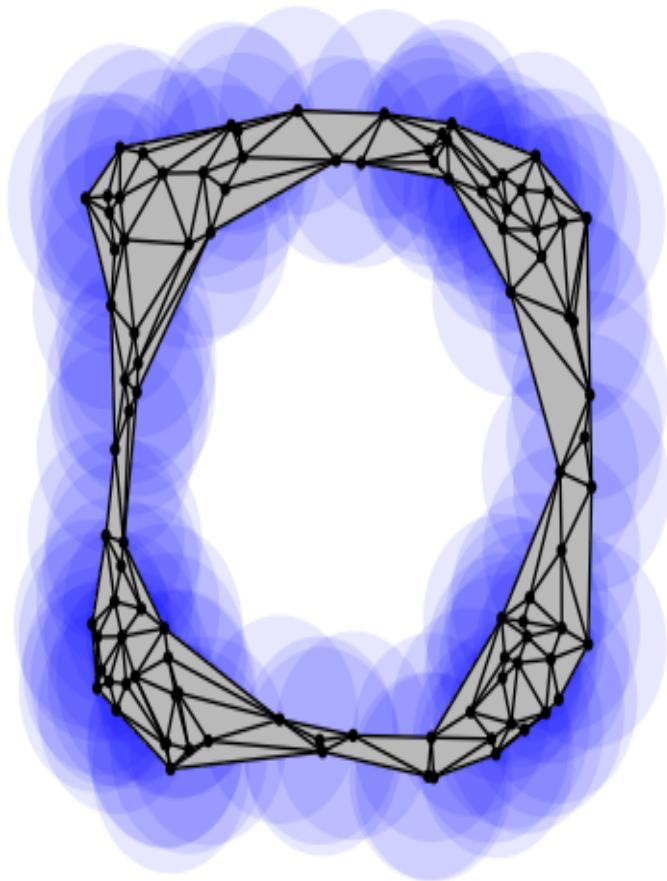


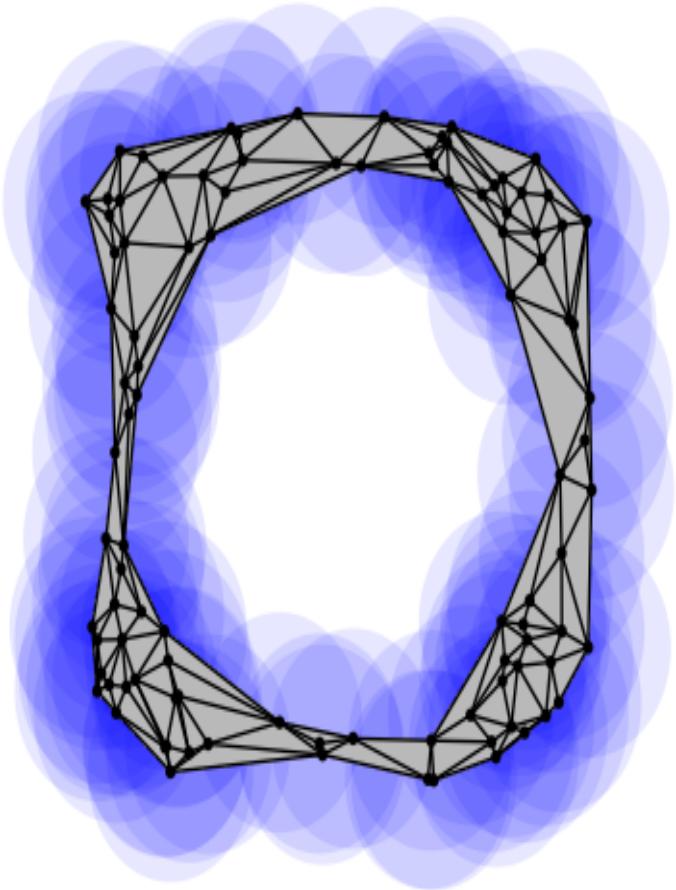


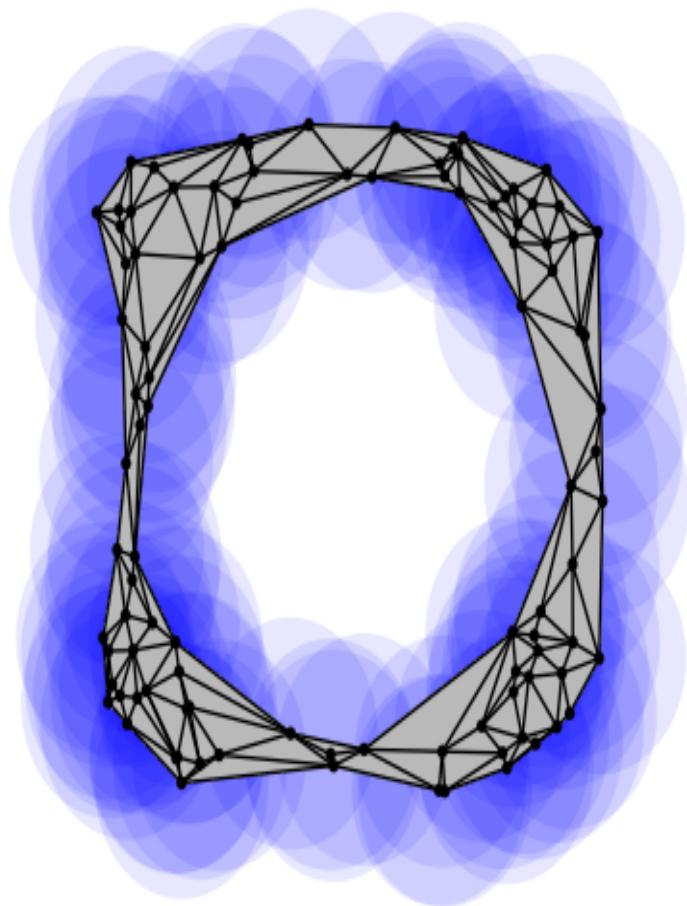


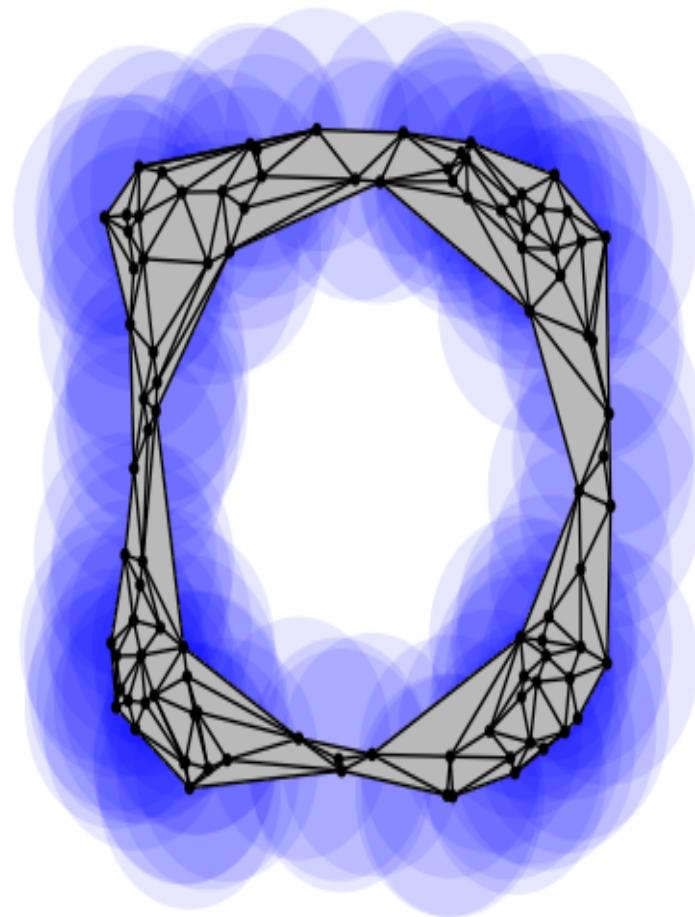


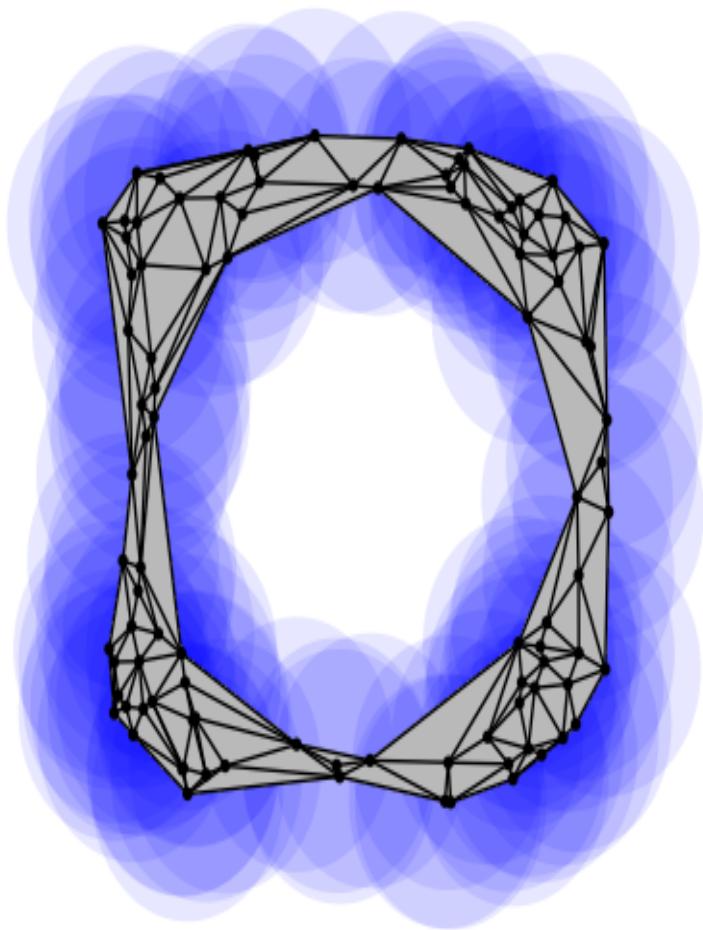


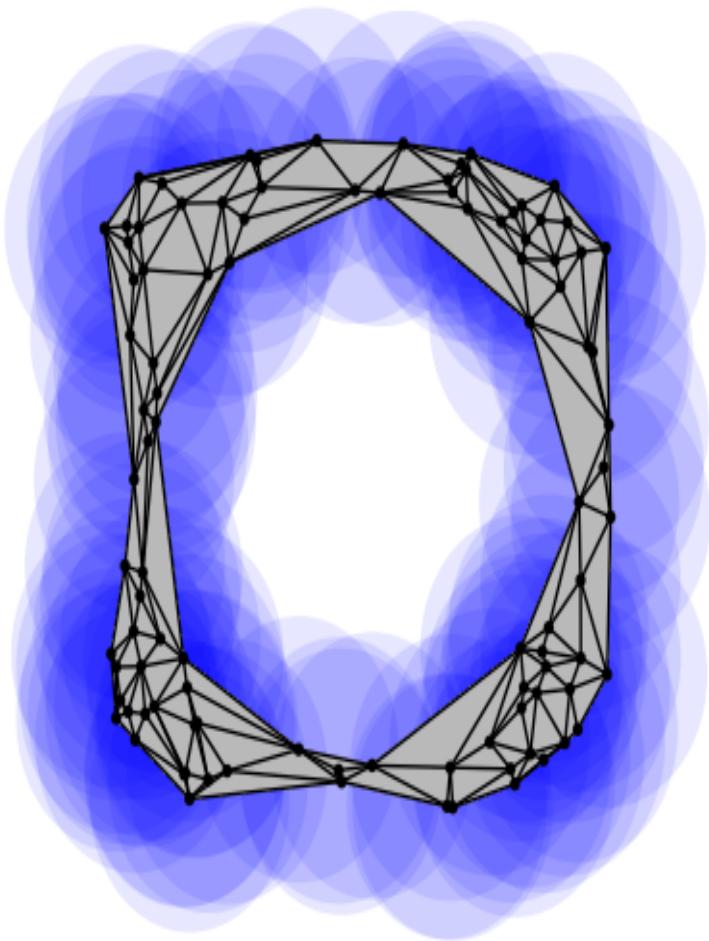


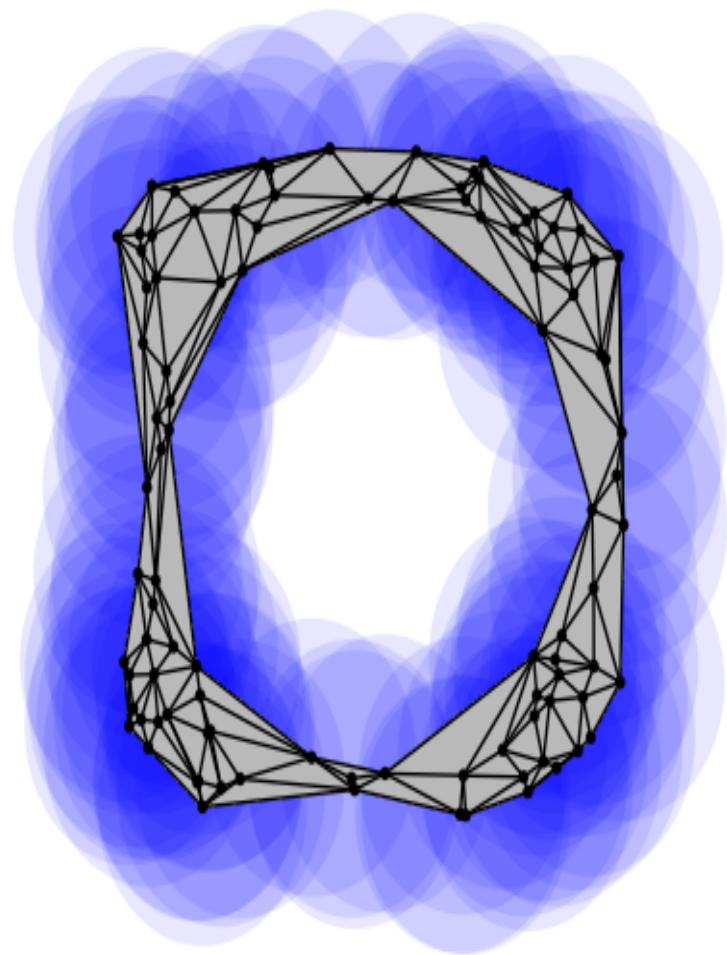


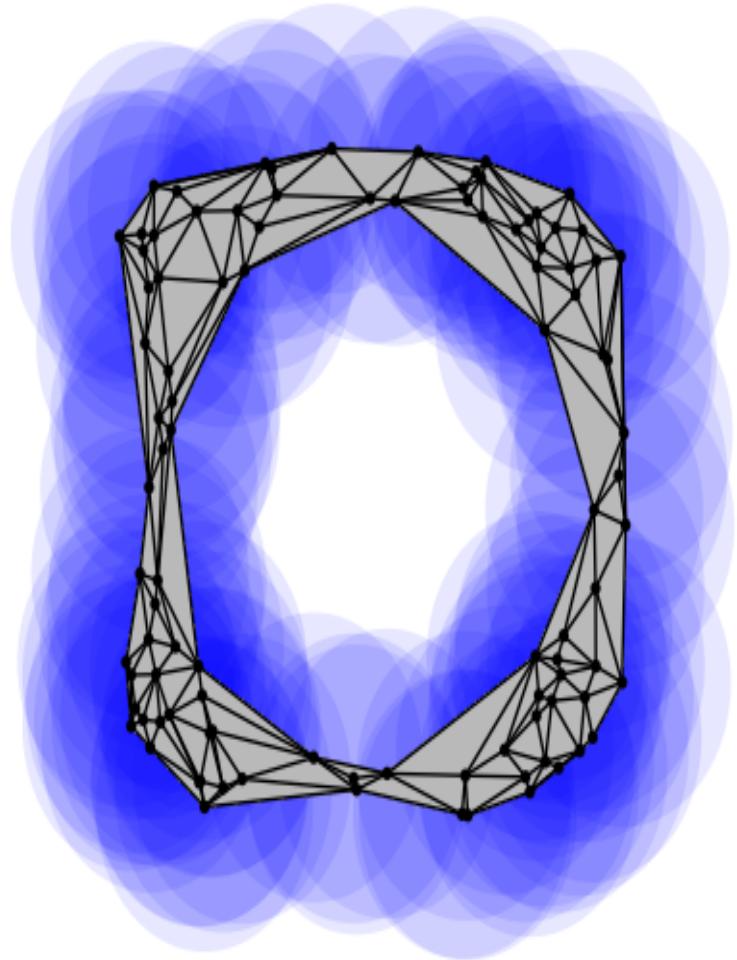


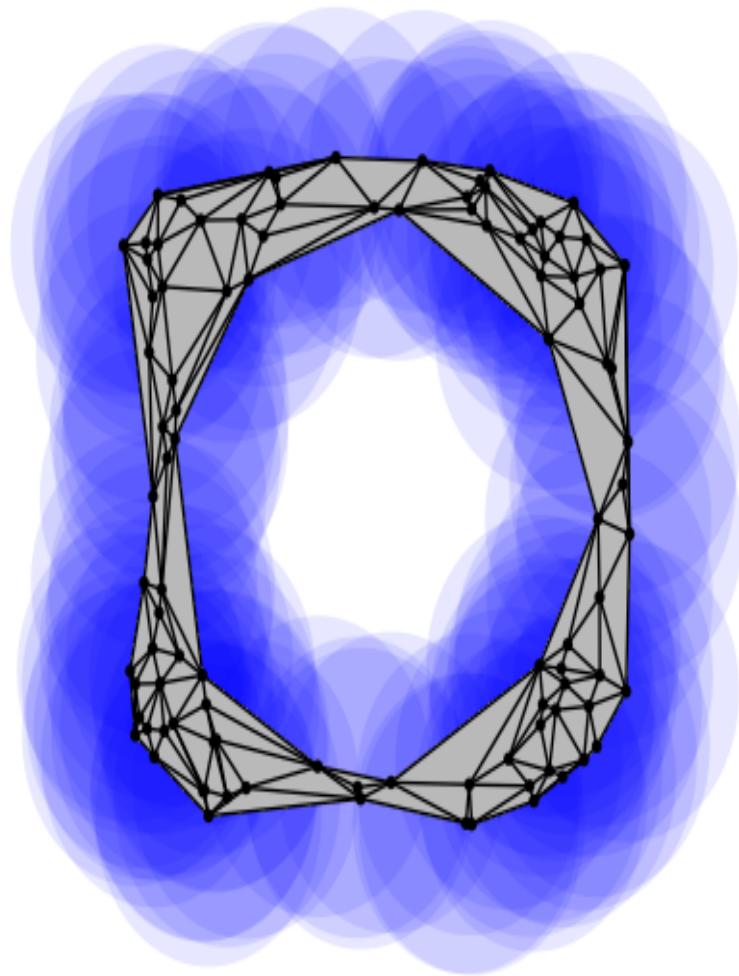


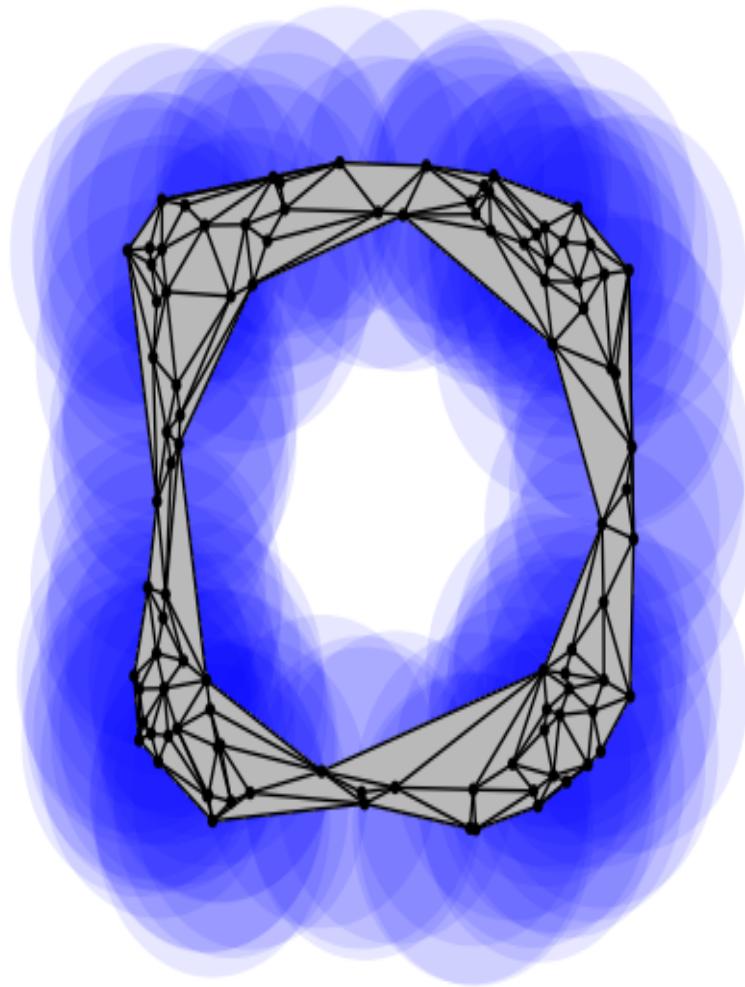


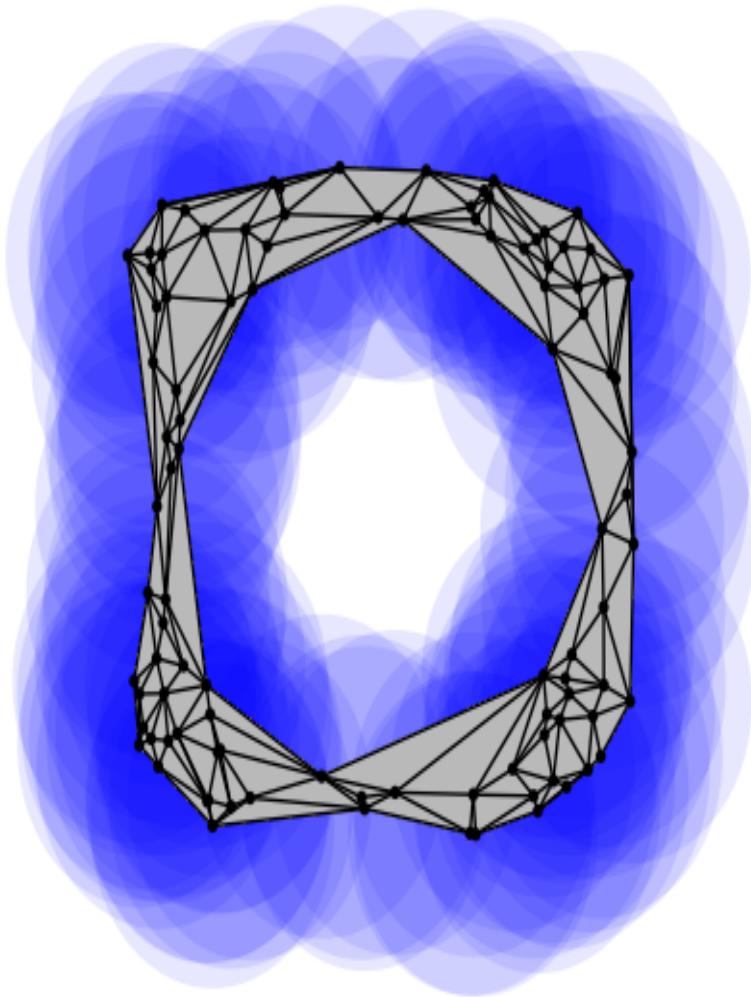


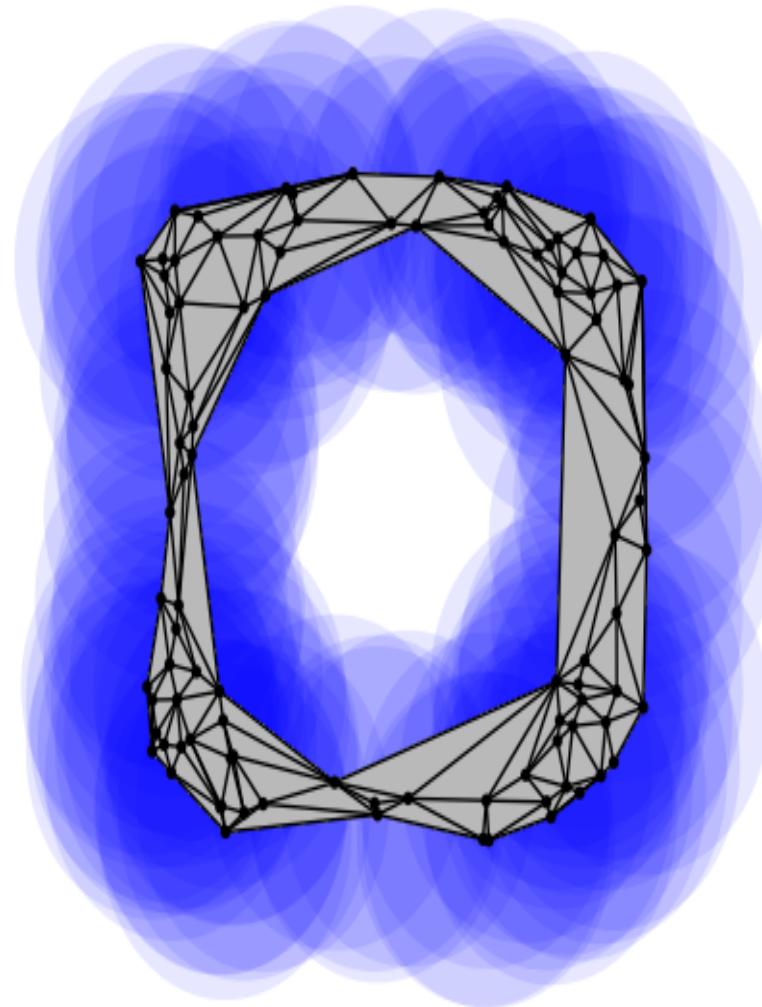


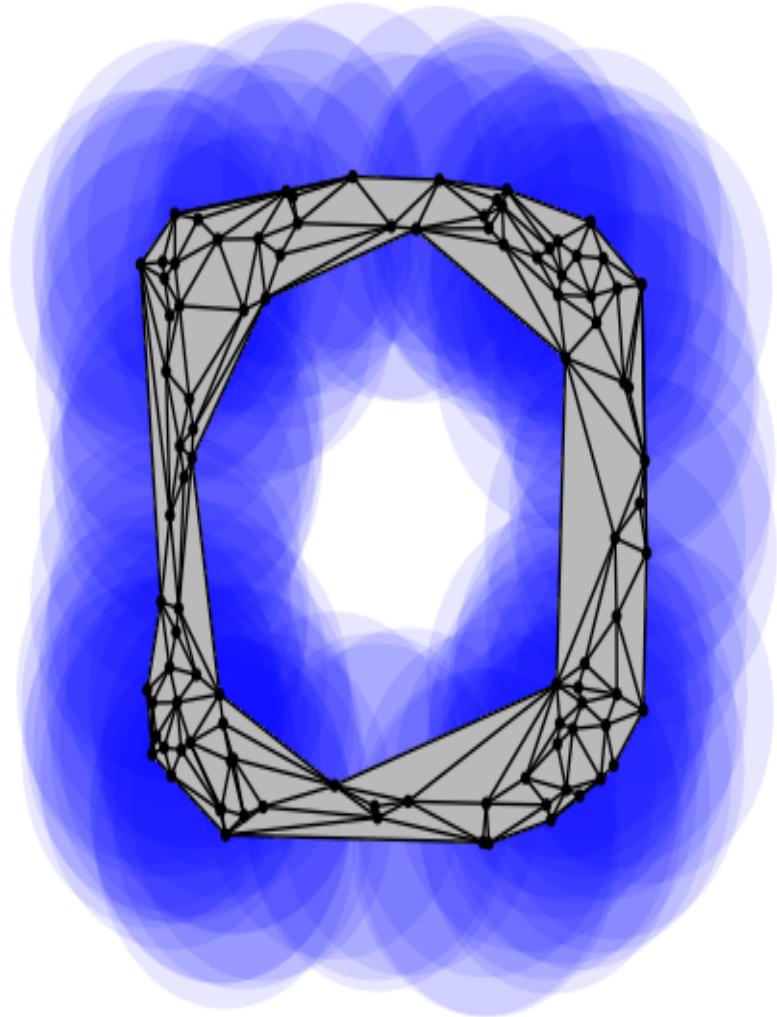


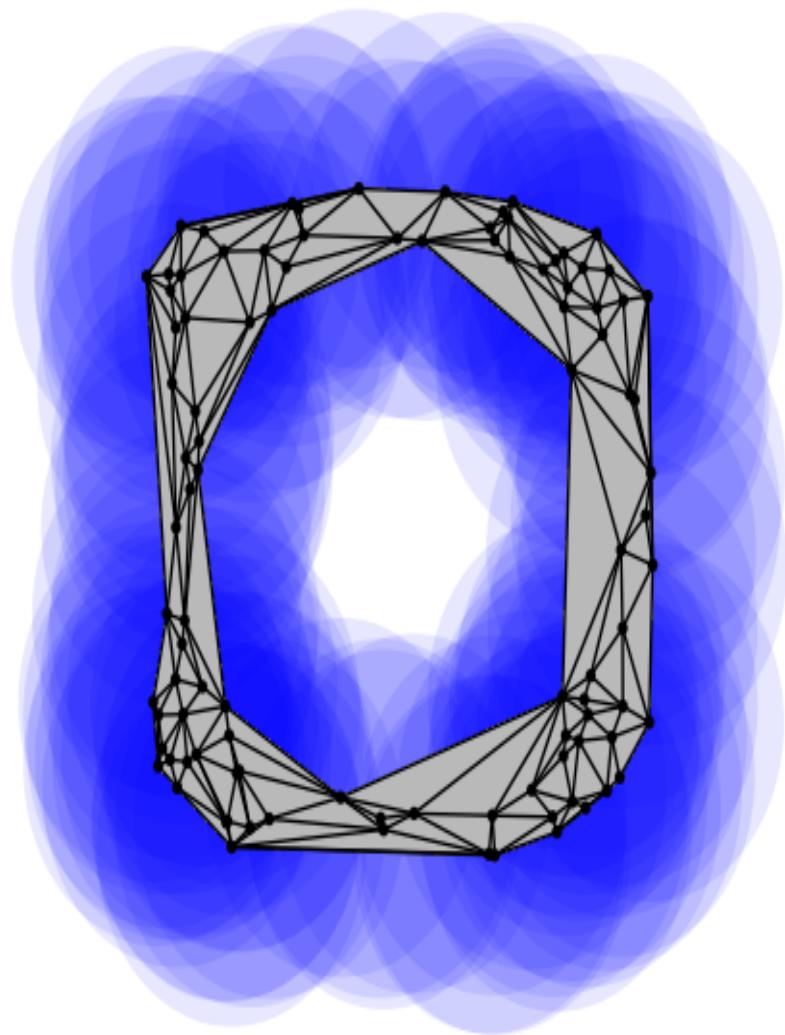


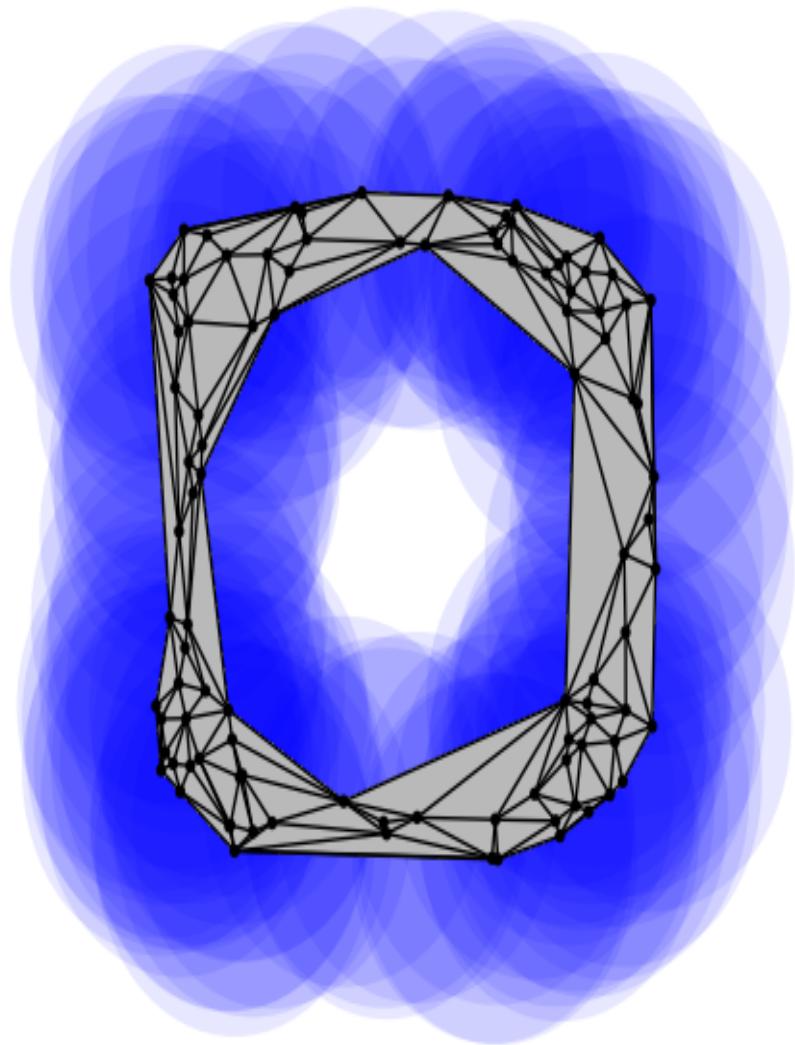


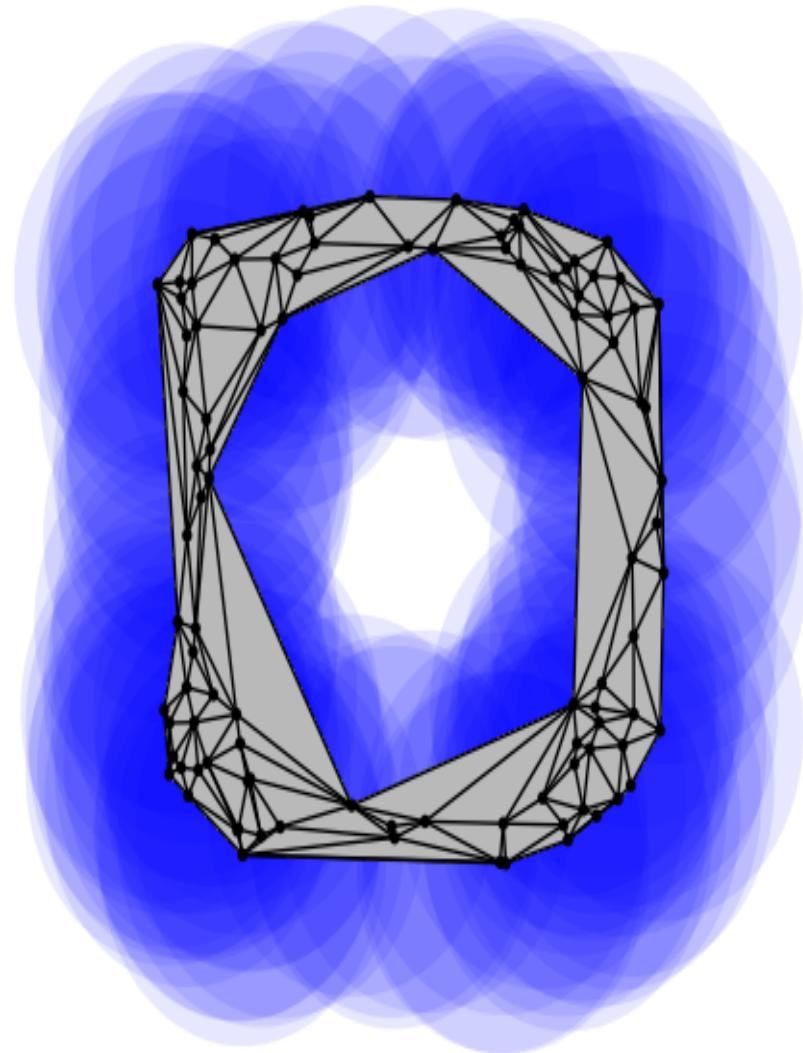


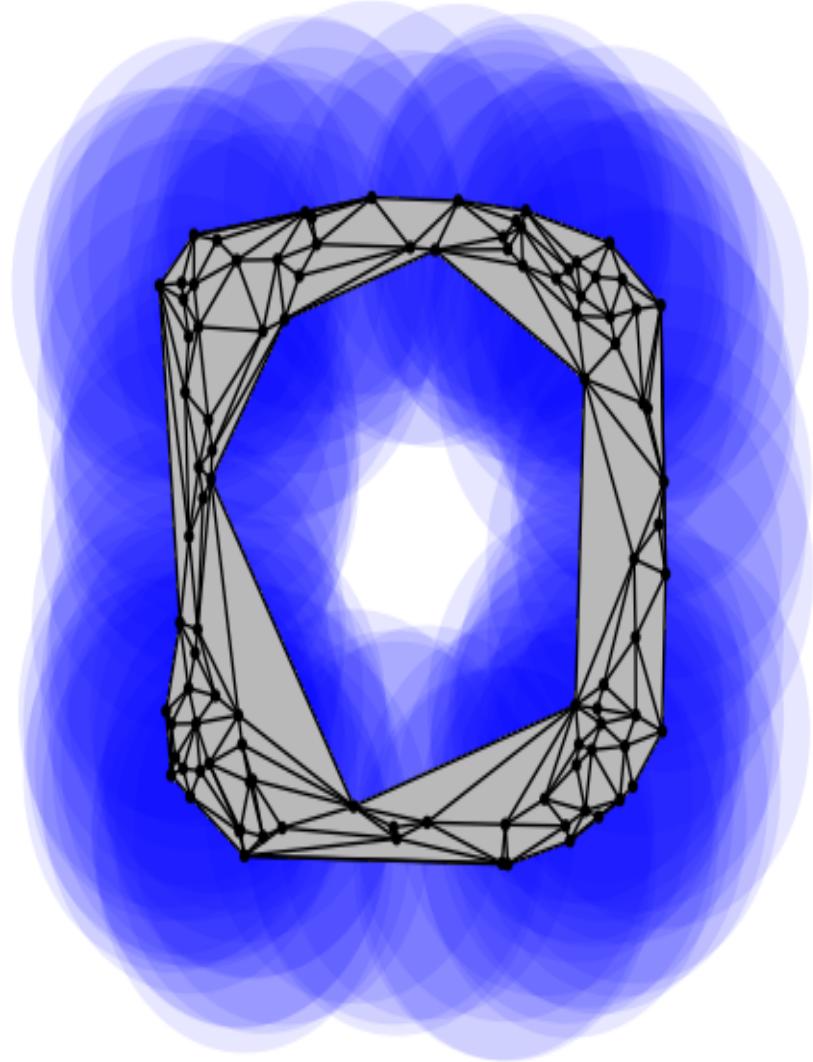


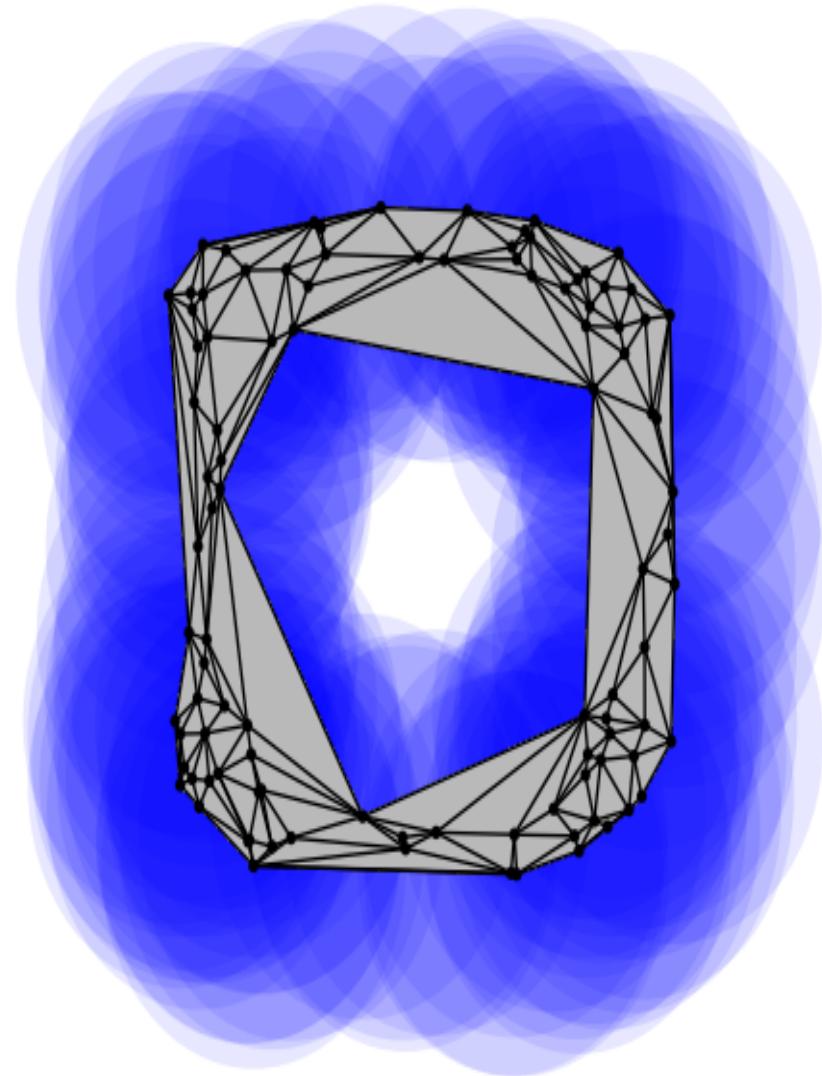


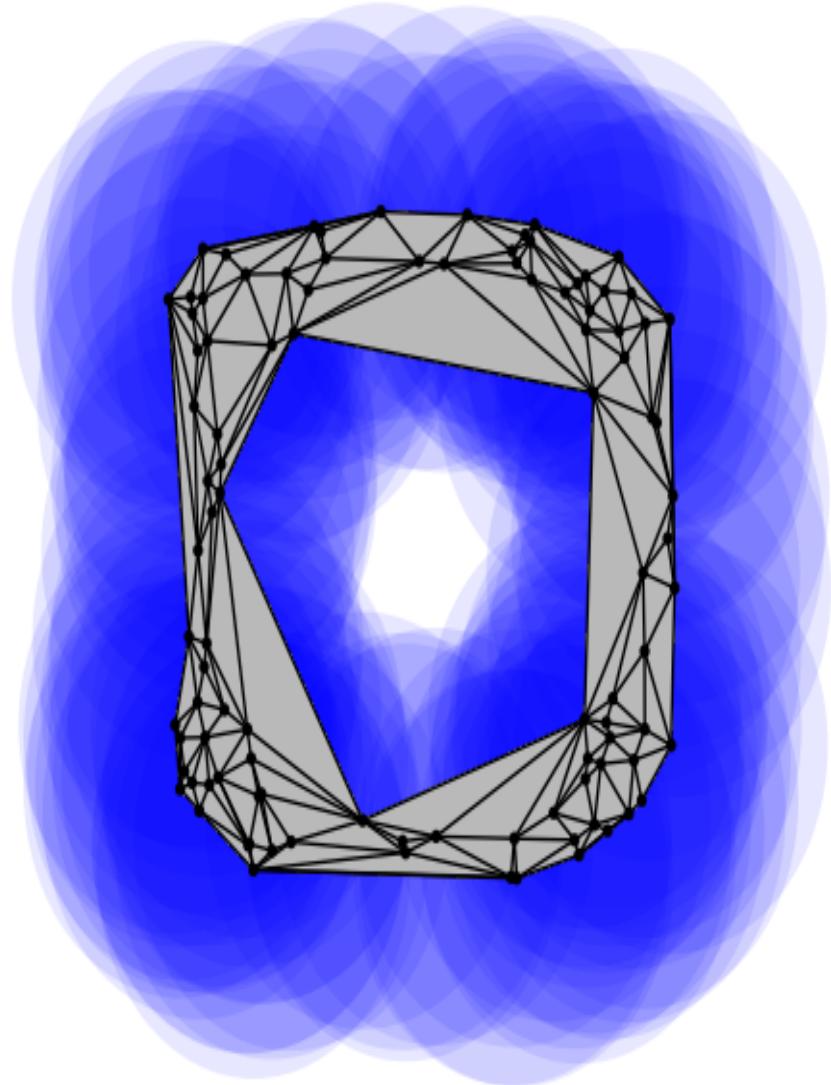


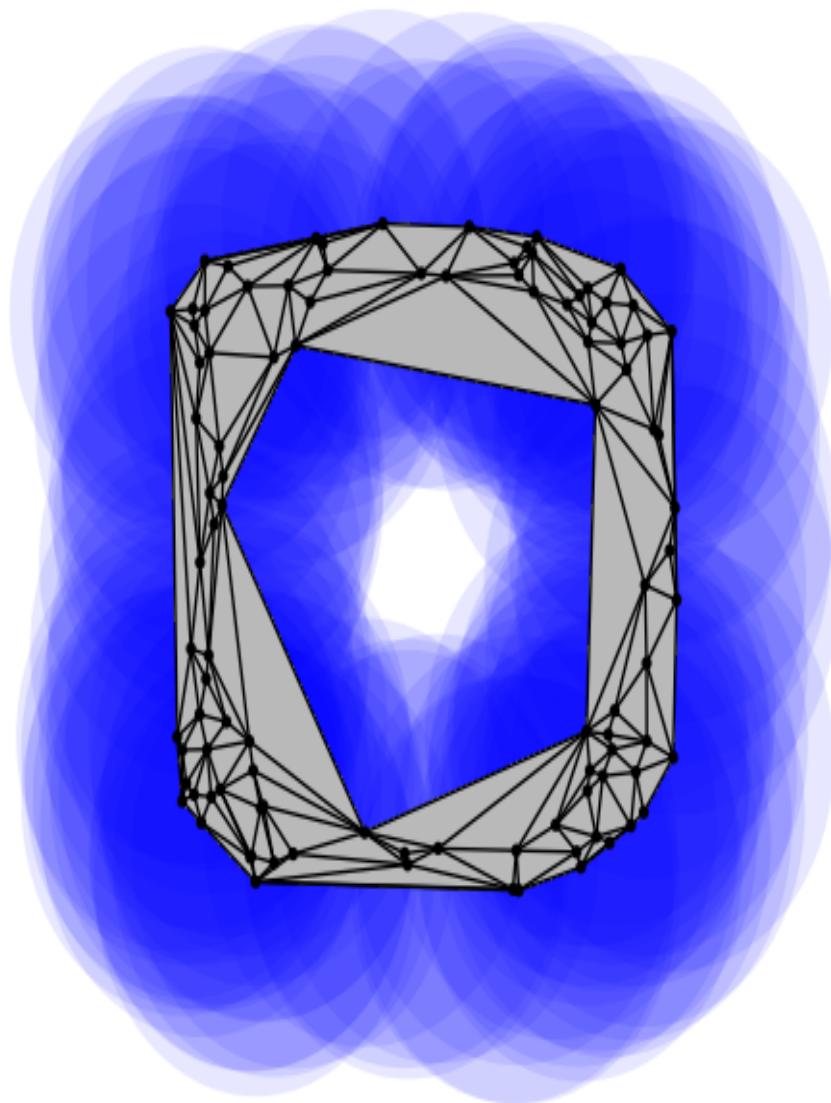


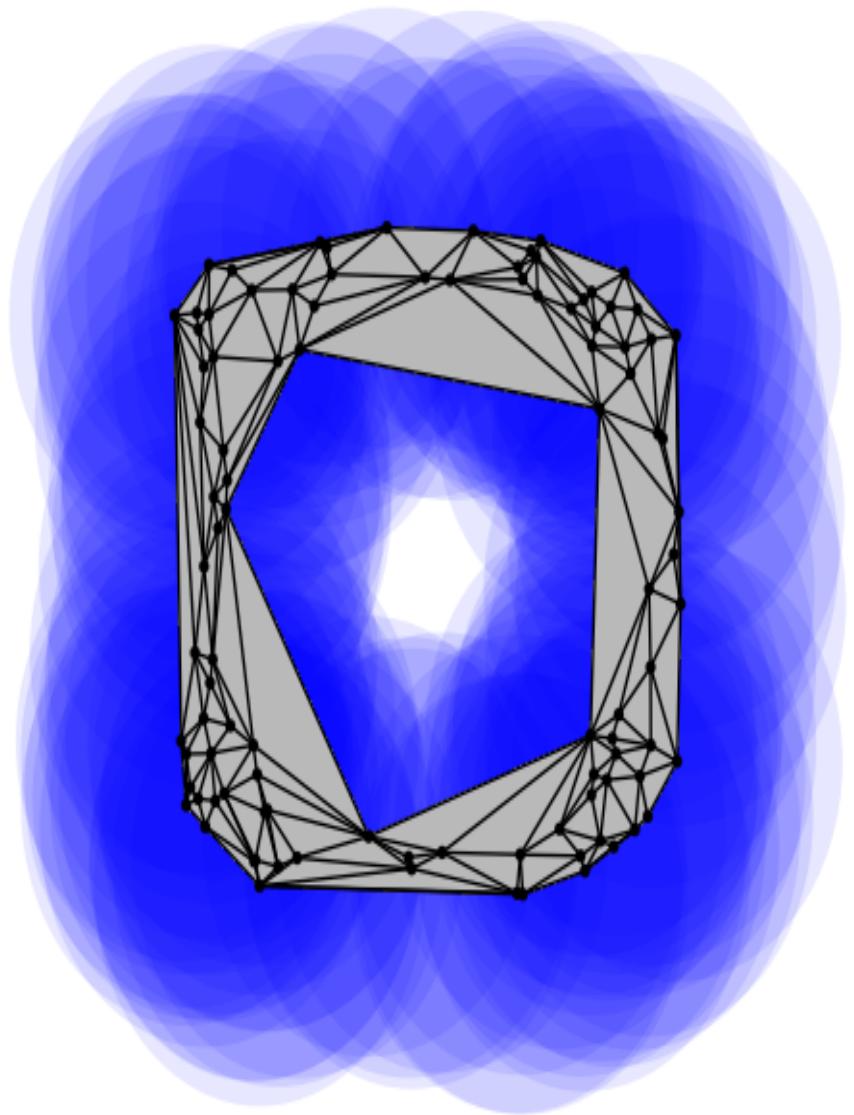


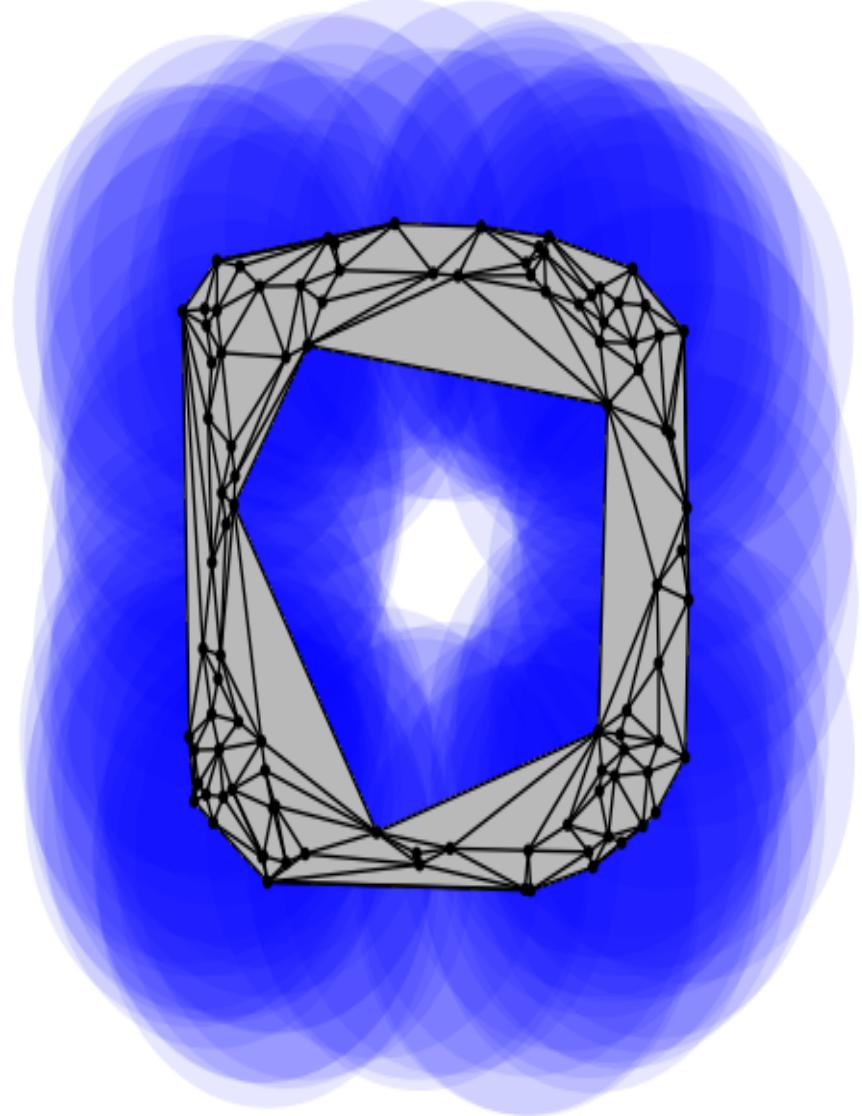


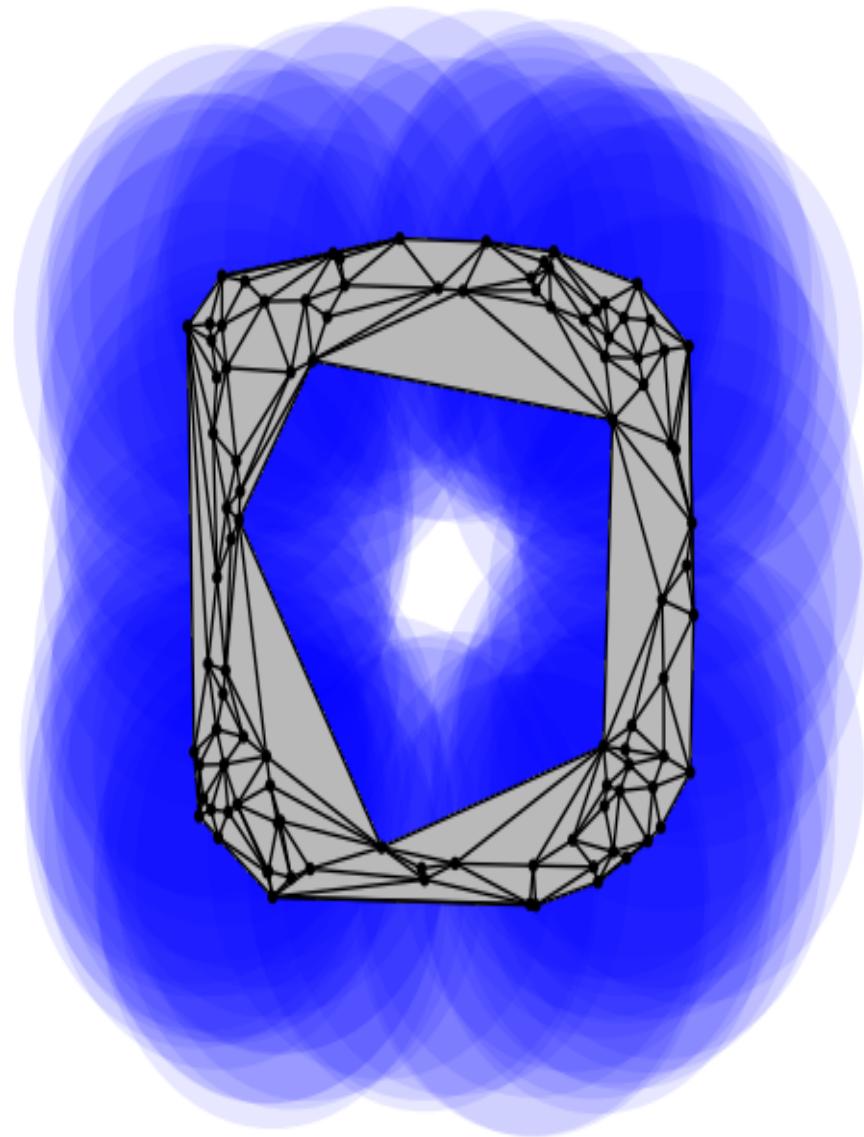


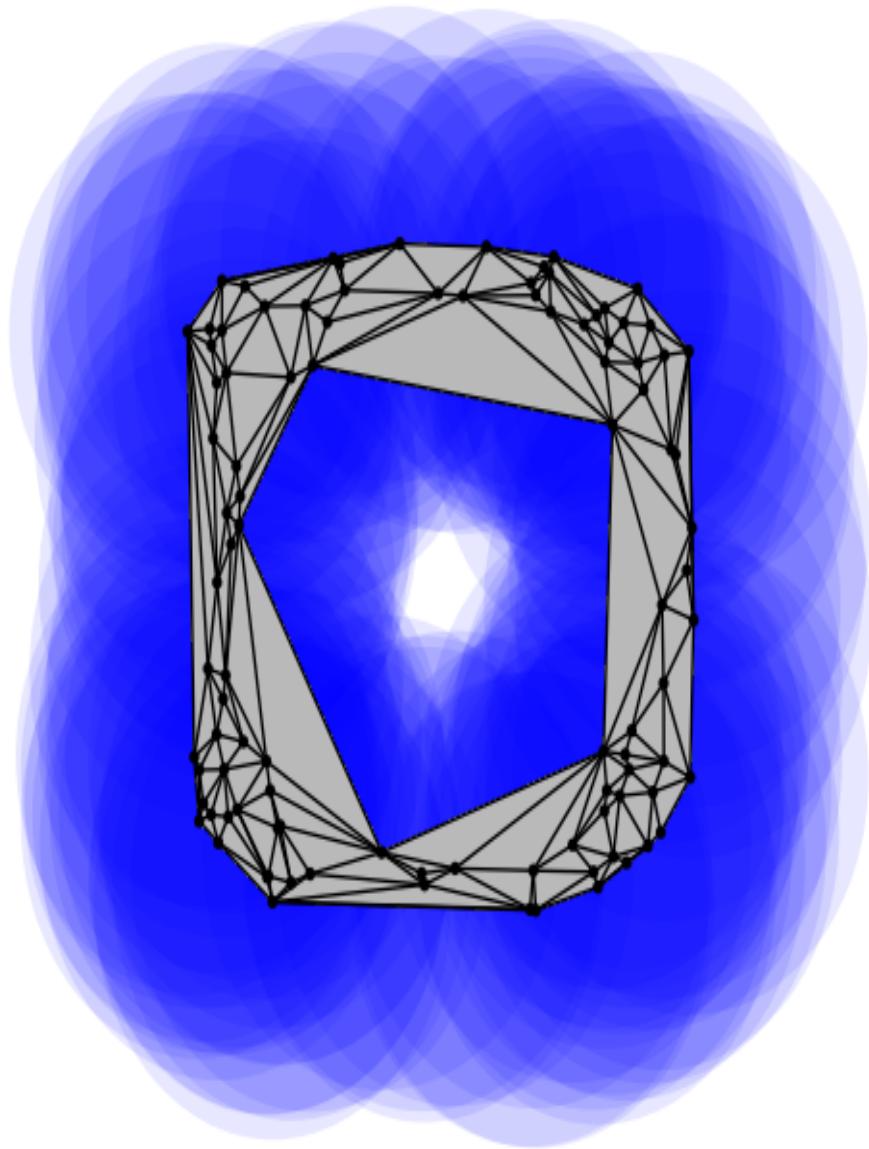


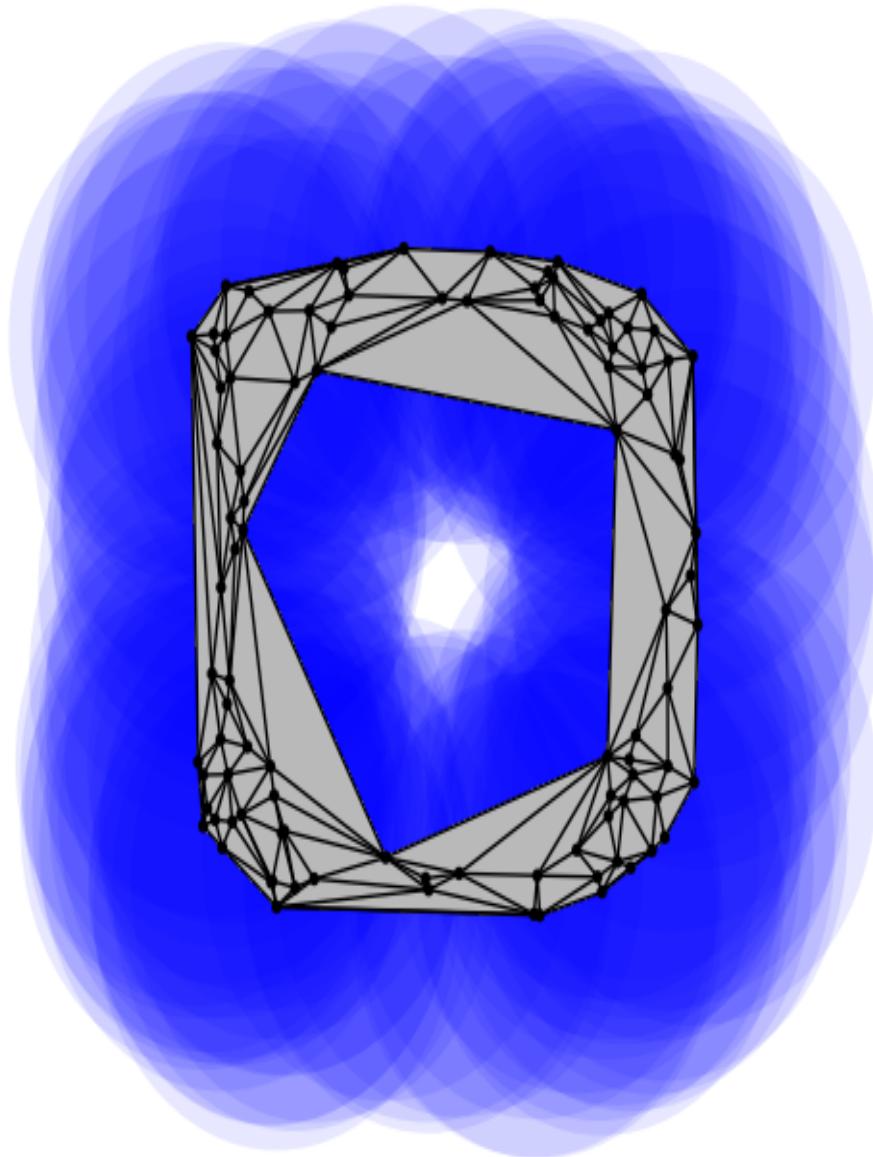


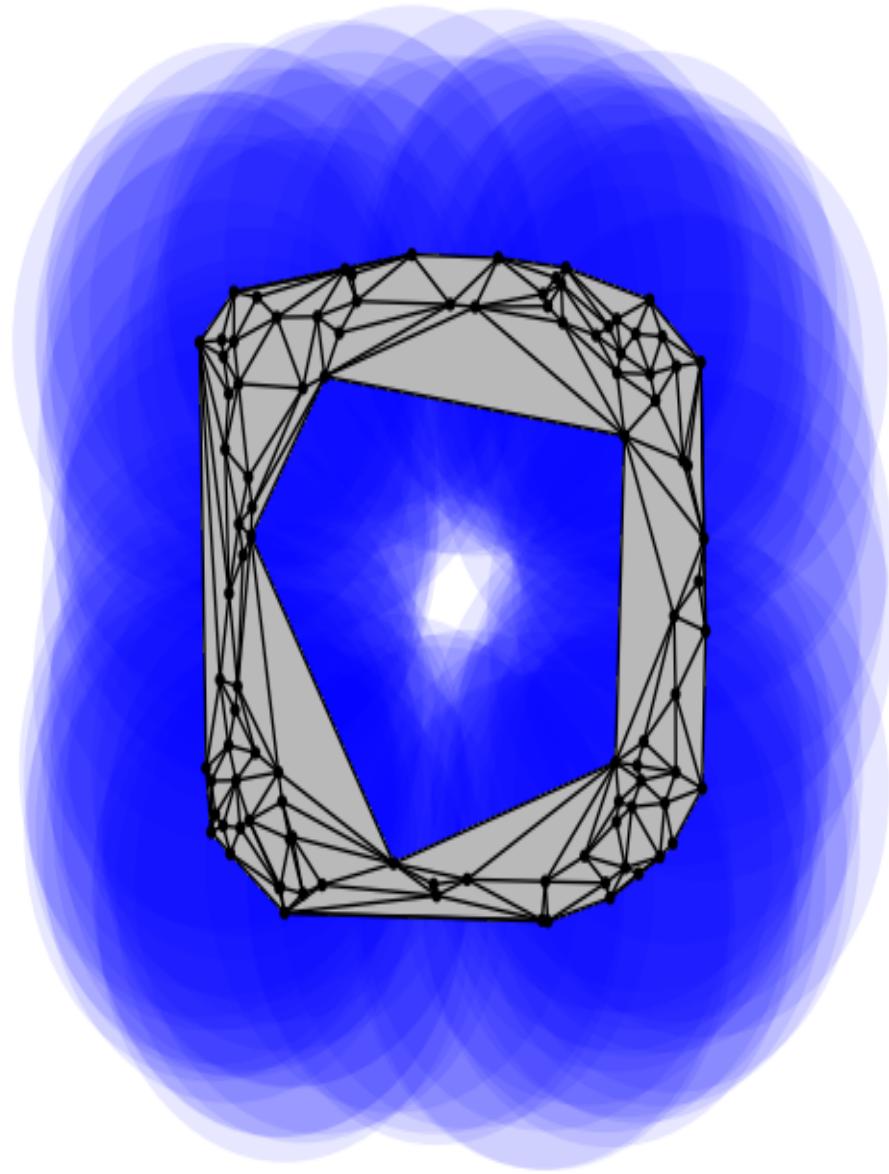


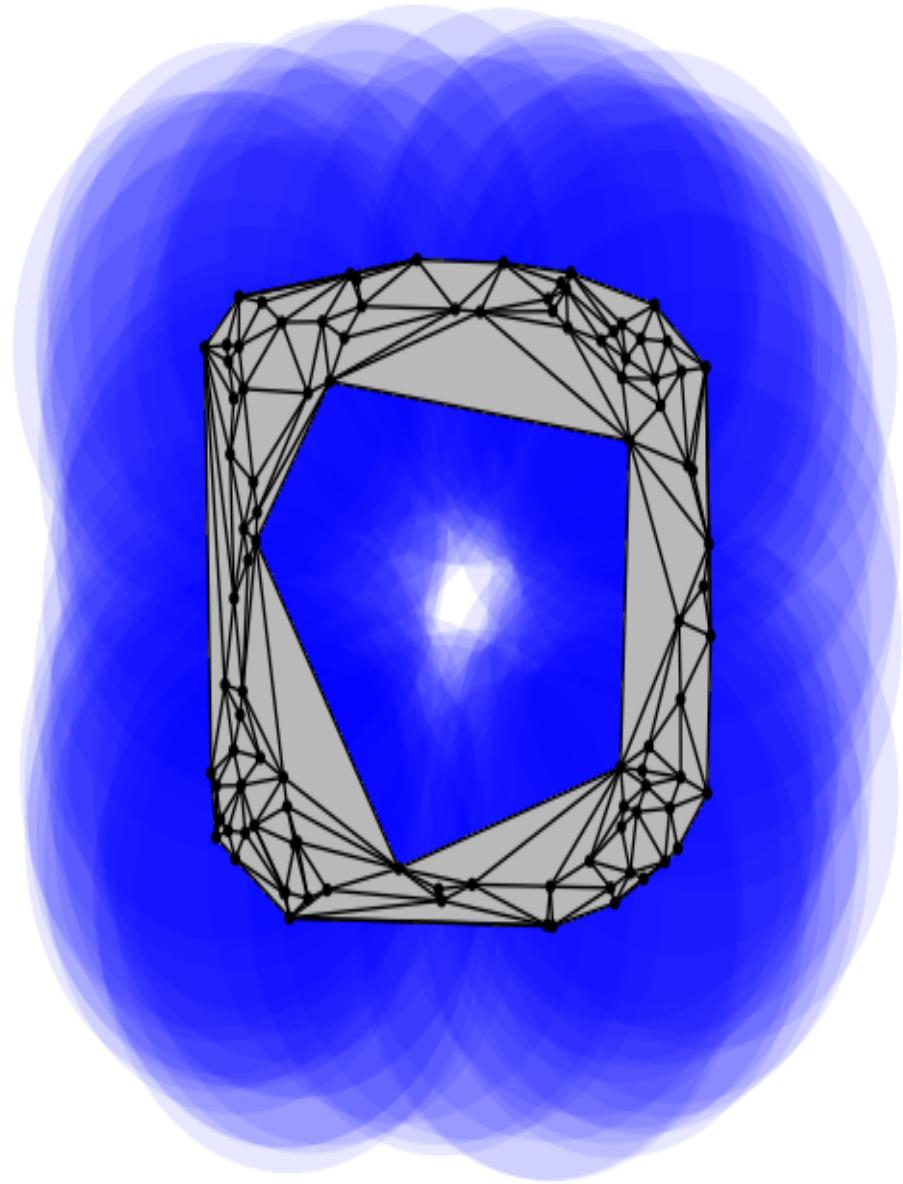


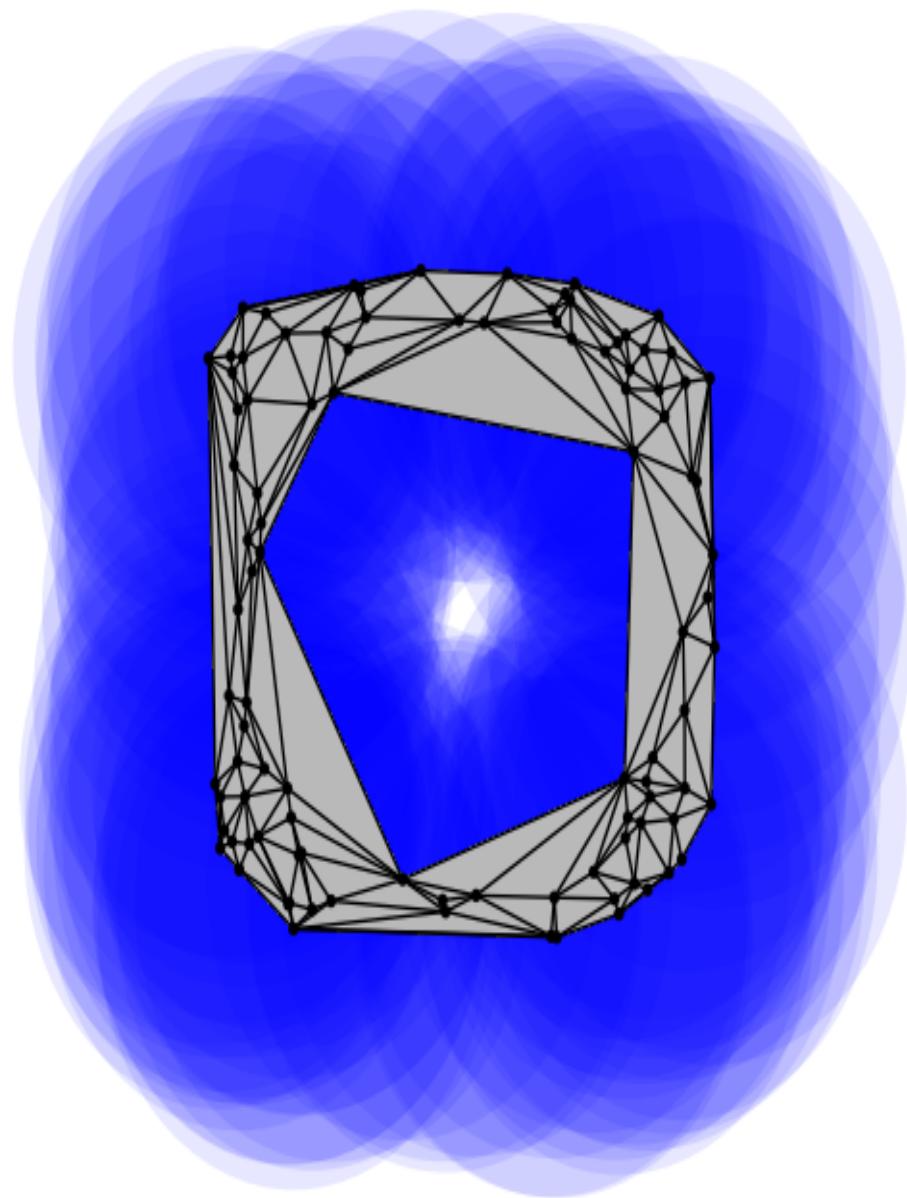


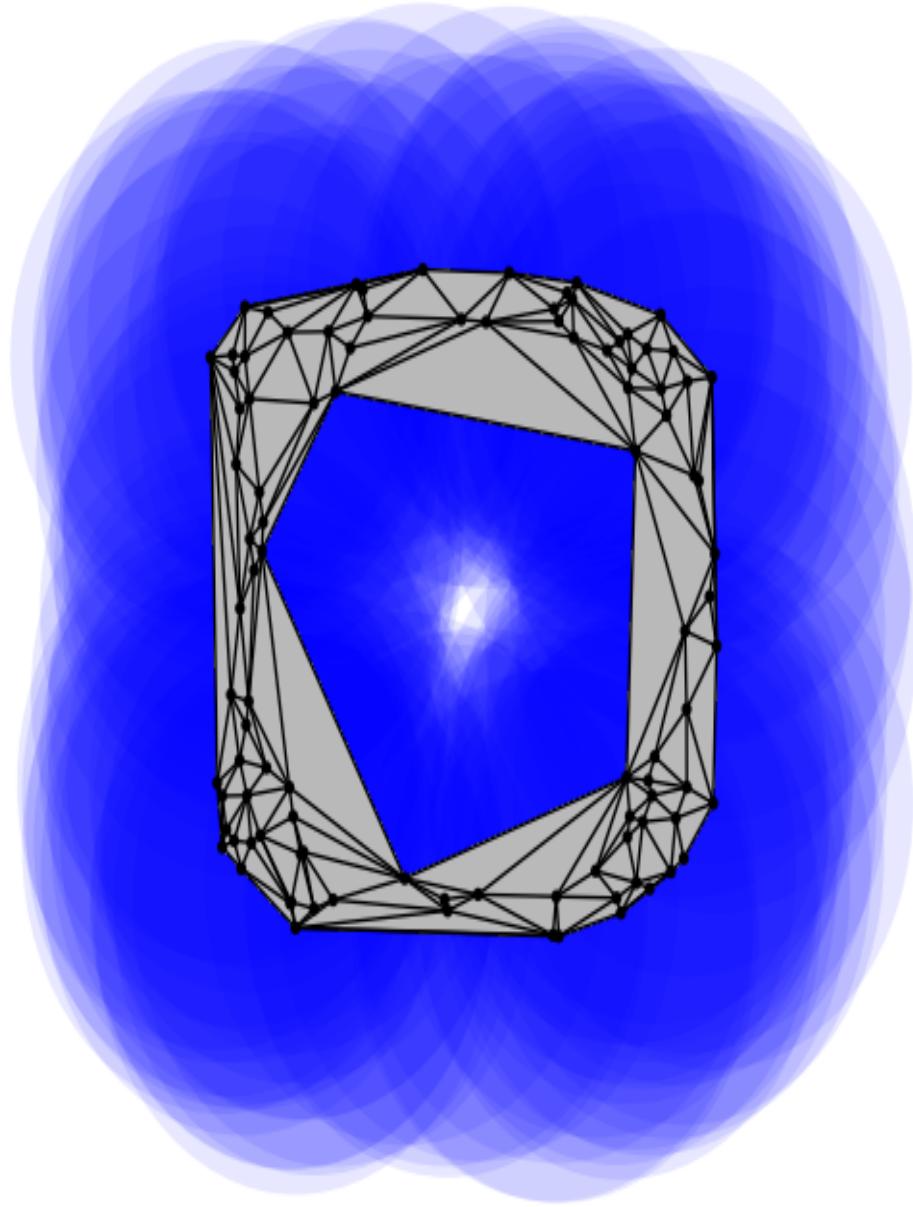


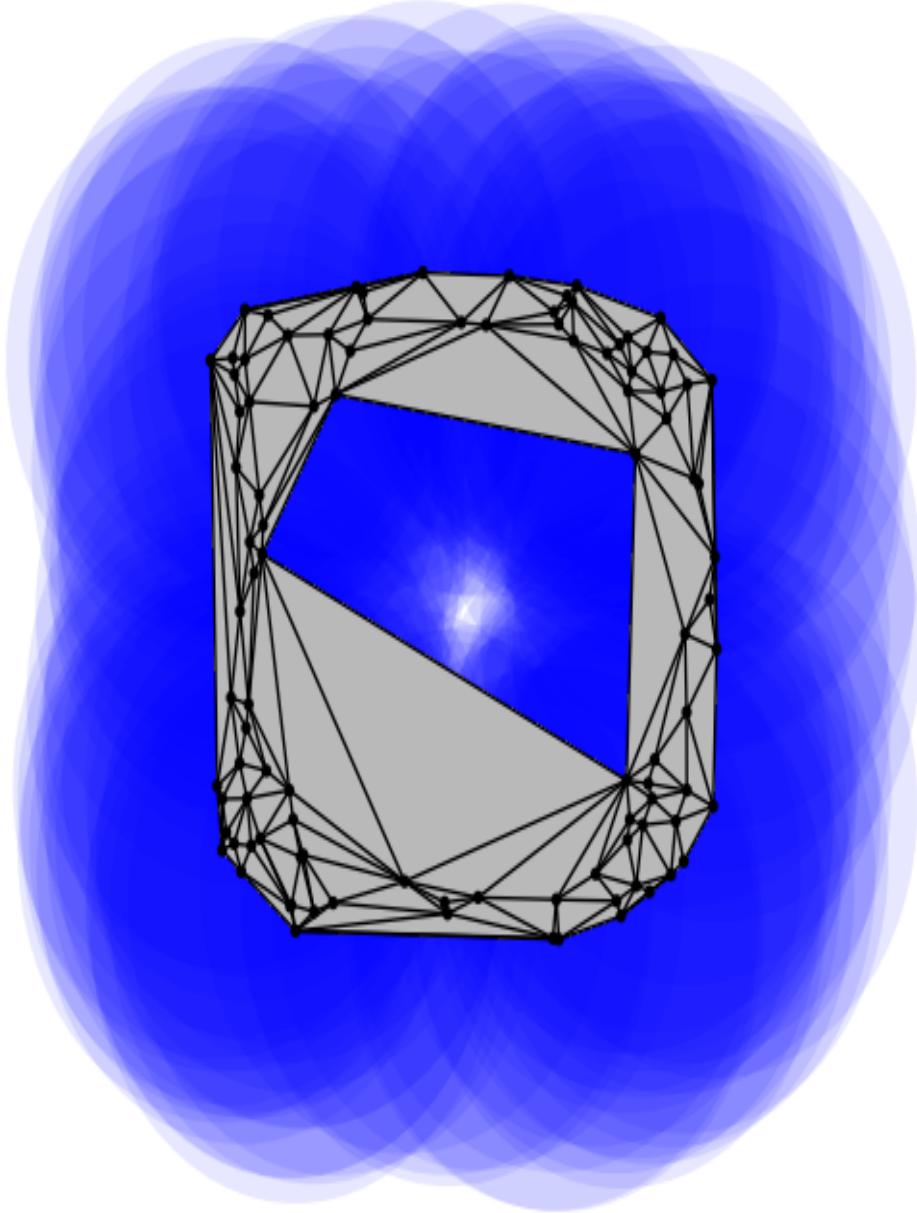


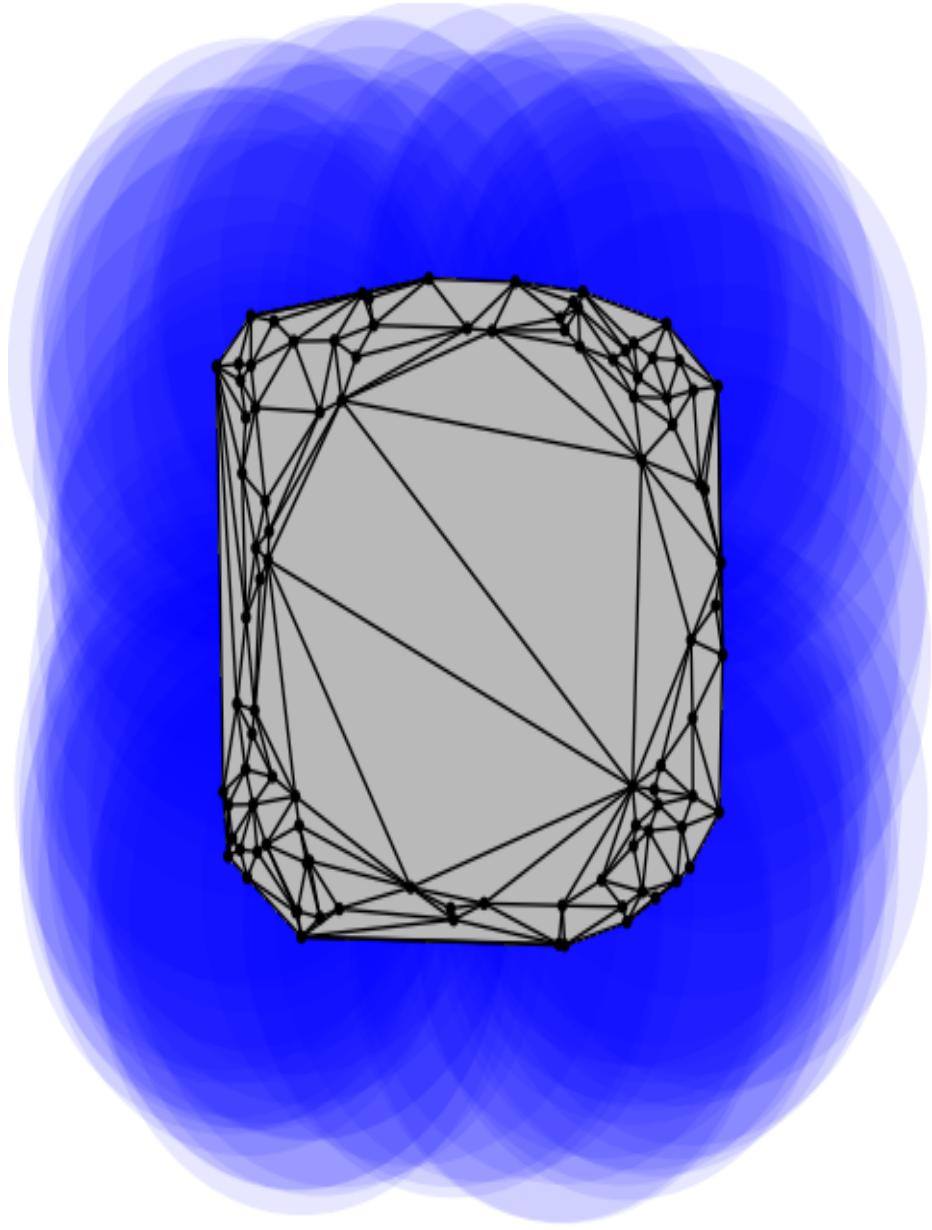


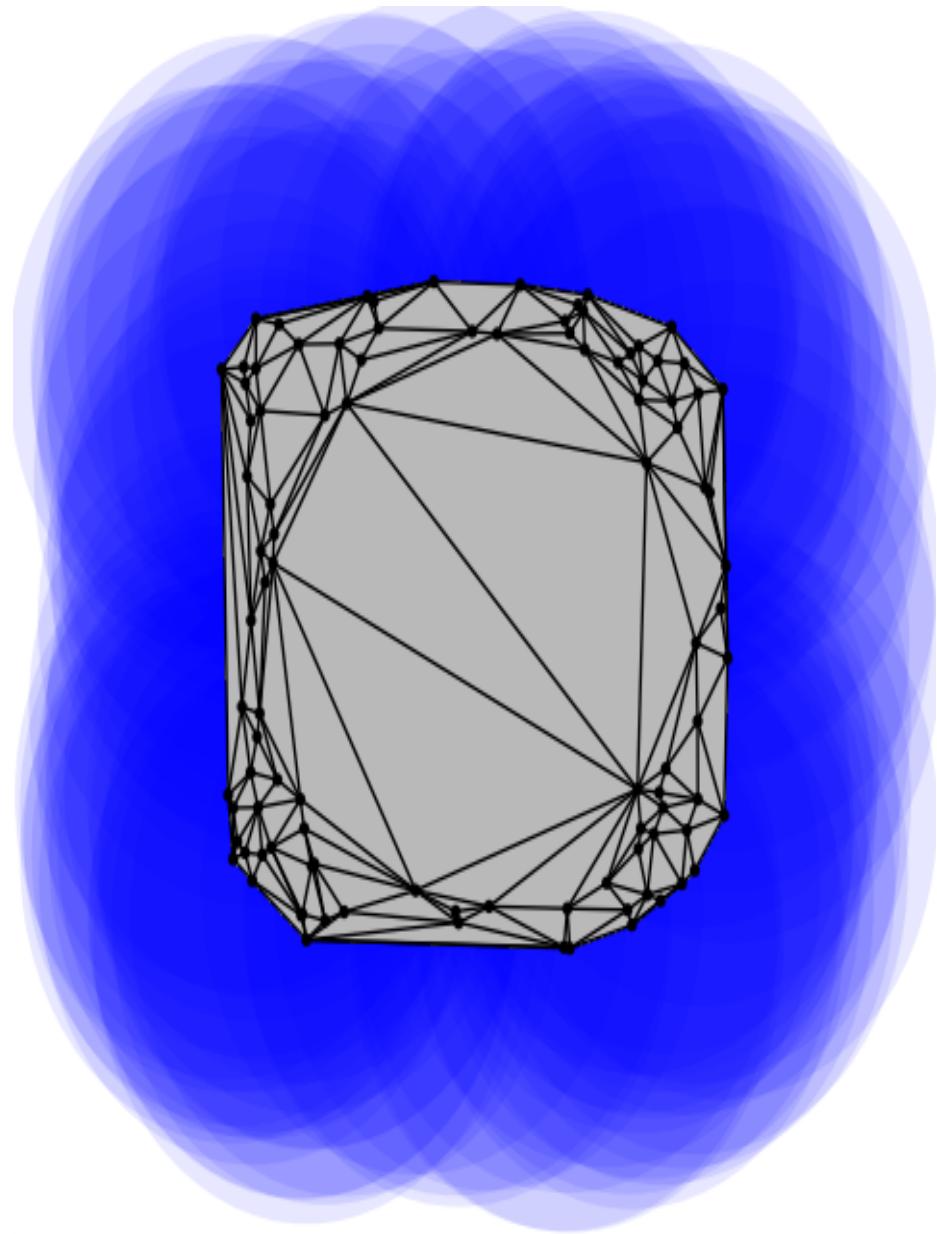


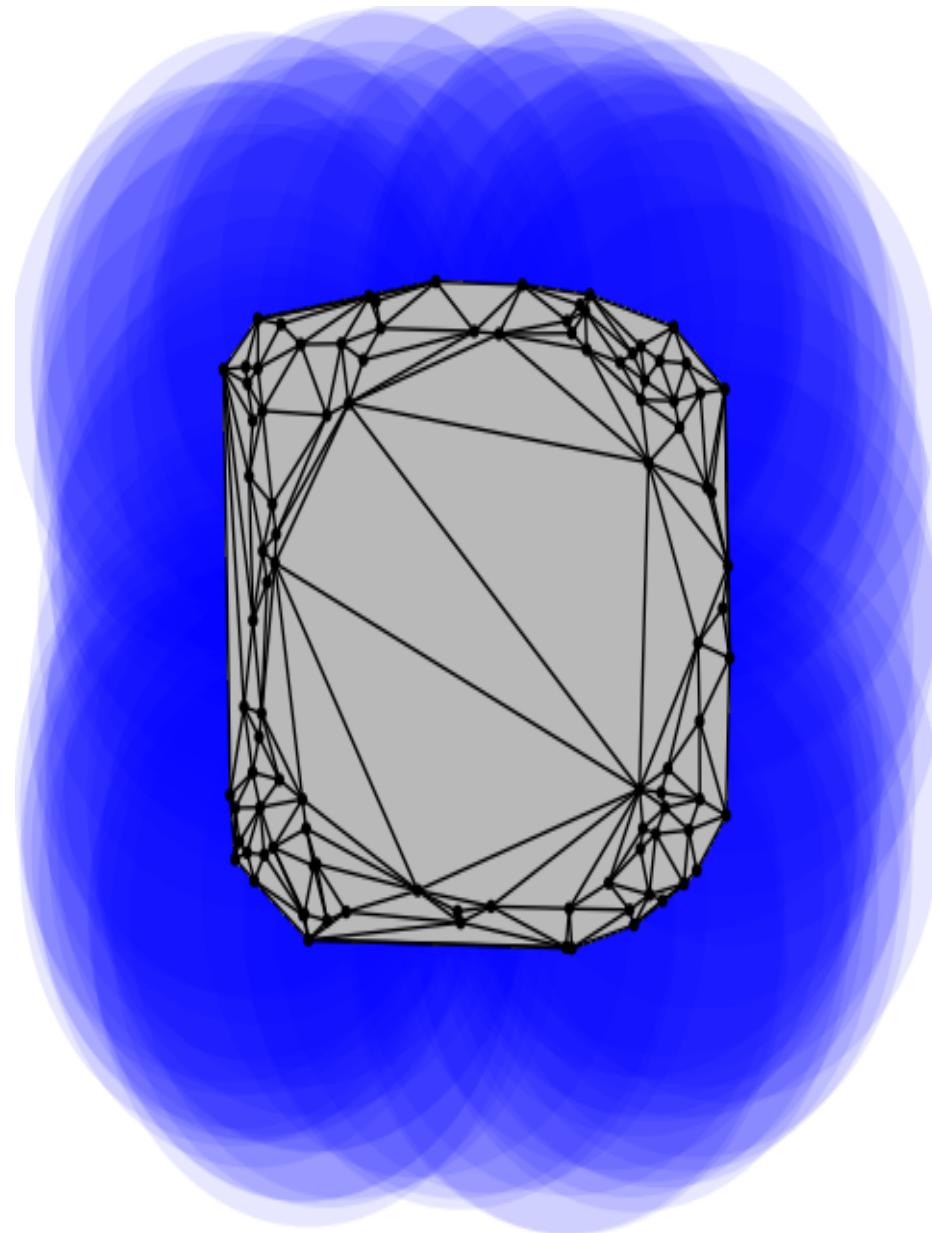




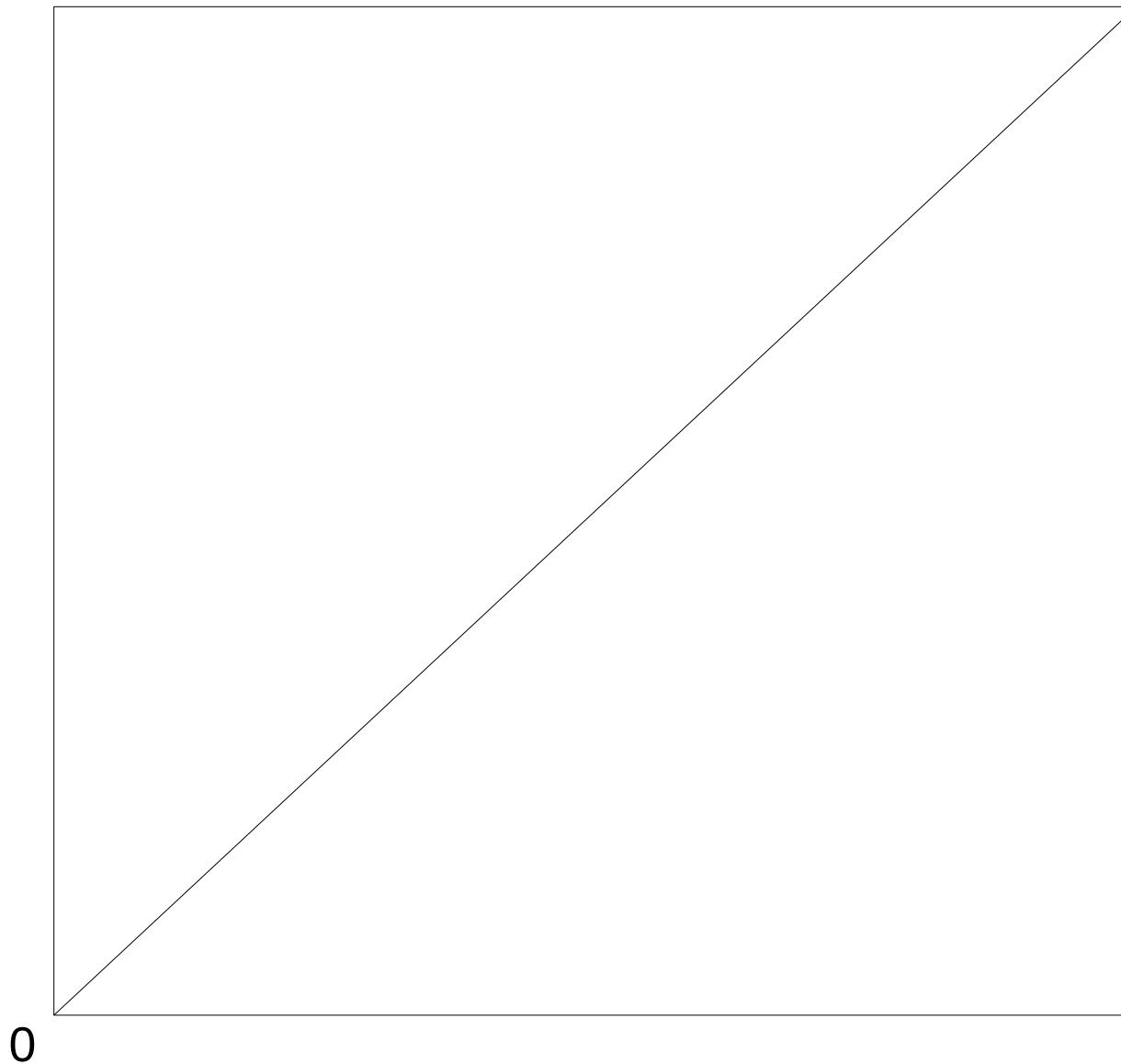




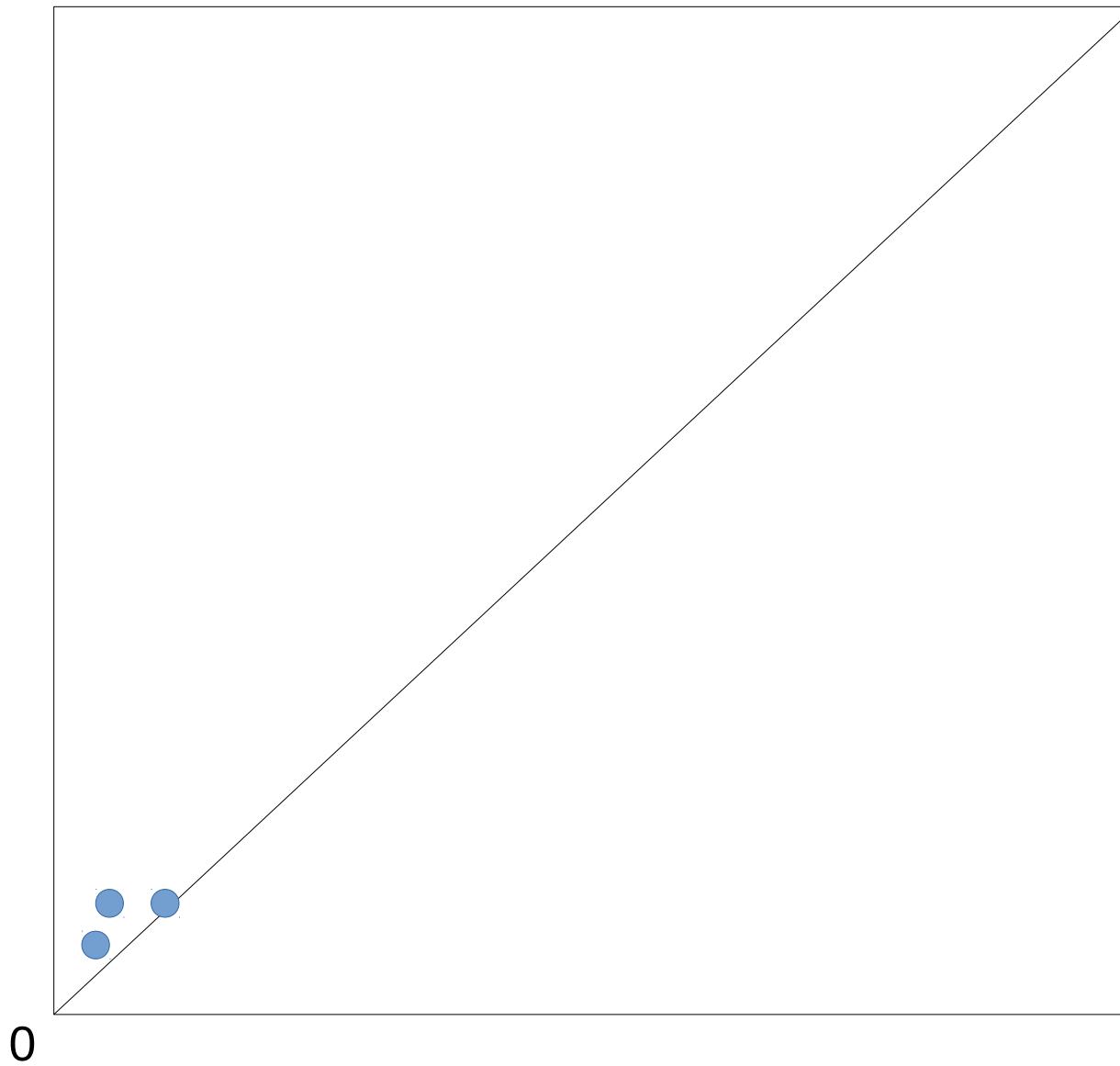




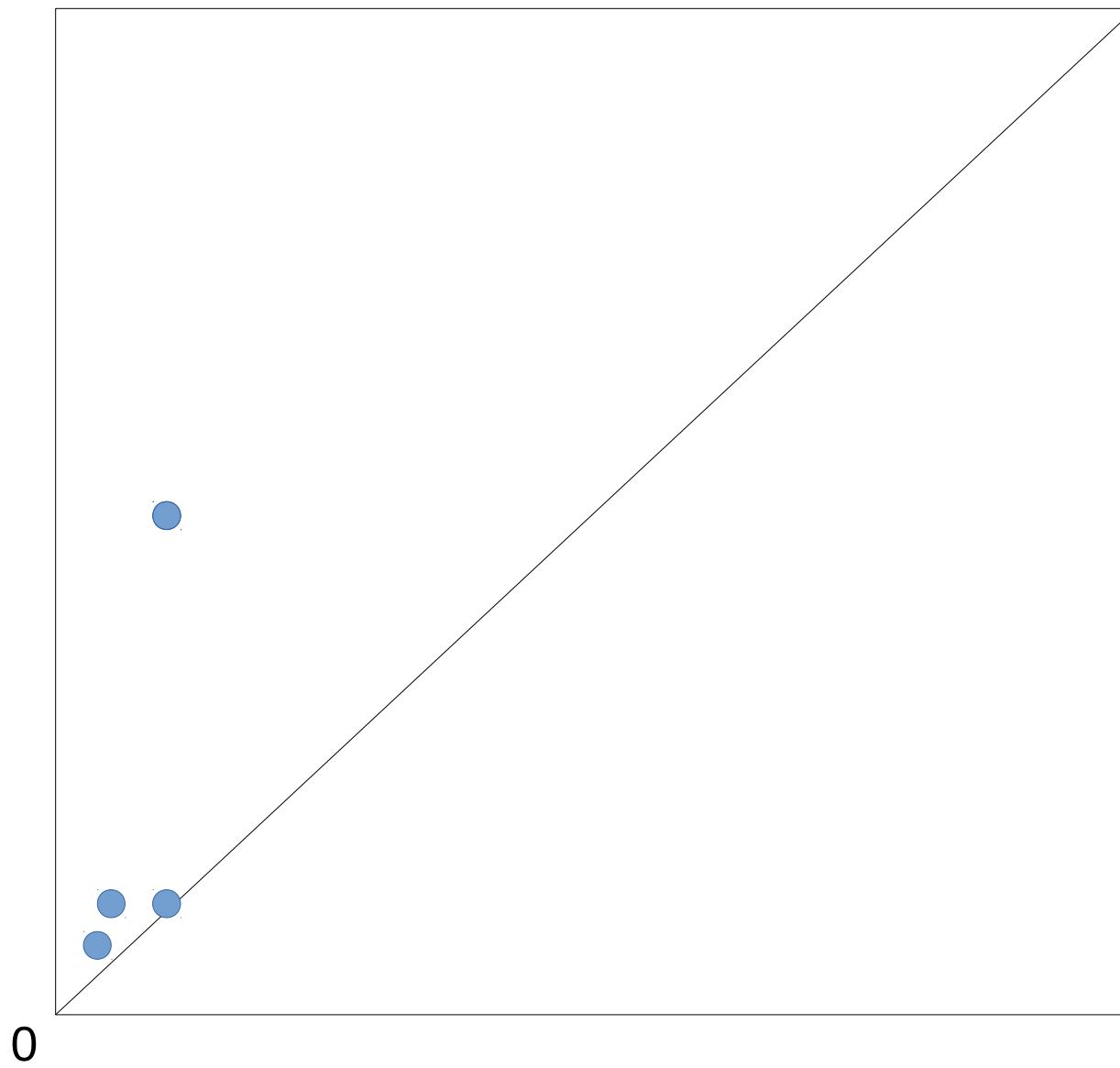
Persistence Diagram



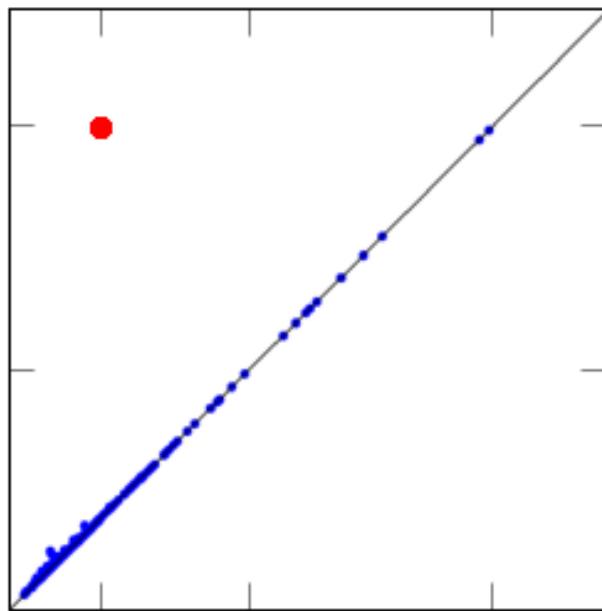
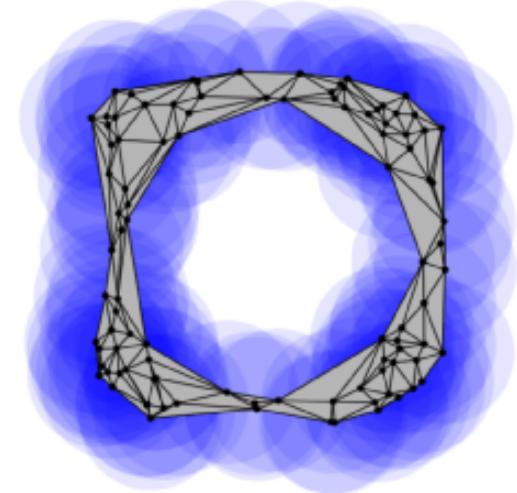
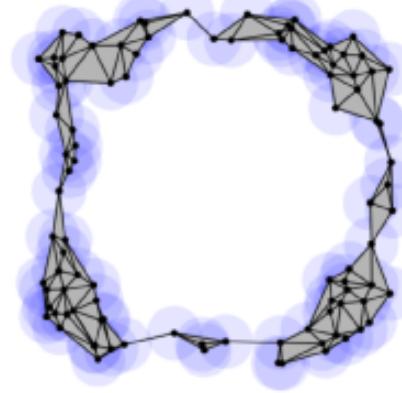
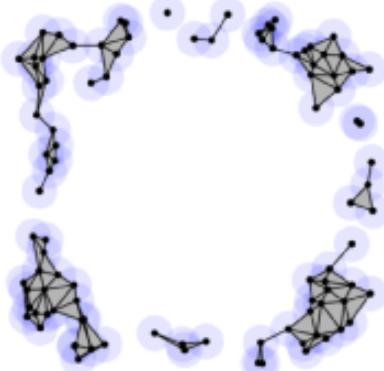
Persistence Diagram



Persistence Diagram



Topological Data Analysis



H. Edelsbrunner, J. Harer: Intro to Computational Topology

G. Carlsson: “Topology and Data”, AMS Bulletin 2009

A. Zomorodian, P. Niyogi, P. Bubenik, F. Chazal, W. Chacholski, U. Bauer,...

Algorithm

```
R = ∂;  
for j = 1 to m do  
    while there exists  $j_0 < j$  with  $low(j_0) = low(j)$  do  
        add column  $j_0$  to column  $j$   
    endwhile  
endfor.
```

Edelsbrunner, Letcher, Zomorodian, 2002
Zomorodian, Carlsson, 2005

Theorem (McMullen '70):

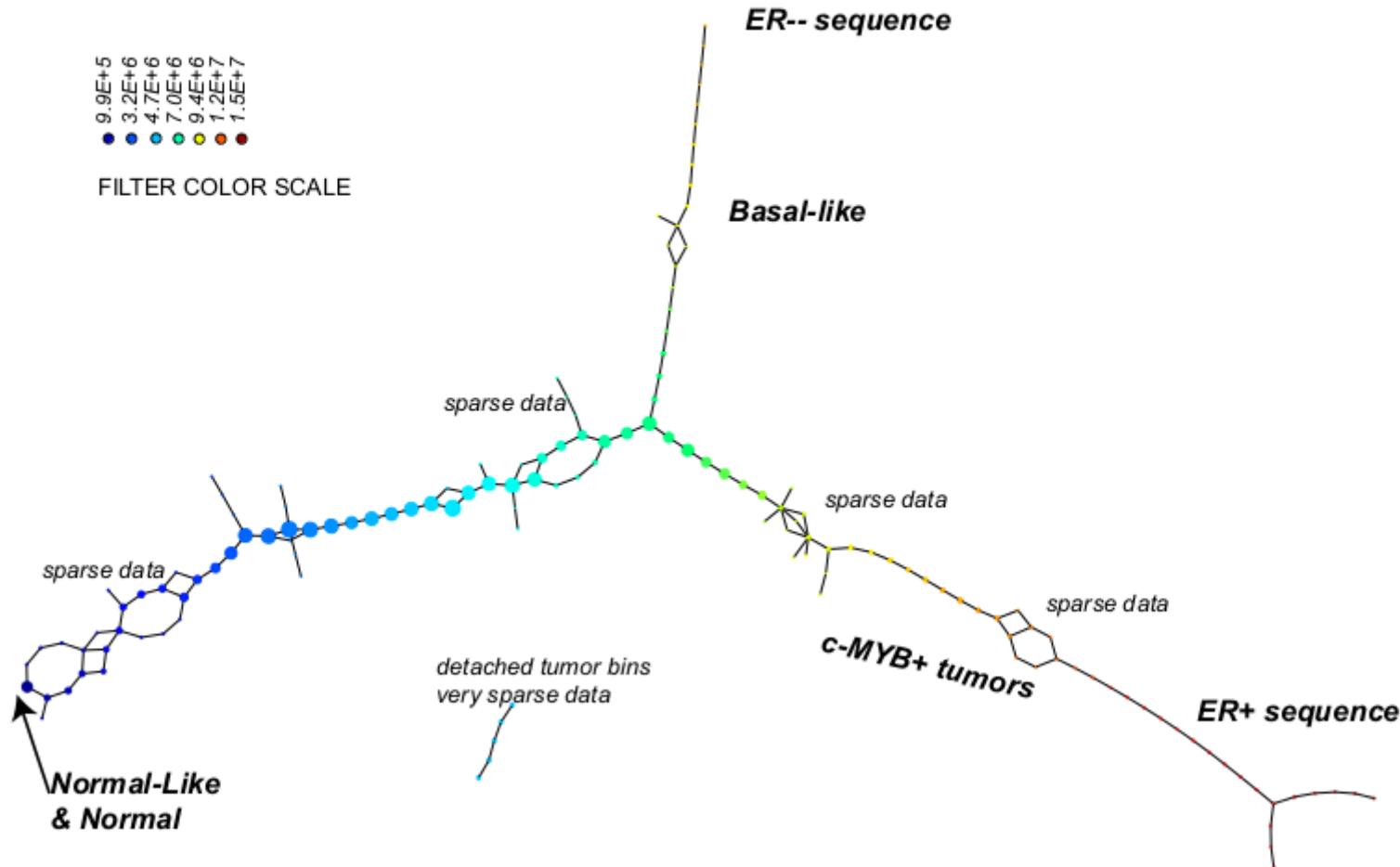
The number of d-simplices in the Delaunay triangulation of n points in d dimensions is at most

$$\binom{n - \lfloor \frac{d+1}{2} \rfloor}{n - d} + \binom{n - \lfloor \frac{d+2}{2} \rfloor}{n - d} = O(n^{\lfloor \frac{d+1}{2} \rfloor})$$

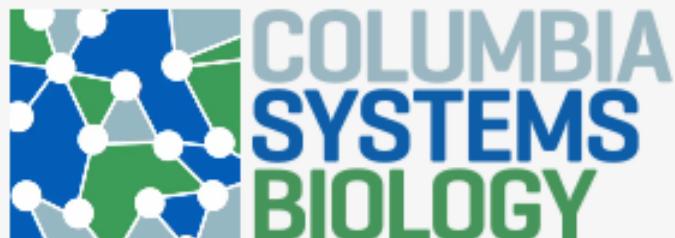
Theorem (Dwyer '91):

The number of d -simplices of n points drawn i.i.d from the unit ball in d dimensions is $O(n)$.

Scientific Applications



M. Nicolau et al, "Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival" PNAS 2010



Center for Topology of Cancer Evolution and Heterogeneity

RESEARCH

CENTERS OF EXCELLENCE

Center for Multiscale Analysis of Genomic & Cellular Networks (MAGNet)

Library of Integrated Network-Based Cellular Signatures (LINCS)

Cancer Target Discovery & Development Center

[Click here](#) to visit the Columbia University Center for Topology of Cancer Evolution and Heterogeneity website.

The Columbia University Center for Topology of Cancer Evolution and Heterogeneity is a member of the [National Cancer Institute's Physical Sciences in Oncology Network](#). Founded in 2009, the

RELATED NEWS

May 21, 2015

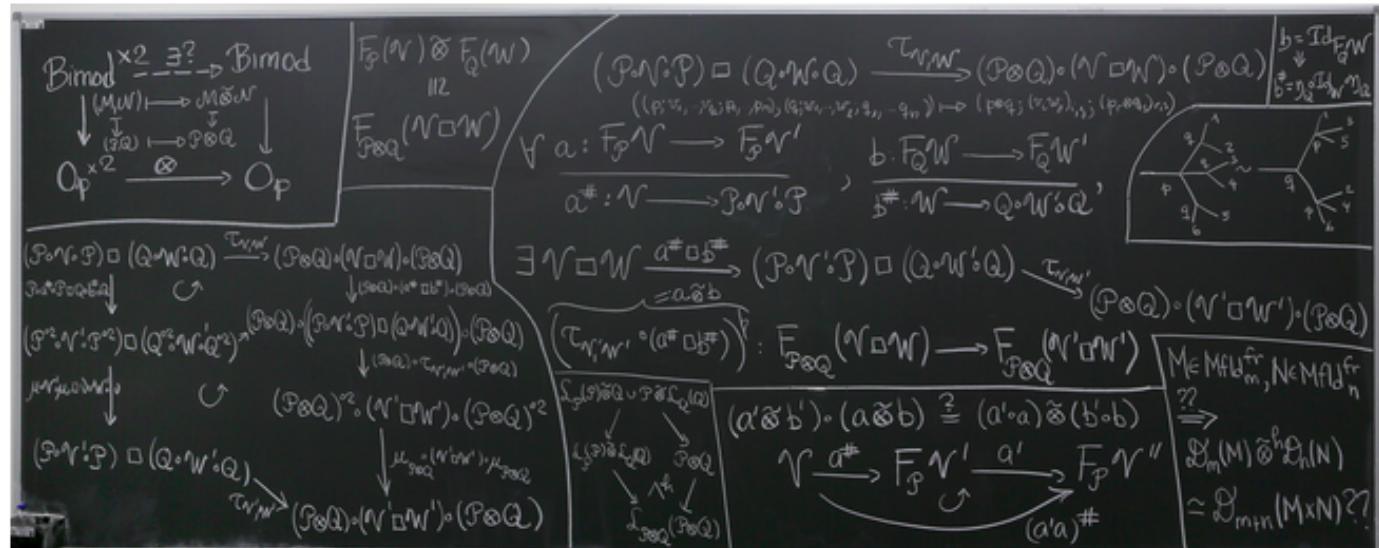
Columbia Investigators Awarded New NCI Physical

LABORATORY FOR TOPOLOGY AND NEUROSCIENCE UPHESS

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Welcome!



Competence or skills

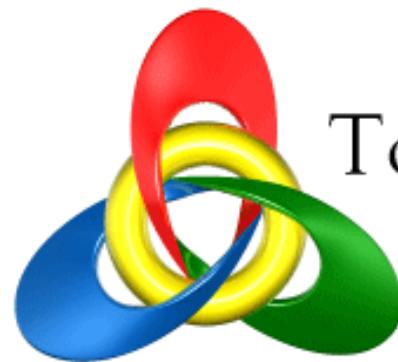
Our laboratory focuses on algebraic topology and its applications, primarily to the life sciences, in particular to neuroscience.

Homotopy theory and category theory are particular areas of expertise in pure mathematics of our research group.

CONTACTS

Group Head

[Prof. Kathryn Hess Bellwald](#)



Topological Materials Science

トポロジーが紡ぐ物質科学のフロンティア

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Norio Kawakami

[Department of Physics, Kyoto University]

Topology & Spacial Reasoning

Learning from Demonstration



Motion Clustering

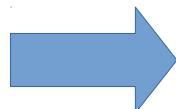
Fréchet distance

Hausdorff distance

L^2 distance

Dynamic Time Warping

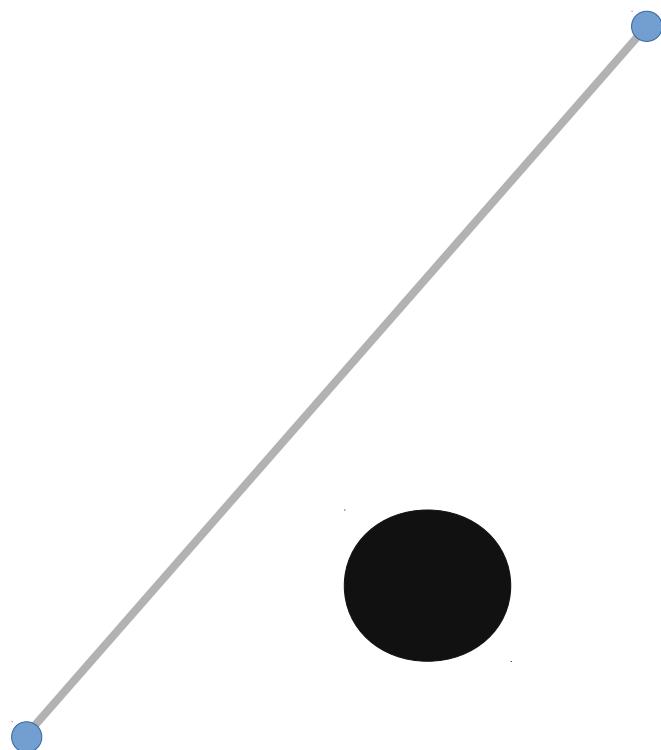
Custom
Featurization/Embedding



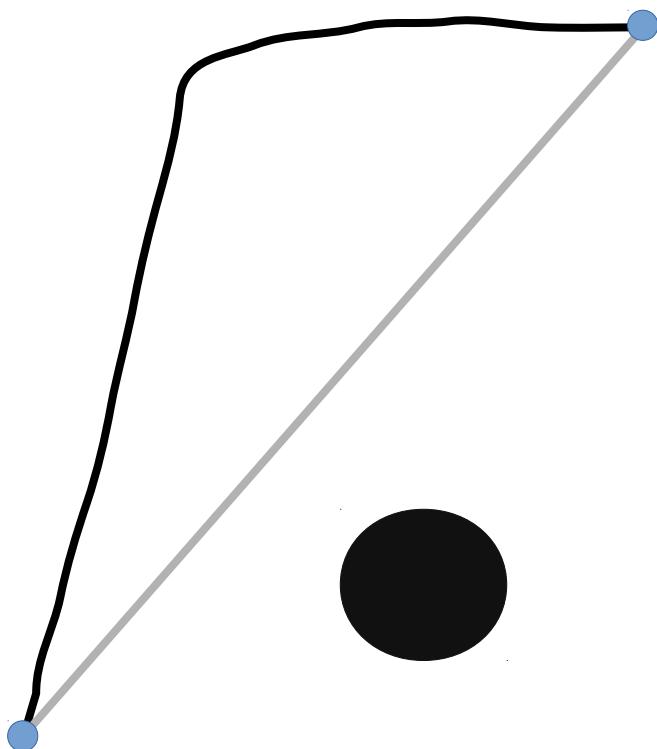
Metric Clustering
(e.g. single linkage)

....

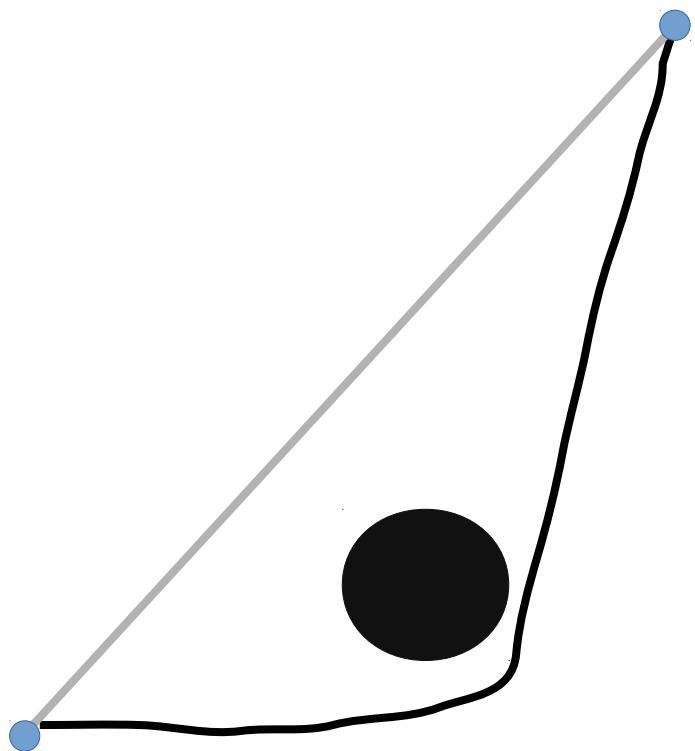
Topology & Spacial Reasoning



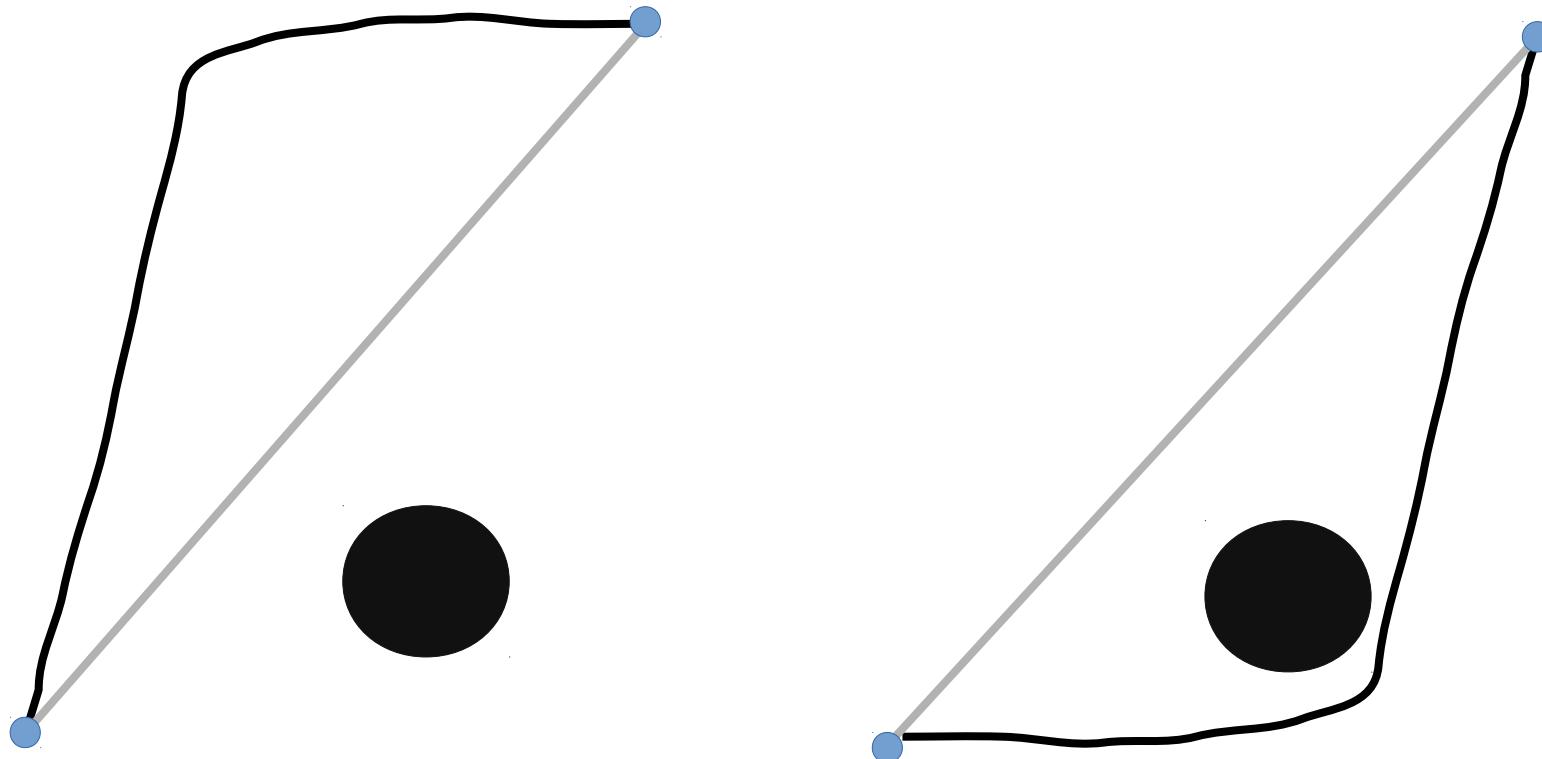
Metric Clustering?



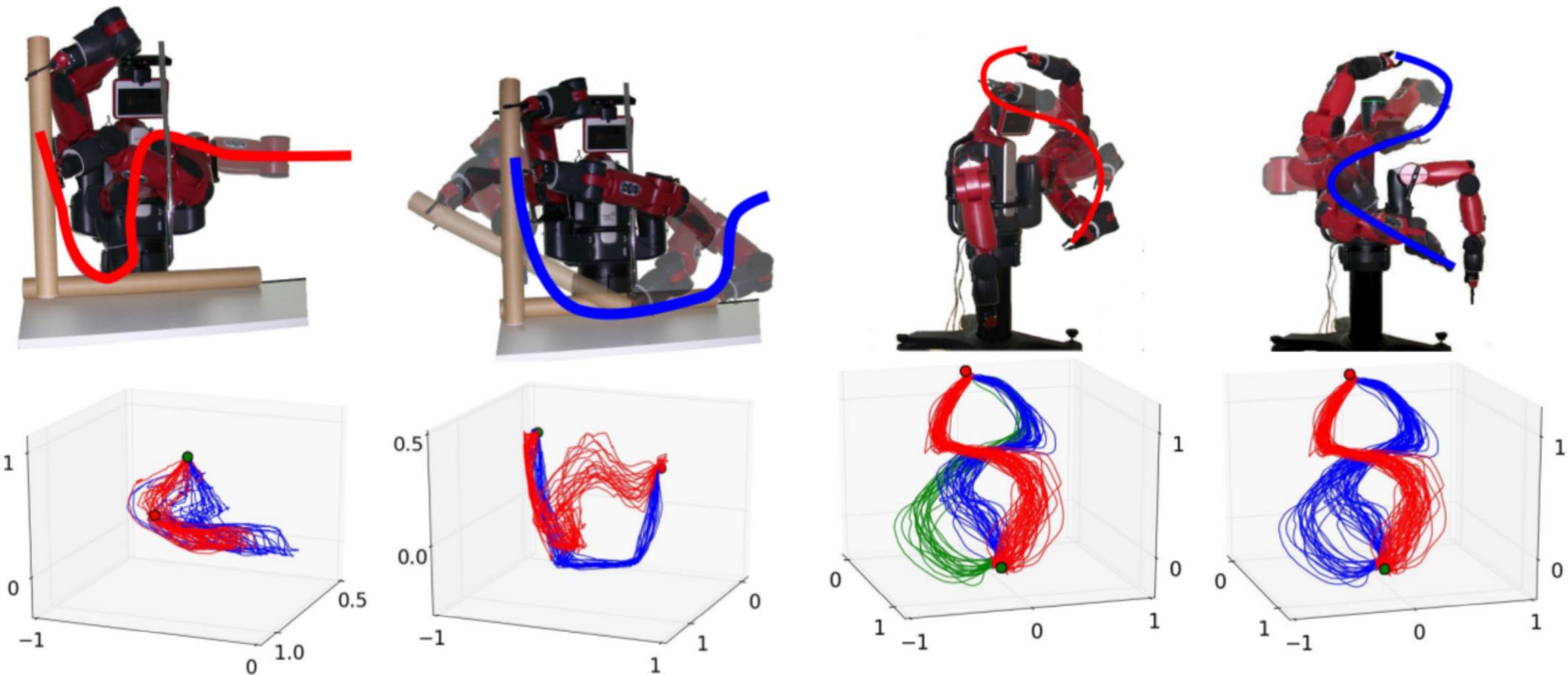
Metric Clustering?



1-Cycles & Clustering

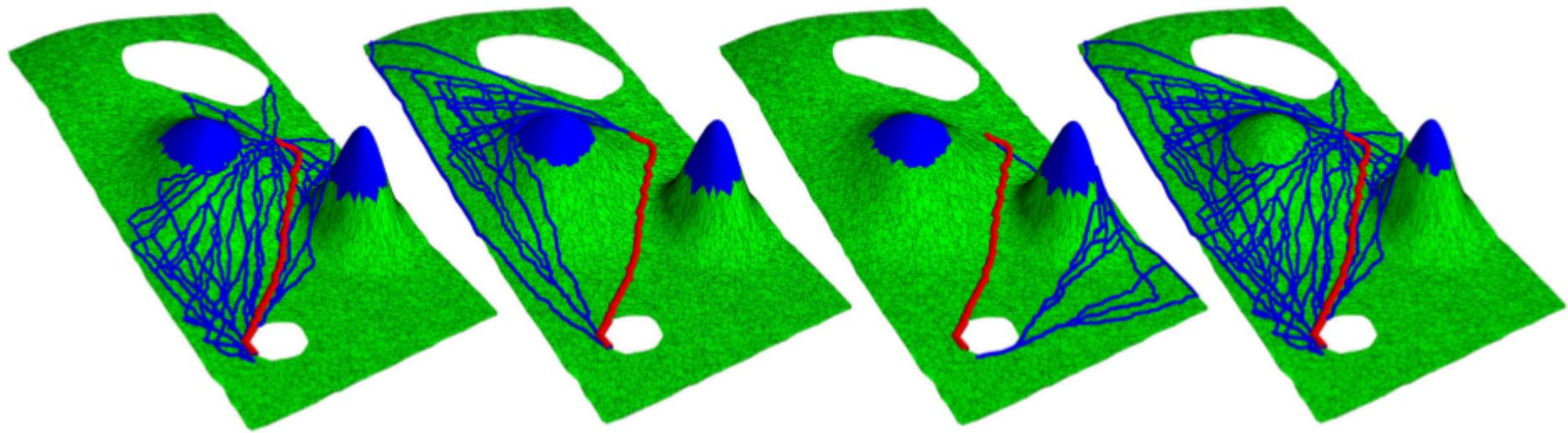


Robotics Applications - Clustering



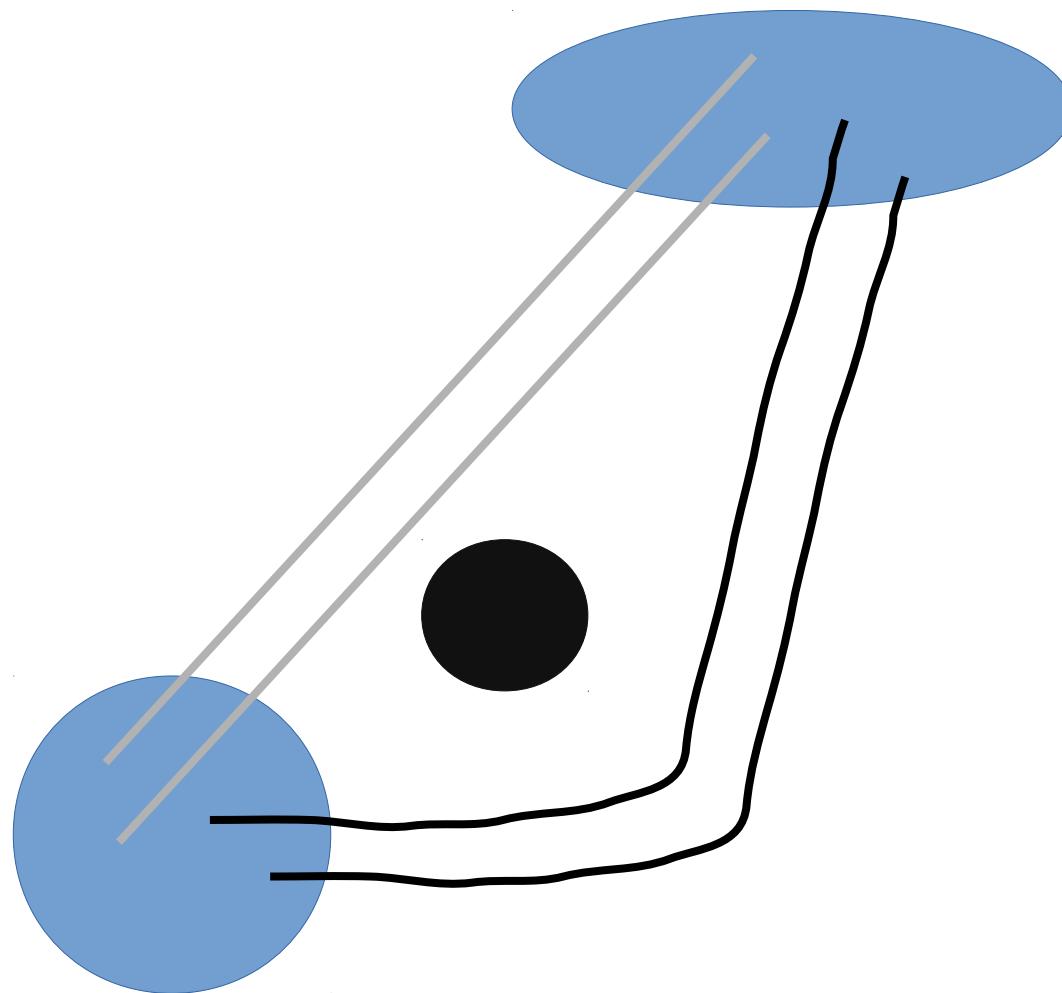
Multiscale Topological Trajectory Classification with Persistent Homology
F. T. Pokorny, M. Hawasly, S. Ramamoorthy, RSS 2014

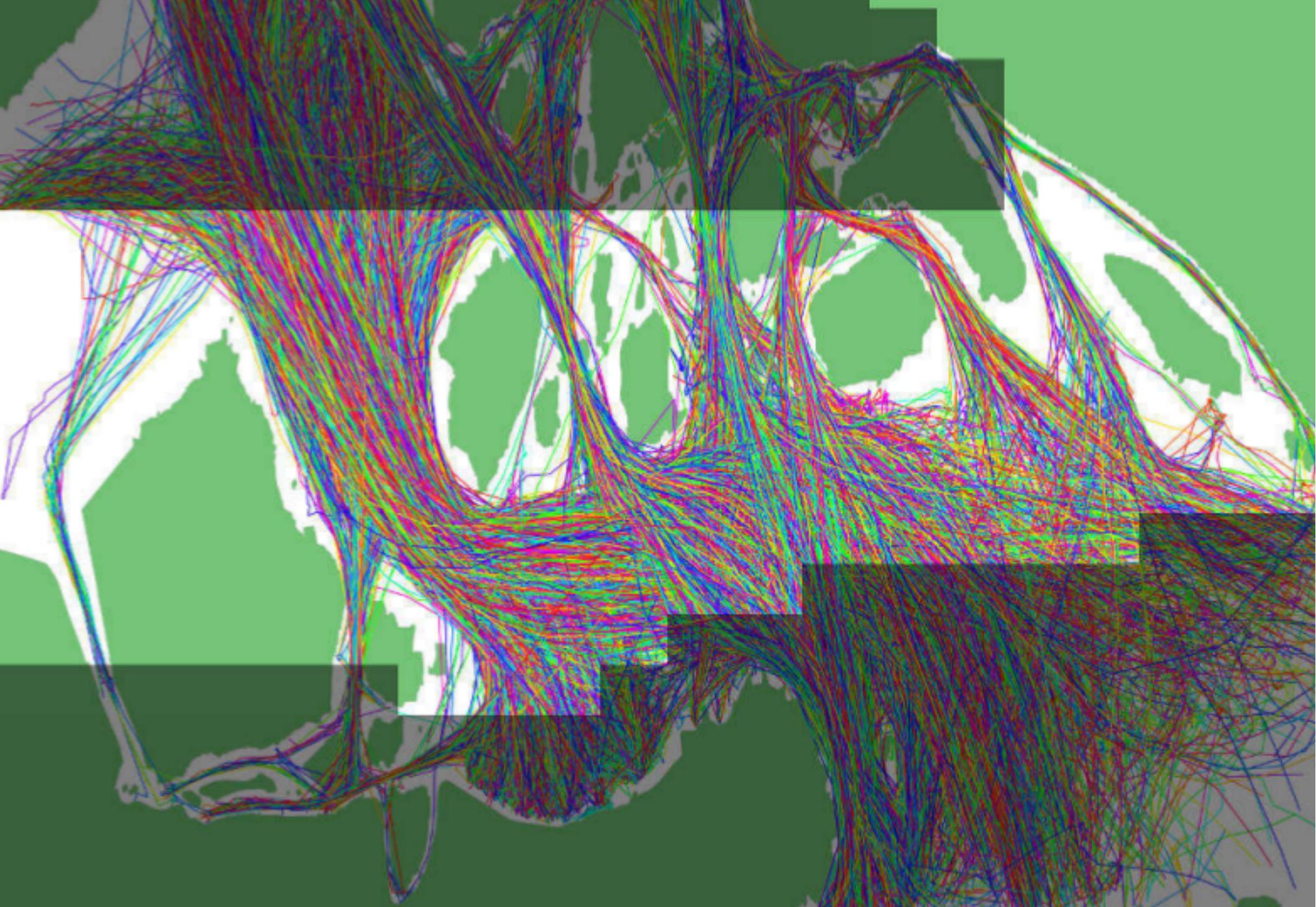
Robotics Applications - Clustering



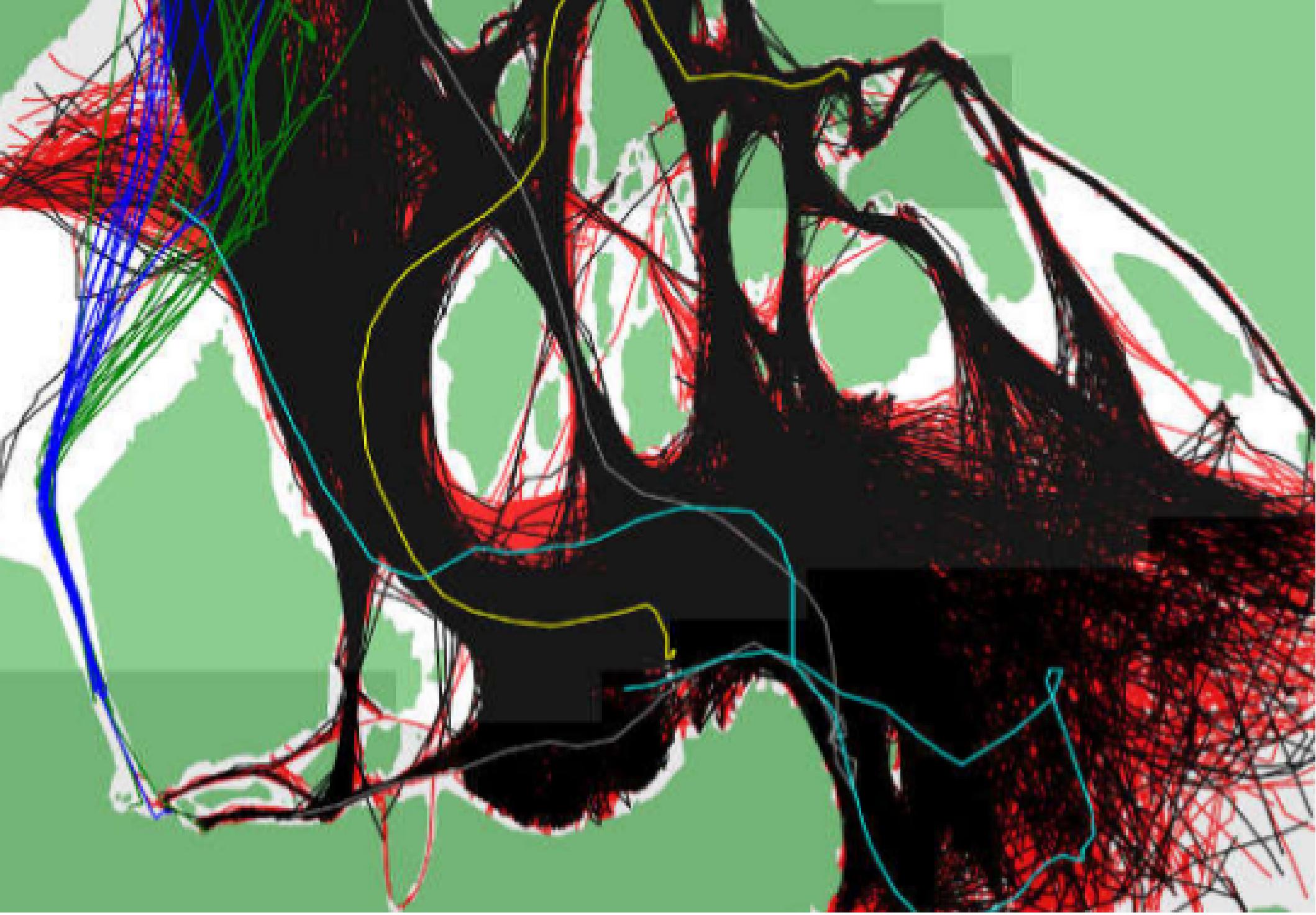
Multiscale Topological Trajectory Classification with Persistent Homology
F. T. Pokorny, M. Hawasly, S. Ramamoorthy, RSS 2014

Generalization to Terminal Regions

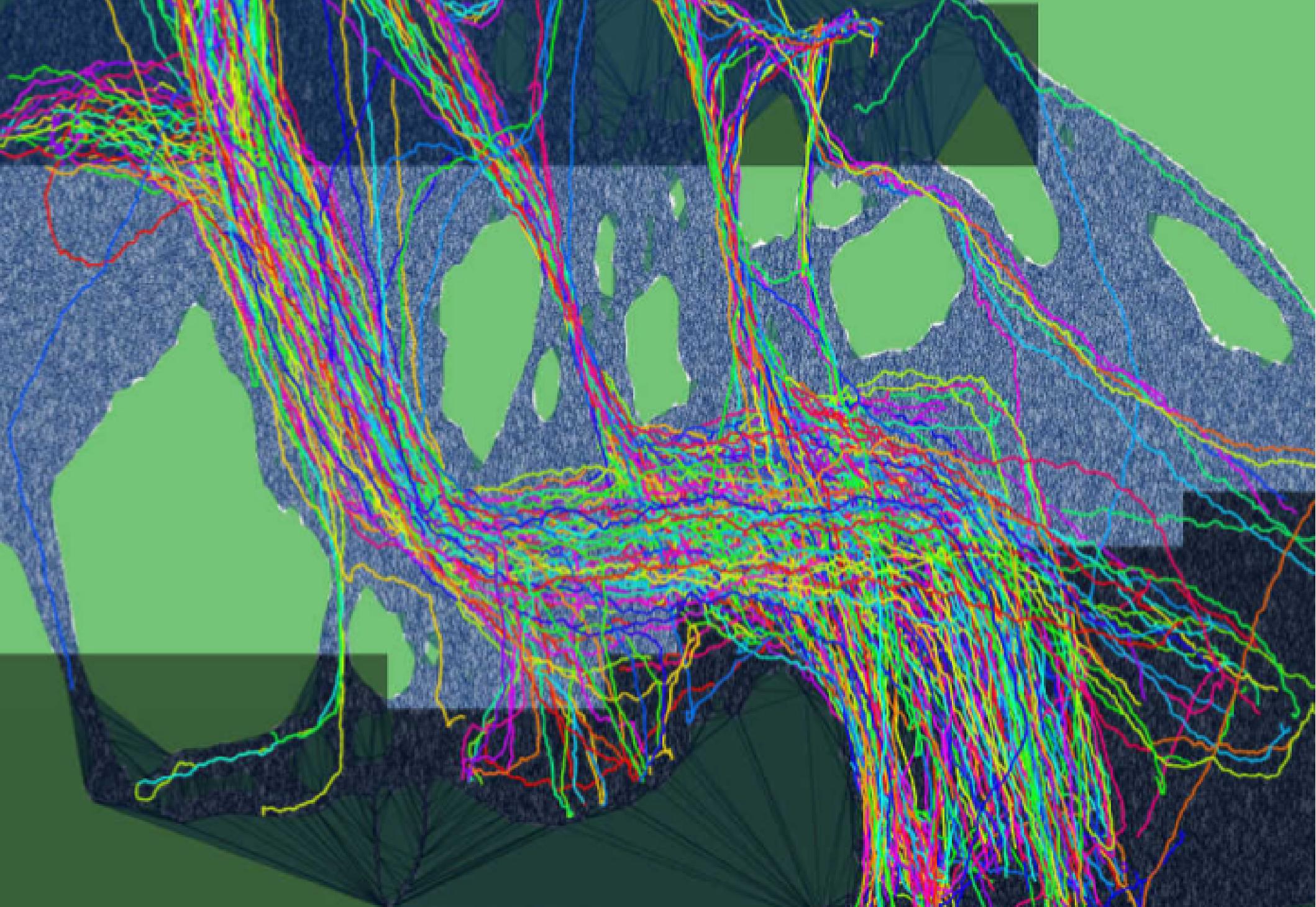




Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

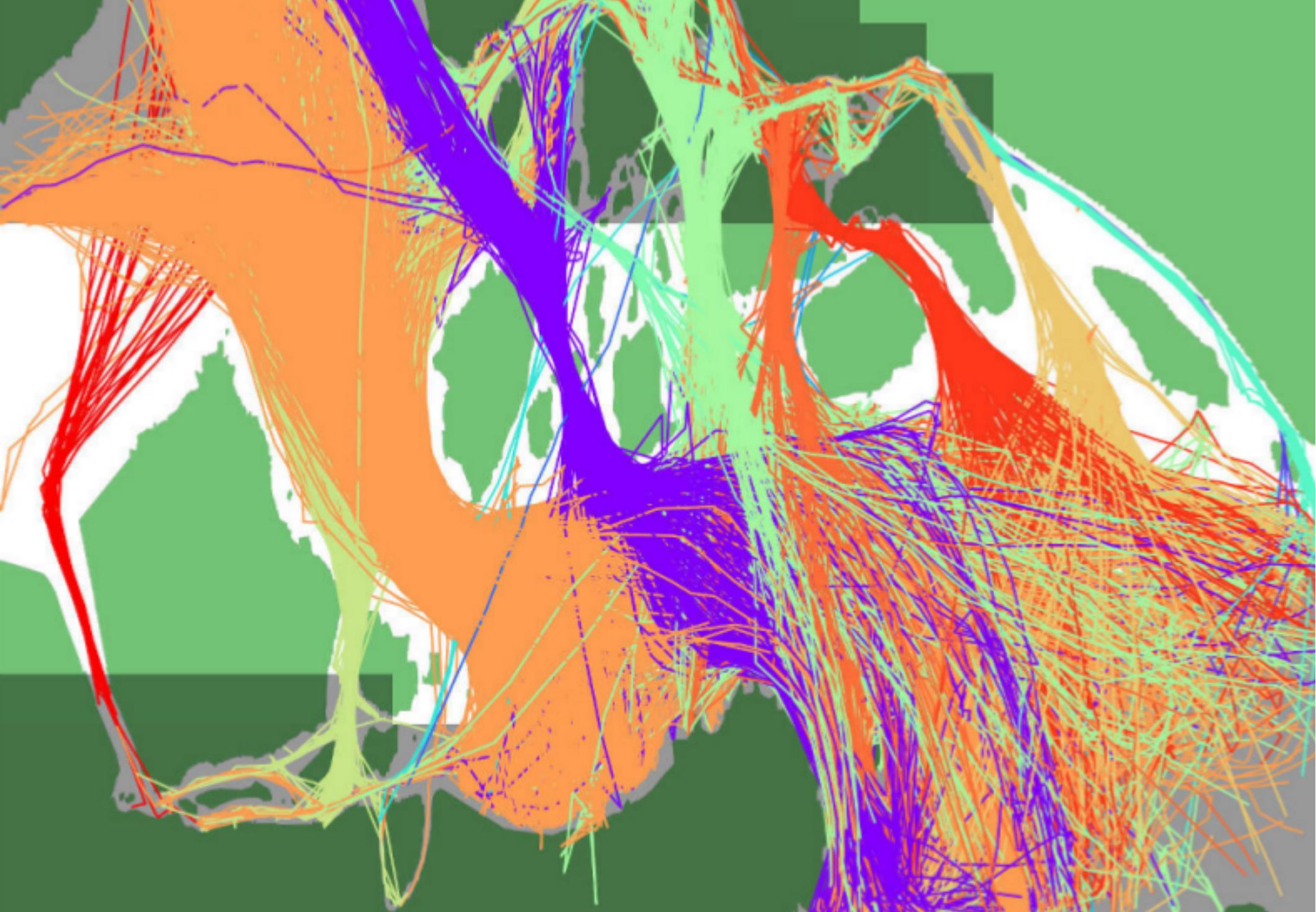


Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016



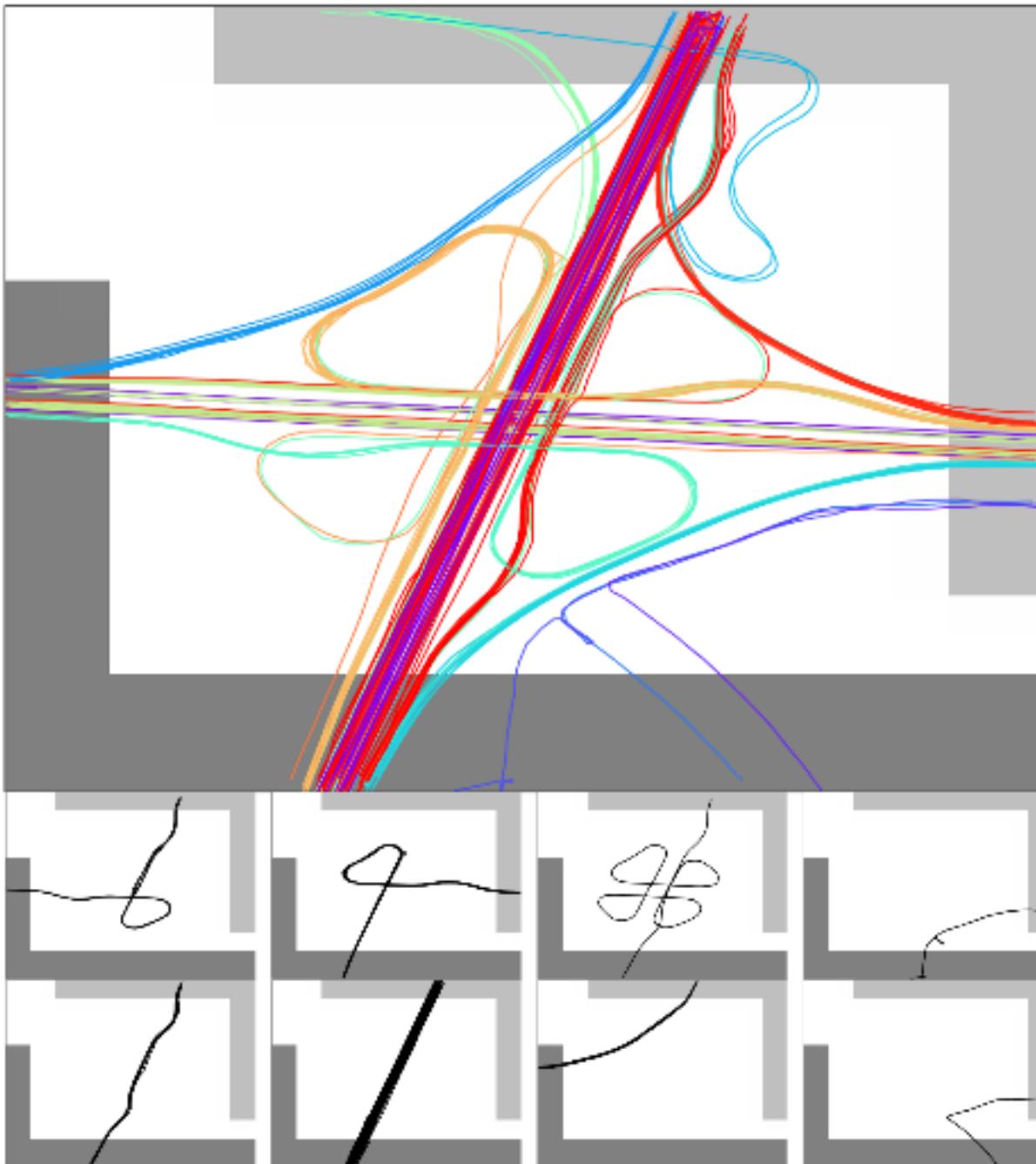
Topological Trajectory Clustering with Relative Persistent Homology

F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

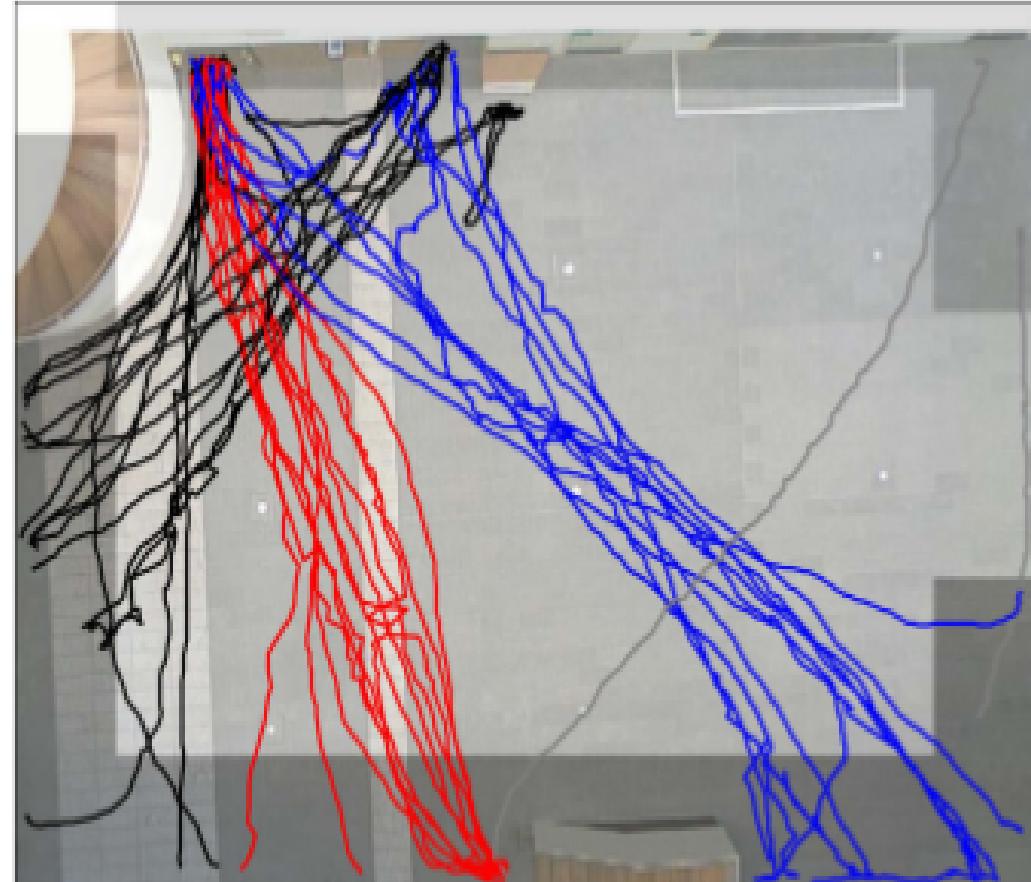
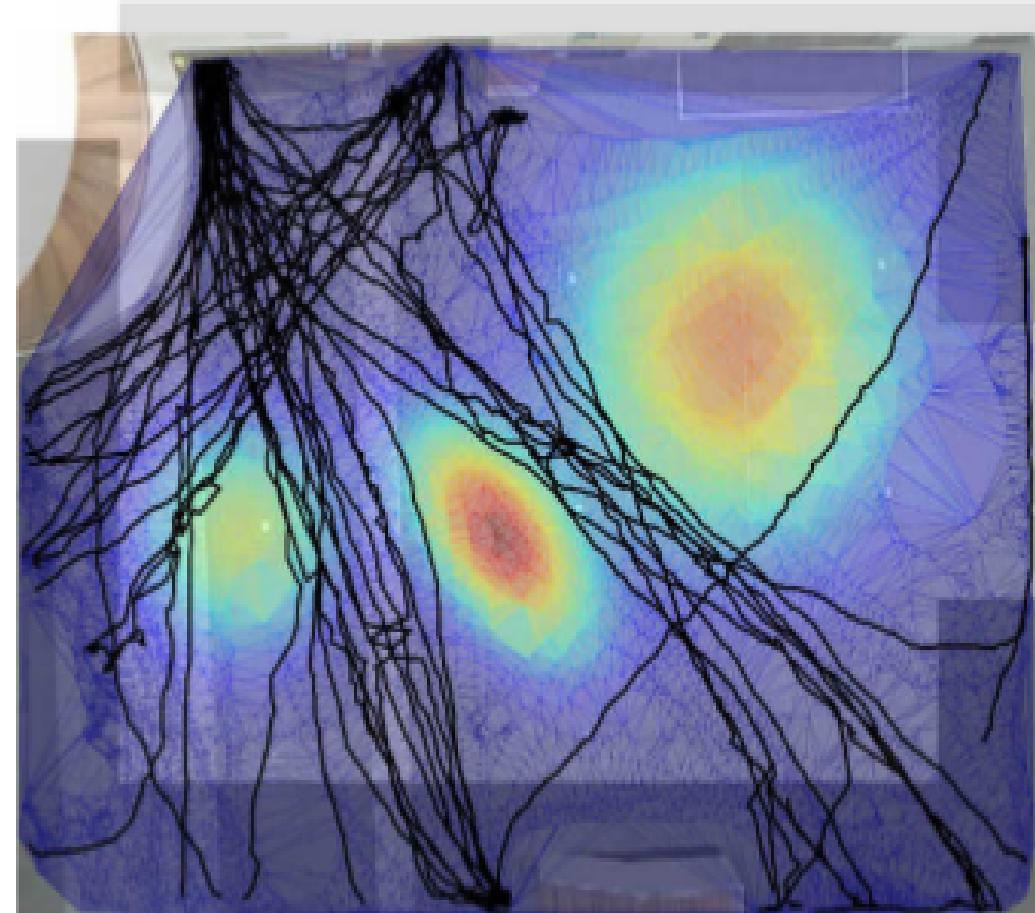


Topological Trajectory Clustering with Relative Persistent Homology

F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

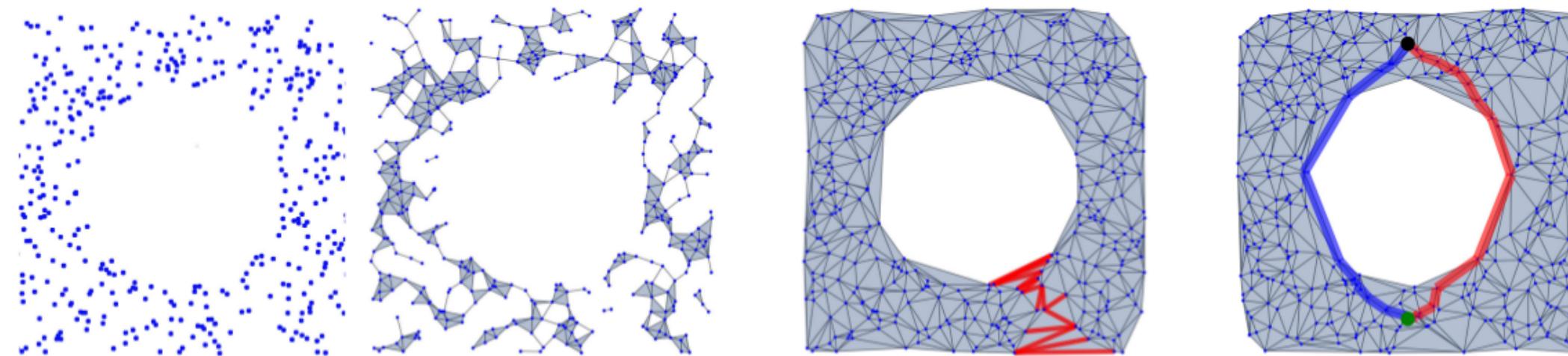
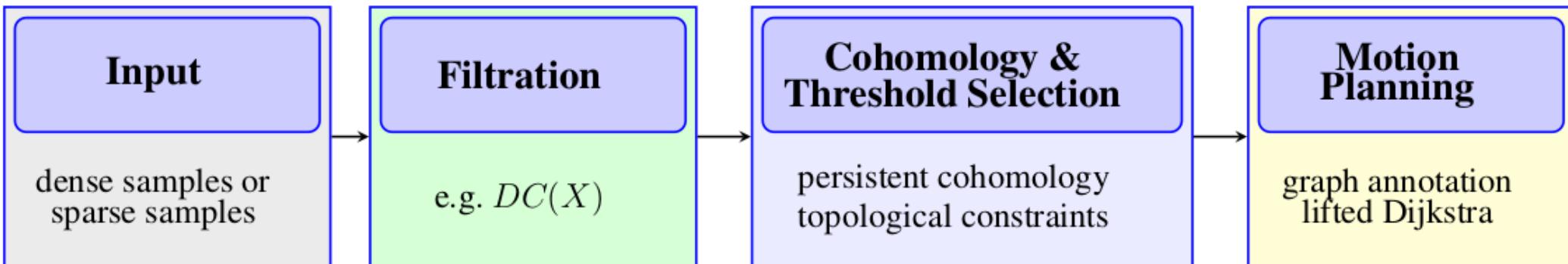


Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

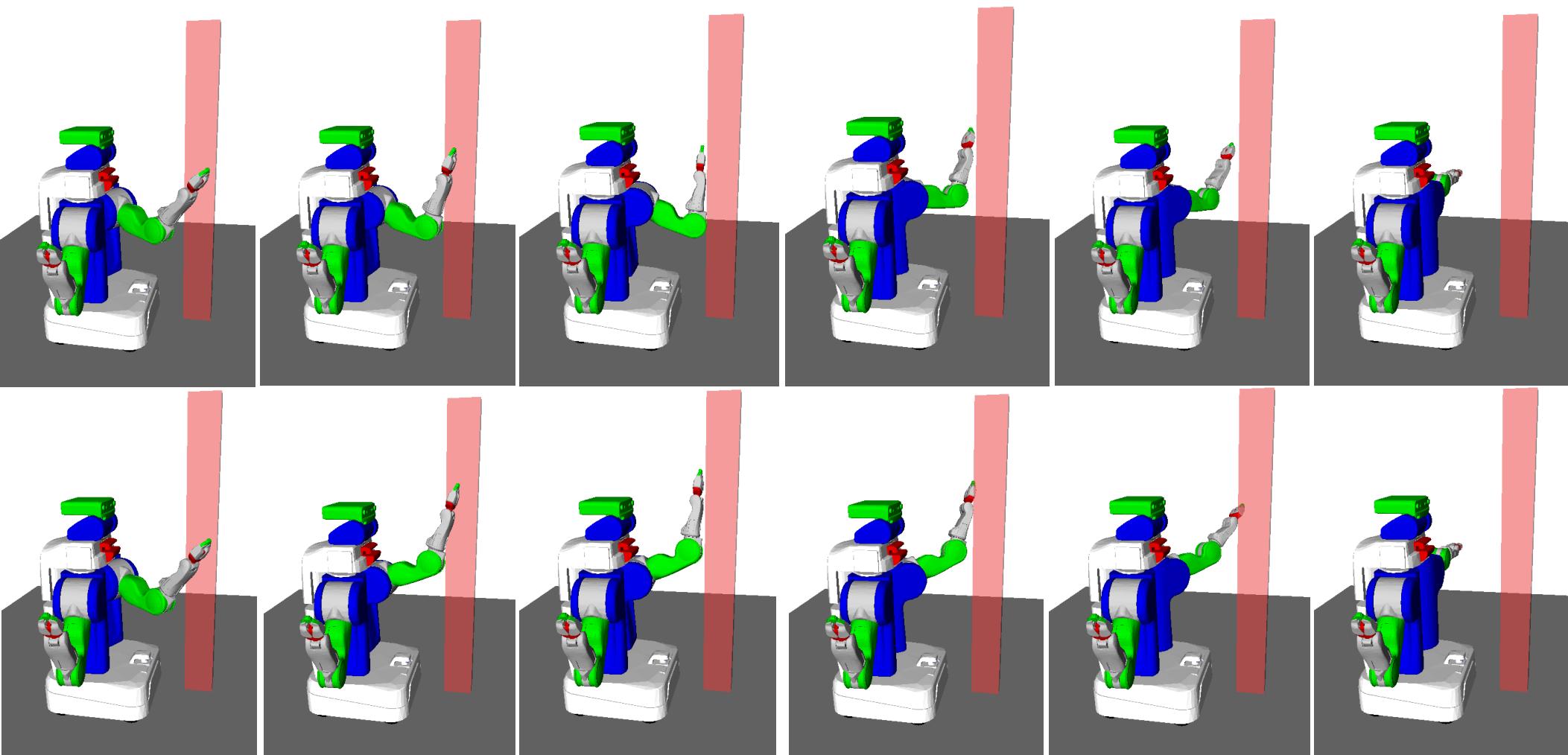


Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

Motion Planning with Cohomology

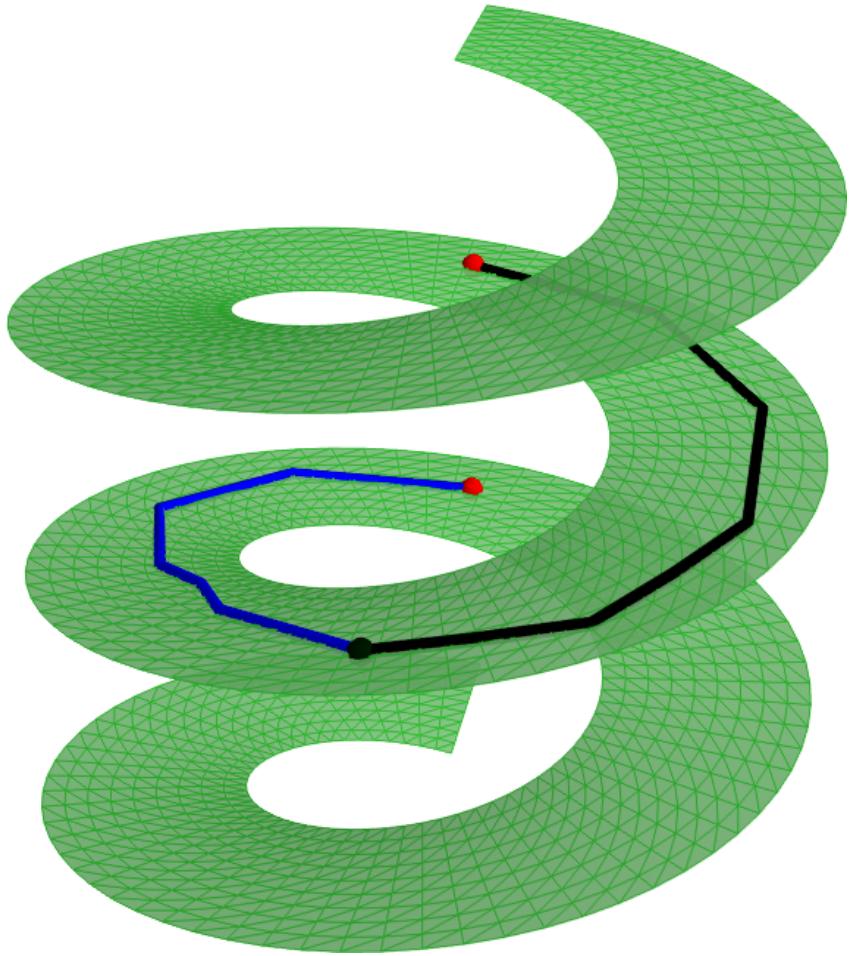
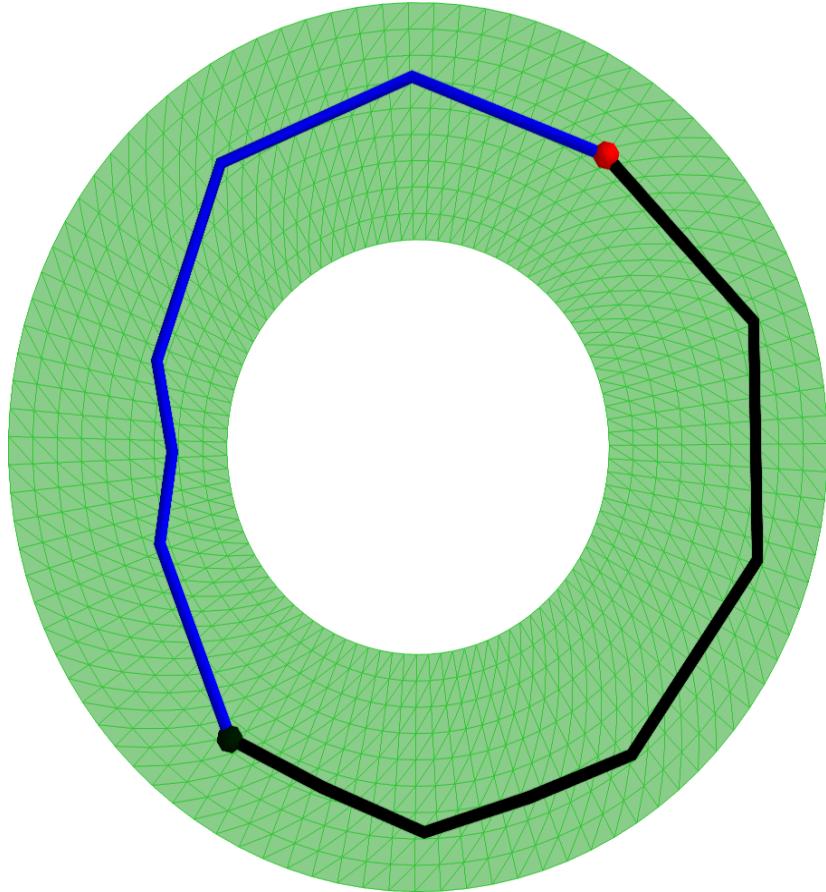


Robotics Applications - Planning

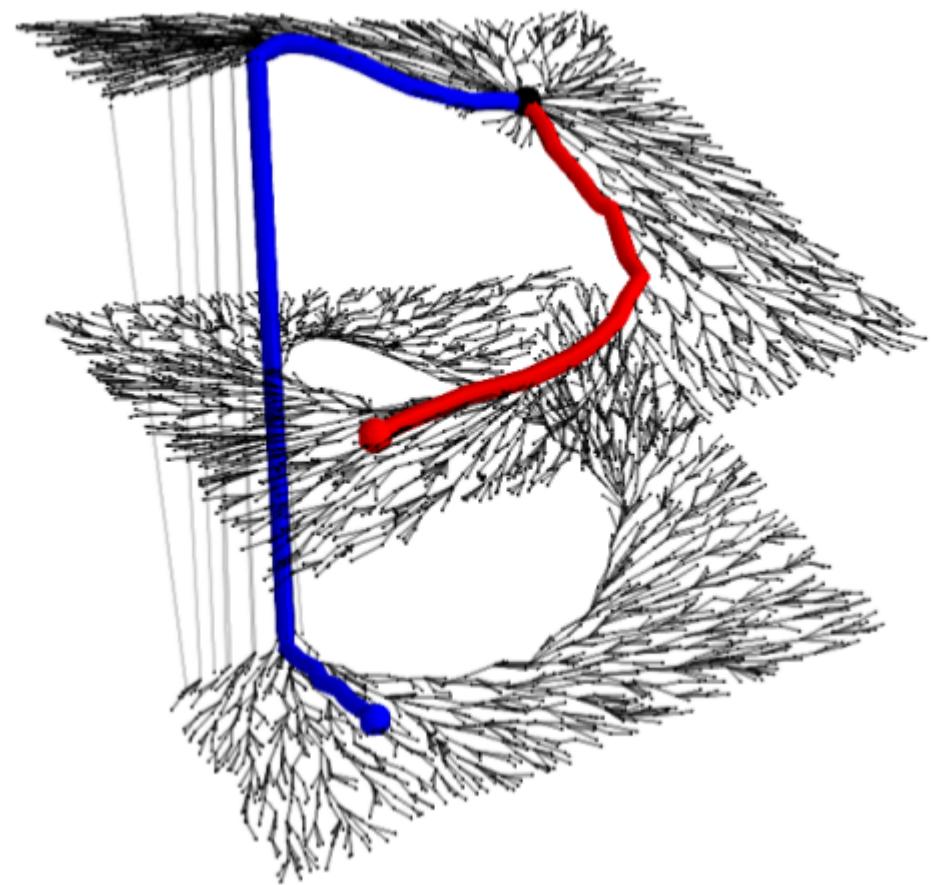
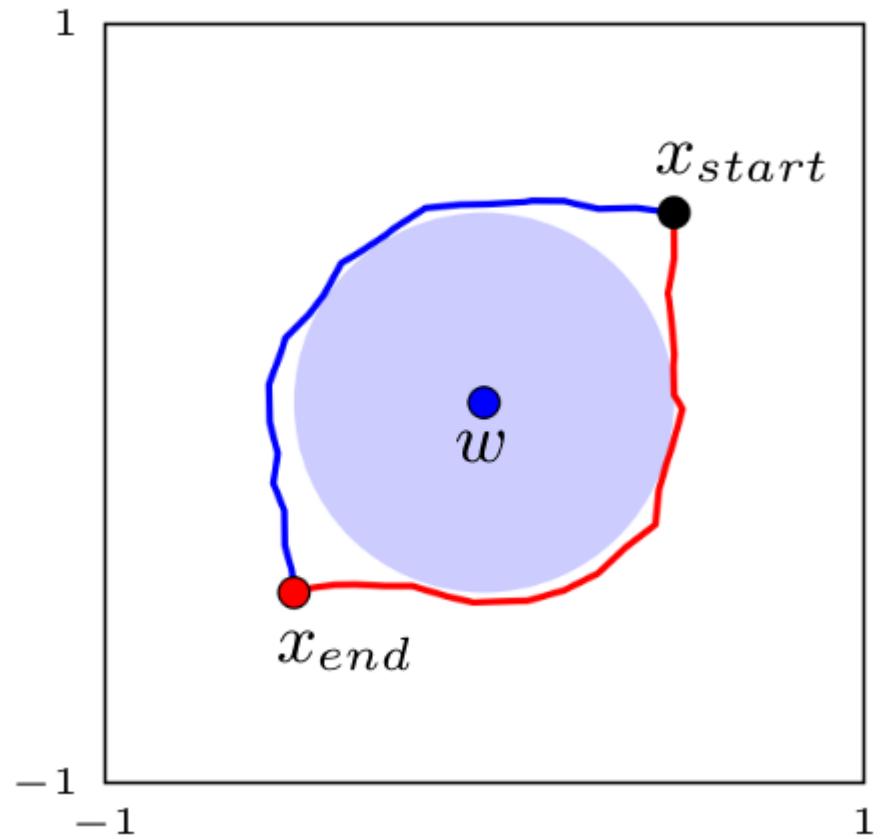


Data Driven Topological Motion Planning with Persistent Cohomology
F. T. Pokorny, D Kragic, RSS 2015

Building on Prior Works on Topological Motion Planning

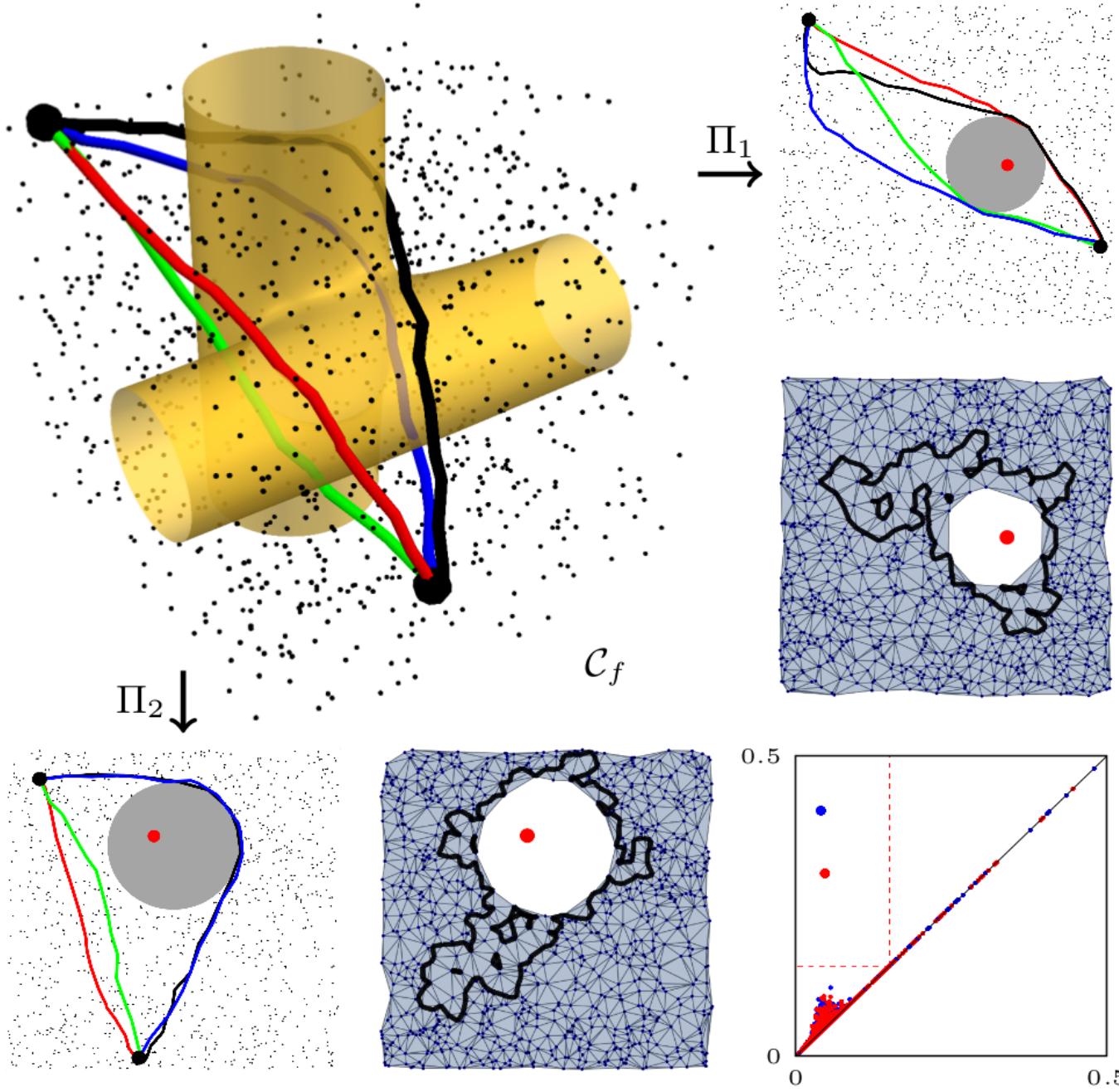


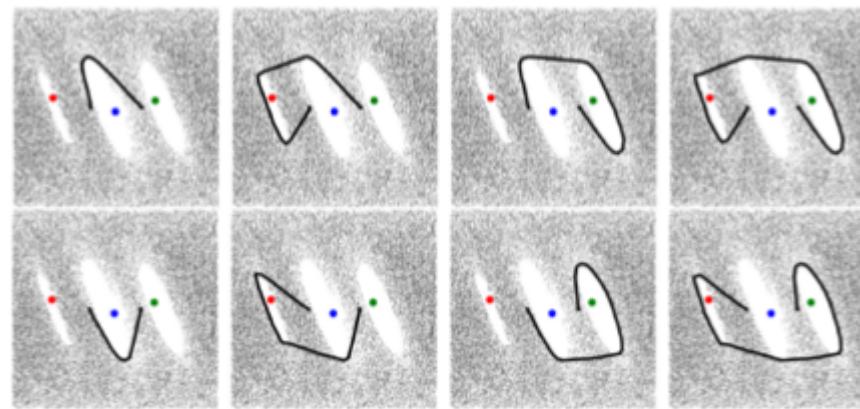
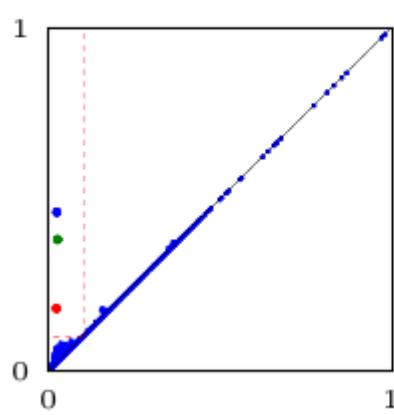
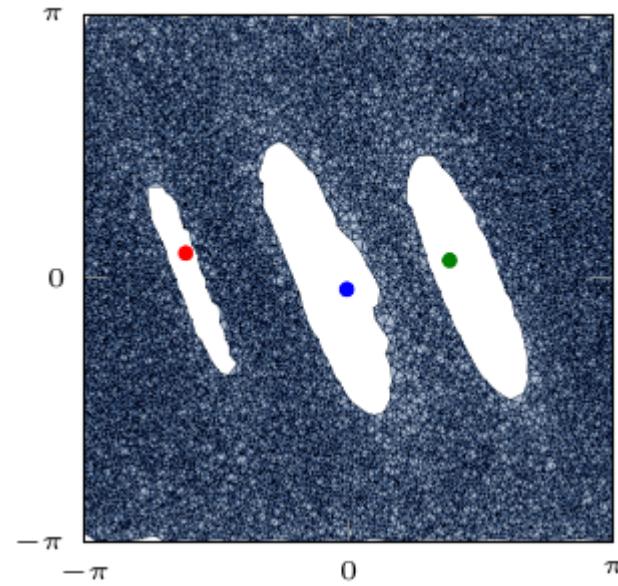
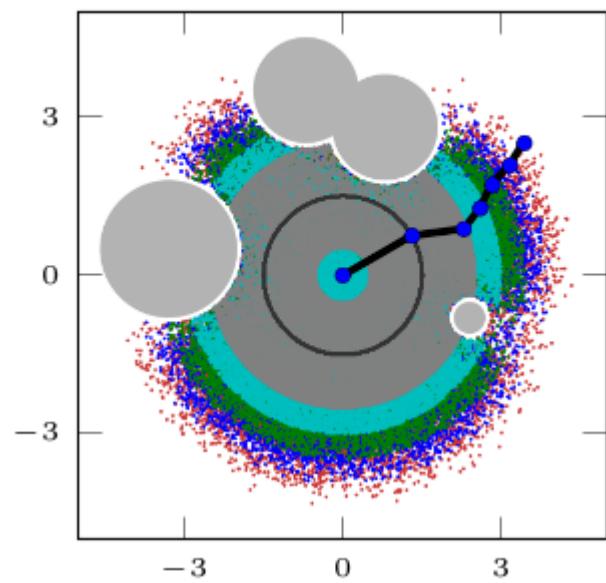
[K. D. Jenkins et al. 1991], [D. Grigoriev et al. 1998], [S. Bhattacharya et al. 2010+], [S. Kim et al. 2012+], [F. T. Pok



High-Dimensional Winding-Augmented Motion Planning with 2D Topological Task Projections and Persistent Homology, ICRA '16, F. T. Pokorny, D. Kragic, L. E. Kavraki, K. Goldberg

Task Projections and Persistence

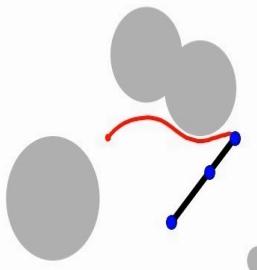




High-Dimensional Winding-Augmented Motion Planning with 2D Topological Task Projections and Persistent Homology, ICRA '16, F. T. Pokorny, D. Kragic, L. E. Kavraki, K. Goldberg

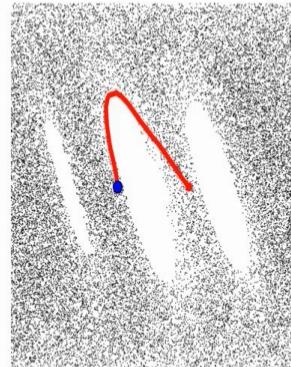
High-Dimensional Winding-Augmented Motion Planning with 2D Topological Task Projections and Persistent Homology

F. T. Pokorny, D. Kragic, L. E. Kavraki, K. Goldberg, ICRA 2016

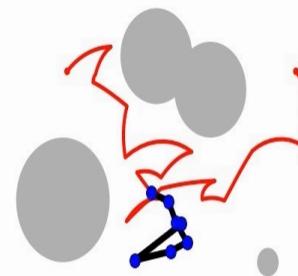


planar linkage

2 DOF linkage, WA-RRT*

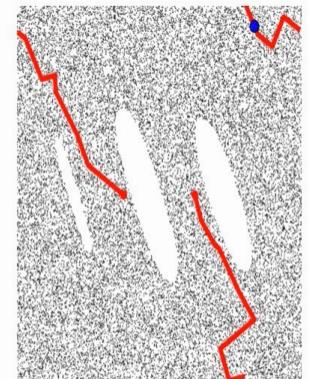


projection to first 2 joint angles



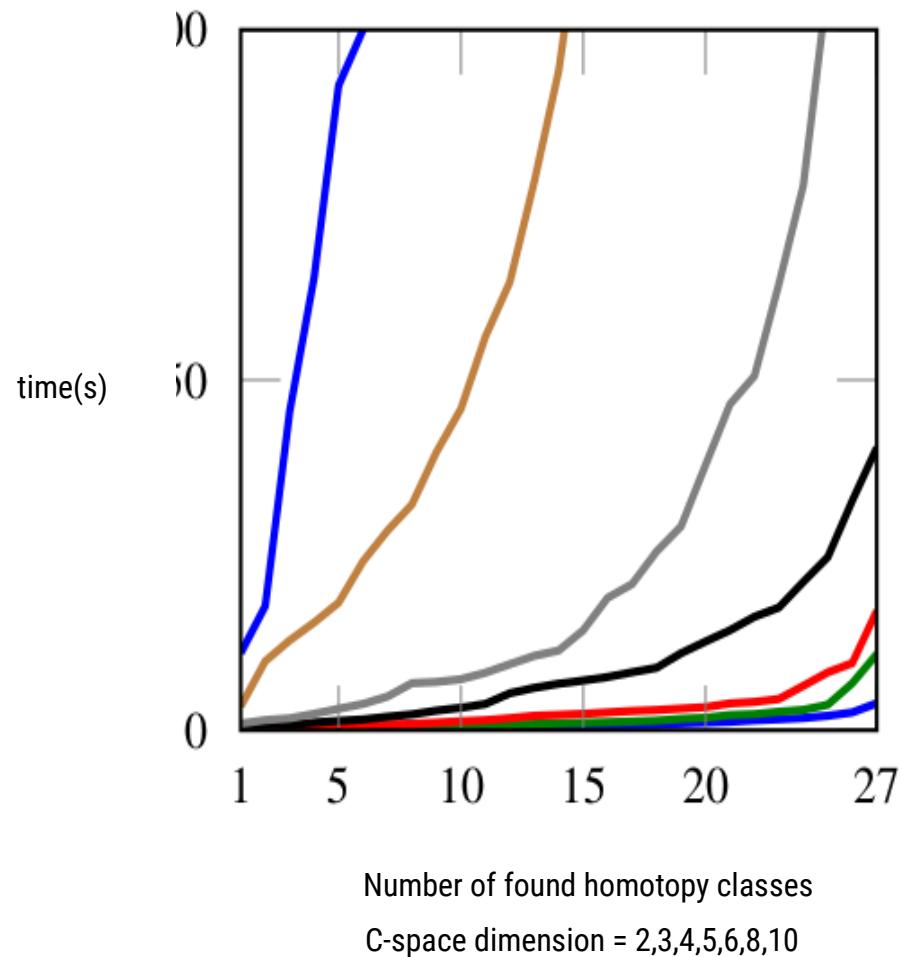
planar linkage

6 DOF linkage, WA-RRT, without self-collisions (torus C-space)



projection to first 2 joint angles

Computational Complexity



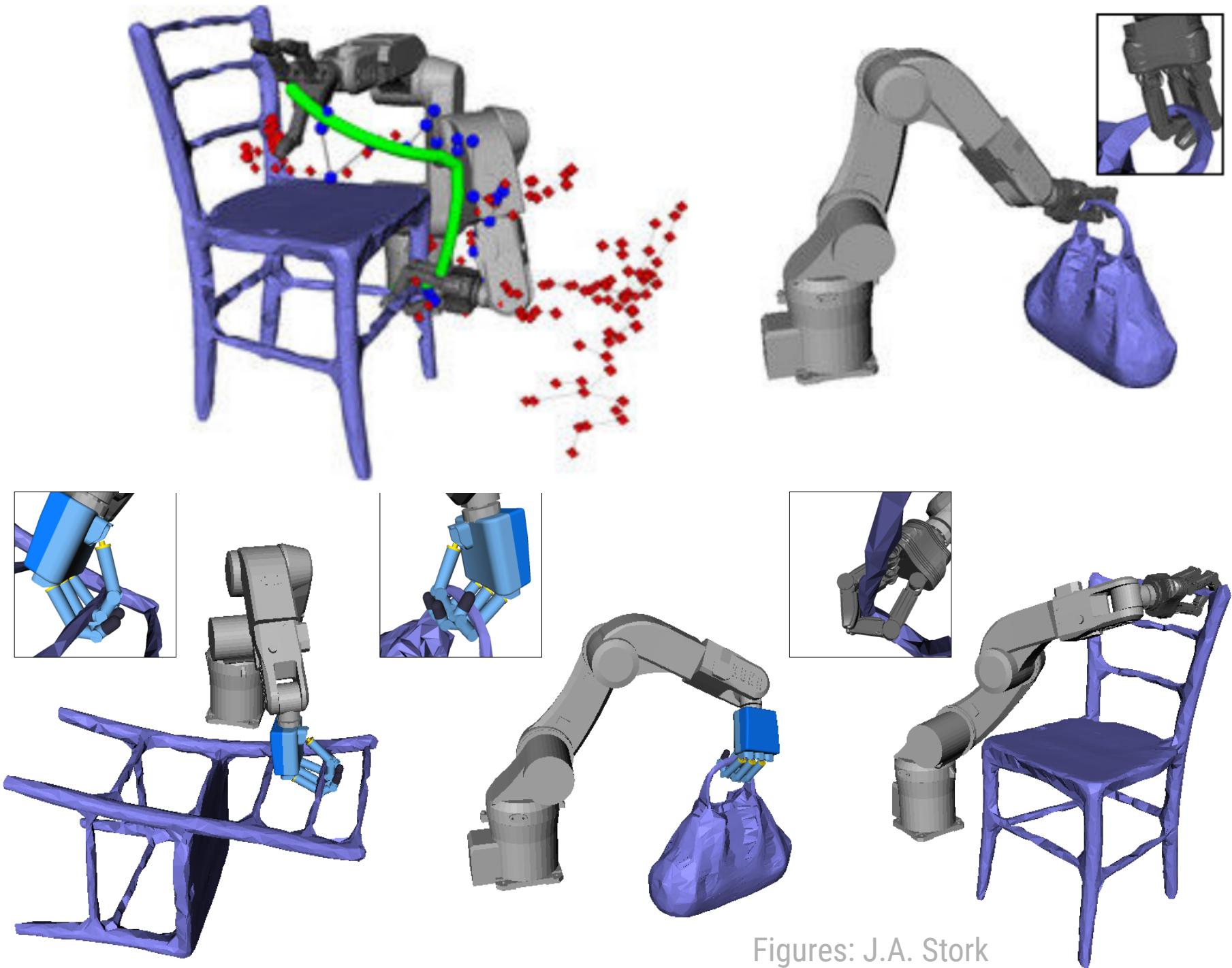
Topology for Manipulation



Grasping objects with holes:



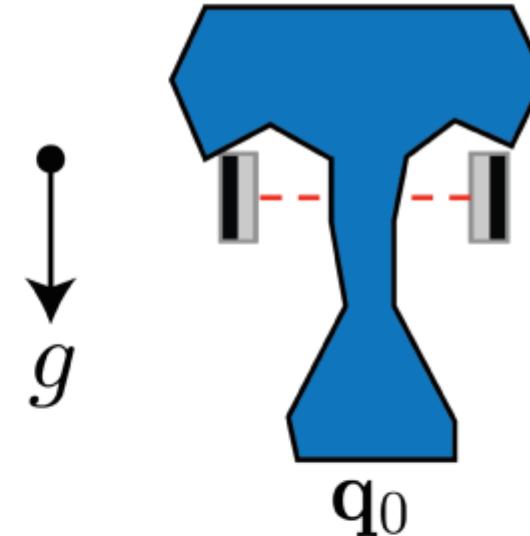
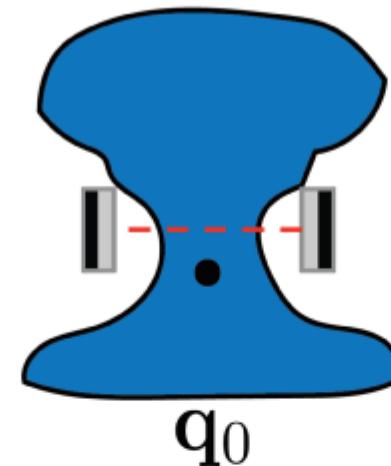
Figures: J.A. Stork



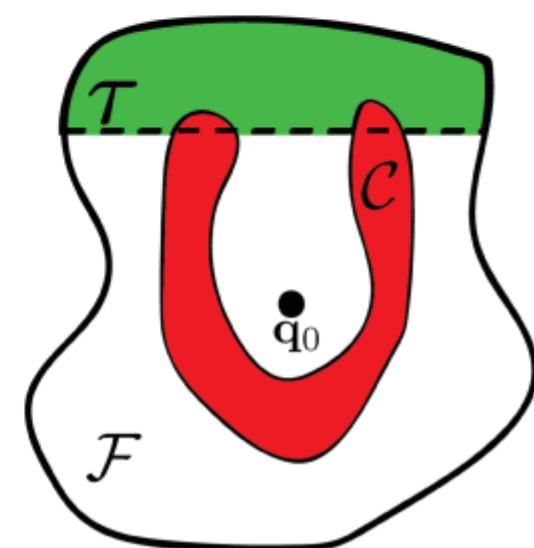
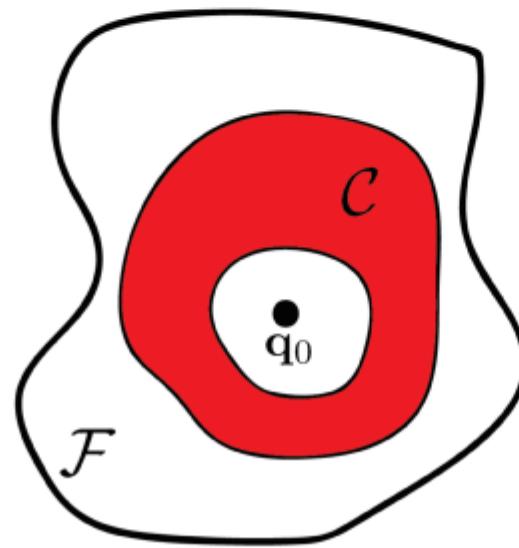
Figures: J.A. Stork

Caging and Gravity

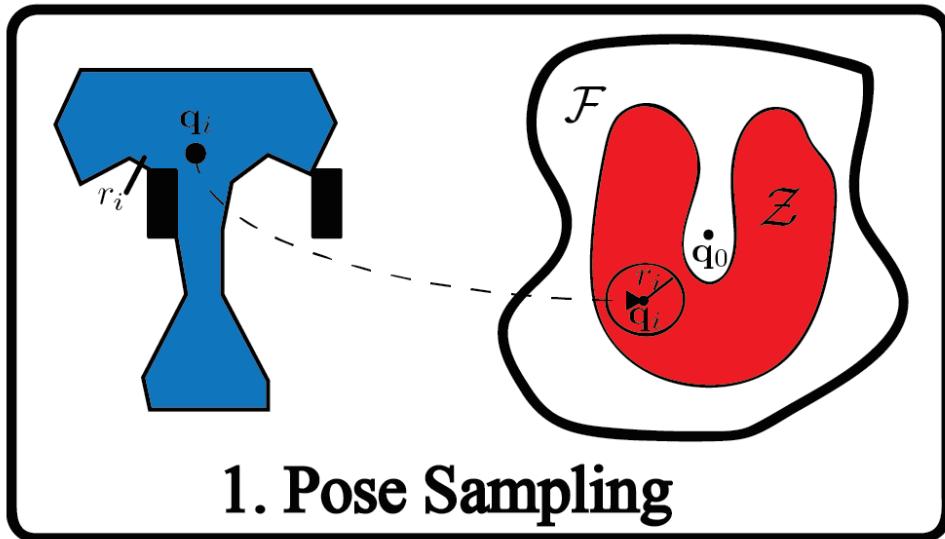
Workspace



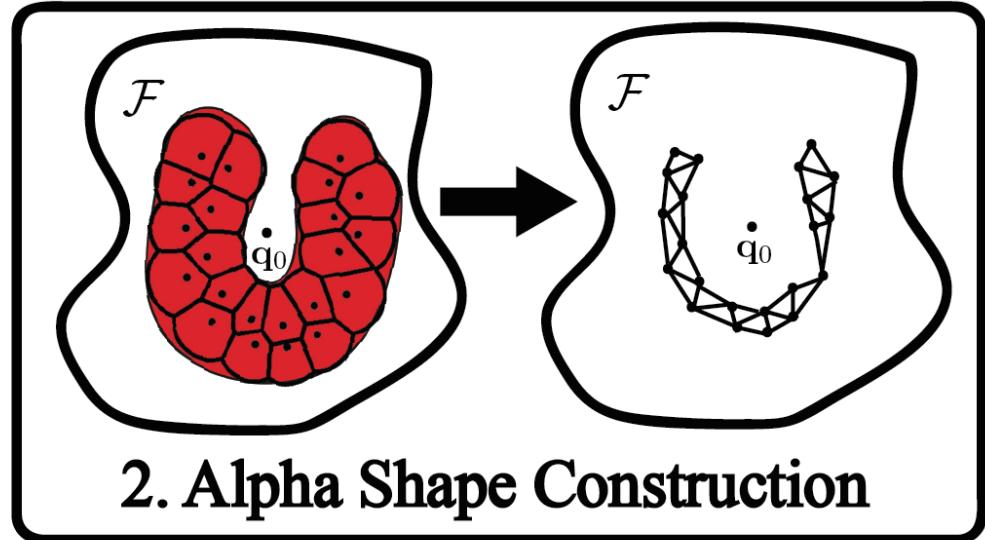
Translational Configuration Space



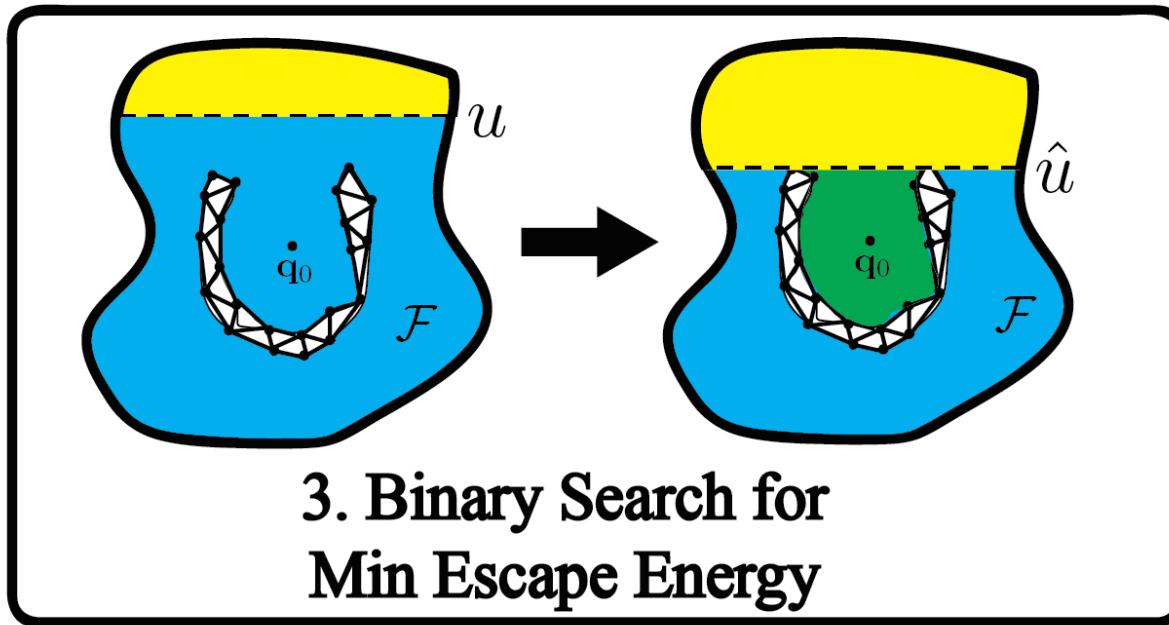
Caging and Gravity



1. Pose Sampling



2. Alpha Shape Construction



3. Binary Search for
Min Escape Energy

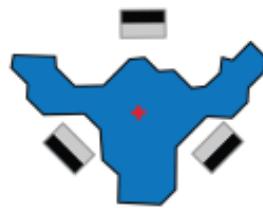
Caging and Gravity

Configuration 1



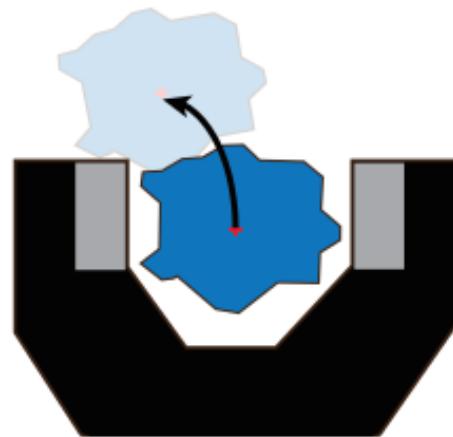
$$\hat{\tau}_n = 6.11$$

Configuration 2



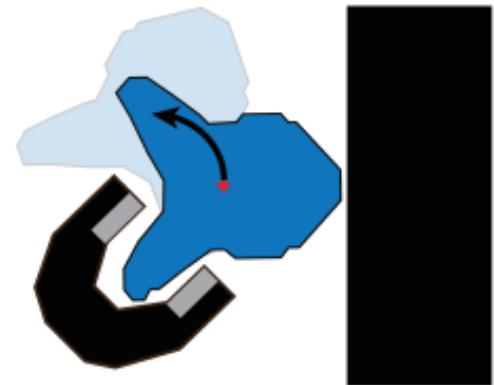
$$\hat{\tau}_n = \infty$$

Configuration 3



$$\hat{\tau}_n = 11.26$$

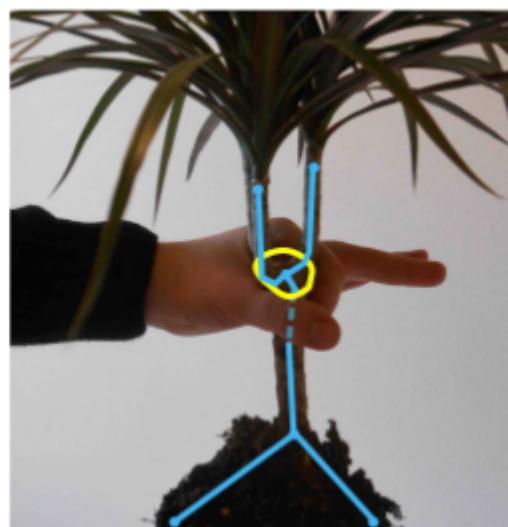
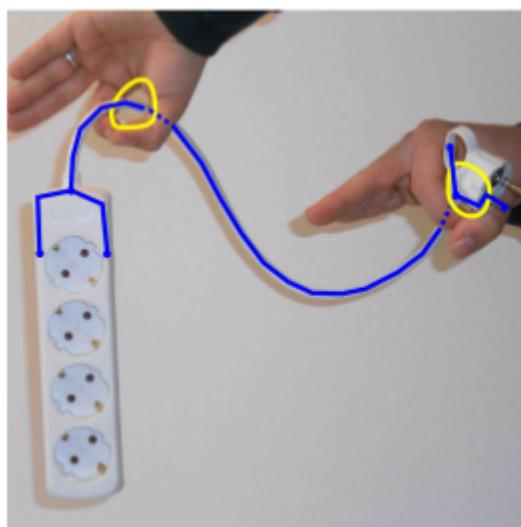
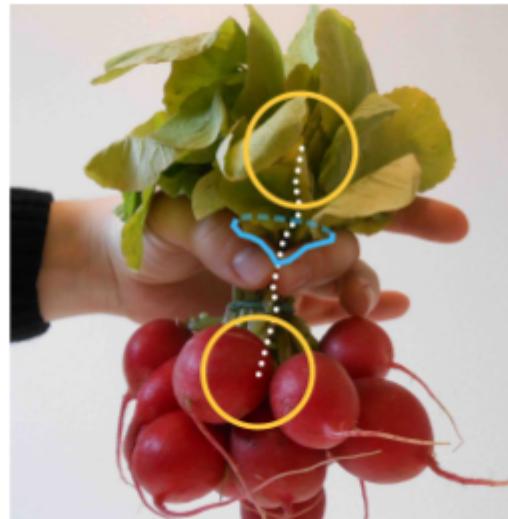
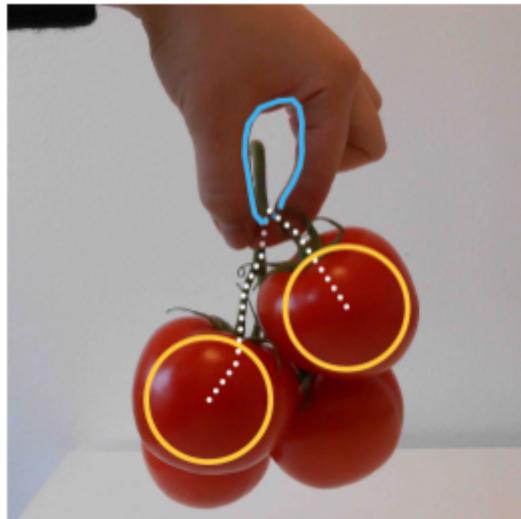
Configuration 4



$$\hat{\tau}_n = 8.79$$

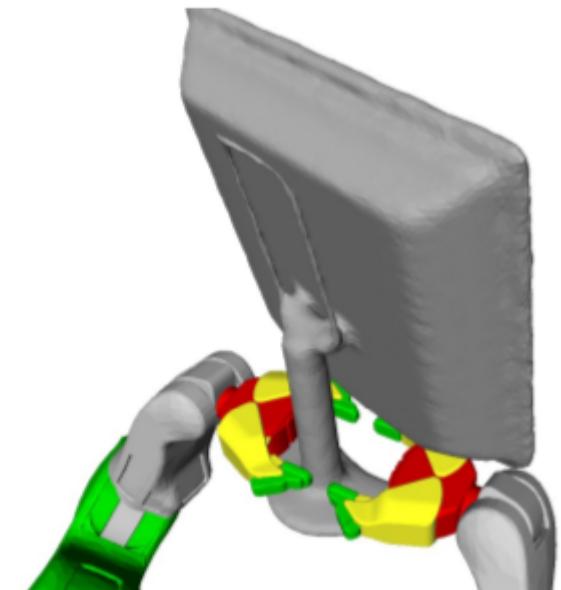
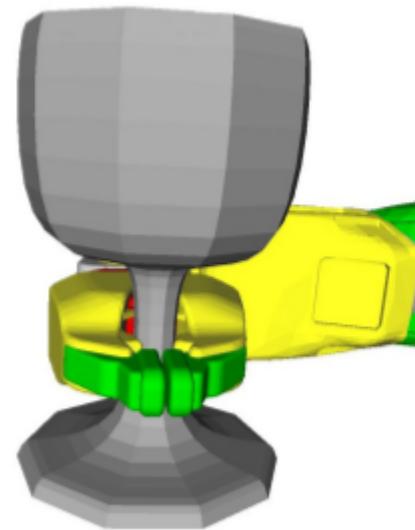
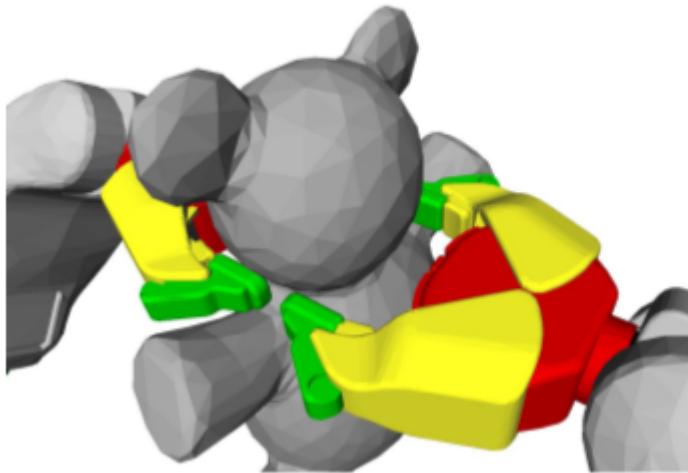


Algebraic Topology & 3D Caging: *deformation classes, isotopies, homotopies*



A. Varava, D. Kragic, F. T. Pokorny, 2016 (conditionally accepted, T-RO)

Algebraic Topology & 3D Caging: *deformation classes, isotopies, homotopies*



A. Varava, D. Kragic, F. T. Pokorny, 2016 (conditionally accepted, T-RO)