Search-based Motion Planning with Topology-based Heuristic Functions

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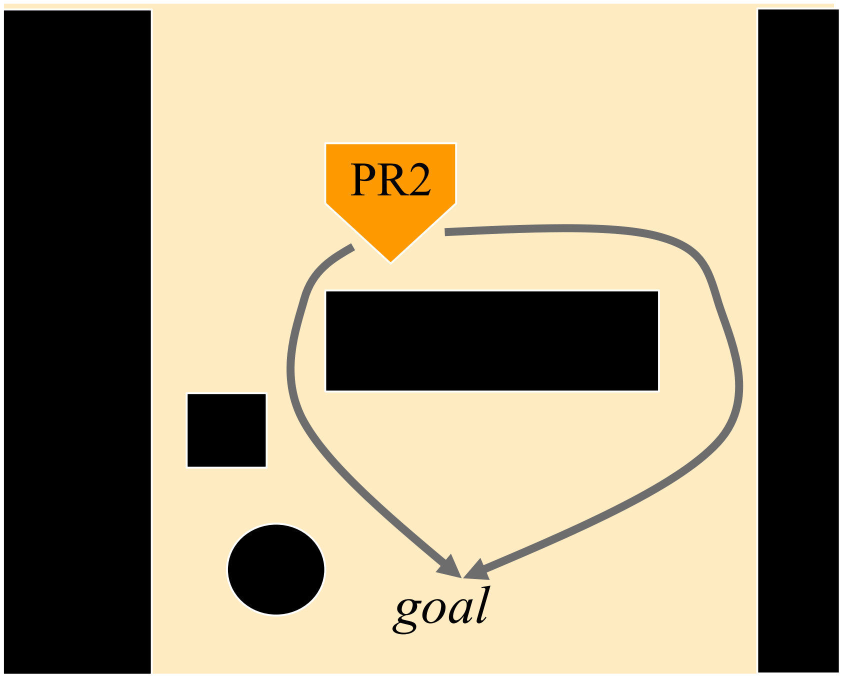
Outline

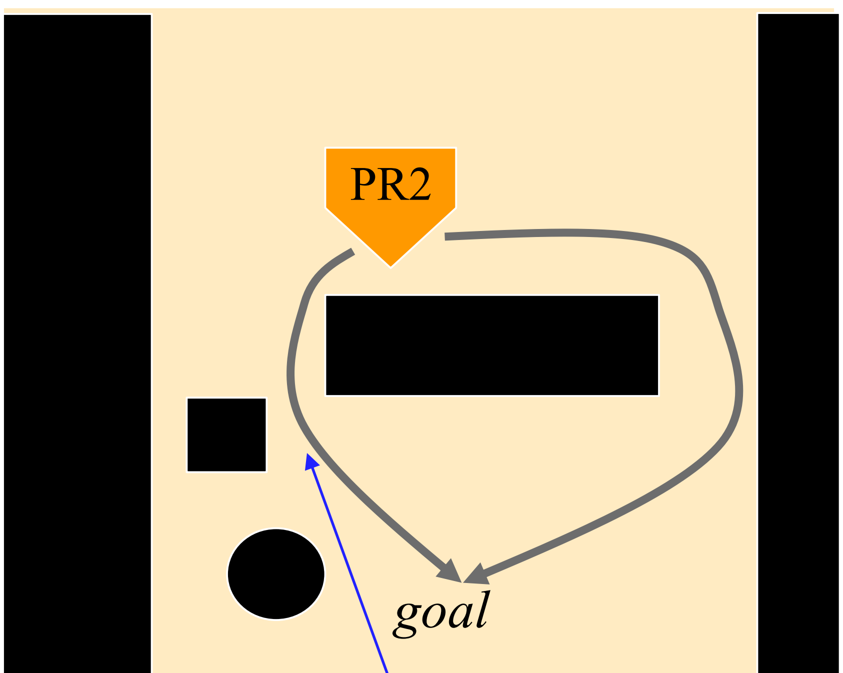
- Topology-based Heuristics
- Multi-Heuristic A* to support the use of Topology-based Heuristics

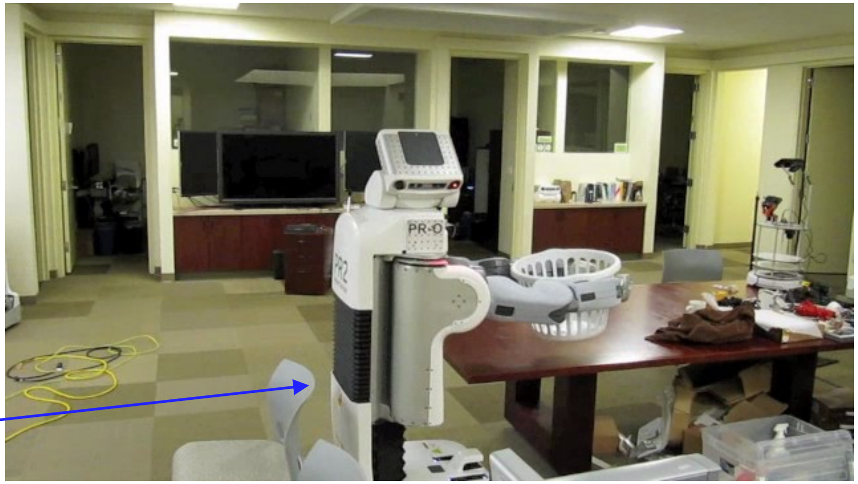
- 3D (x,y,Θ) path planning with **full body collision checking**



Multiple Hypotheses in Planning

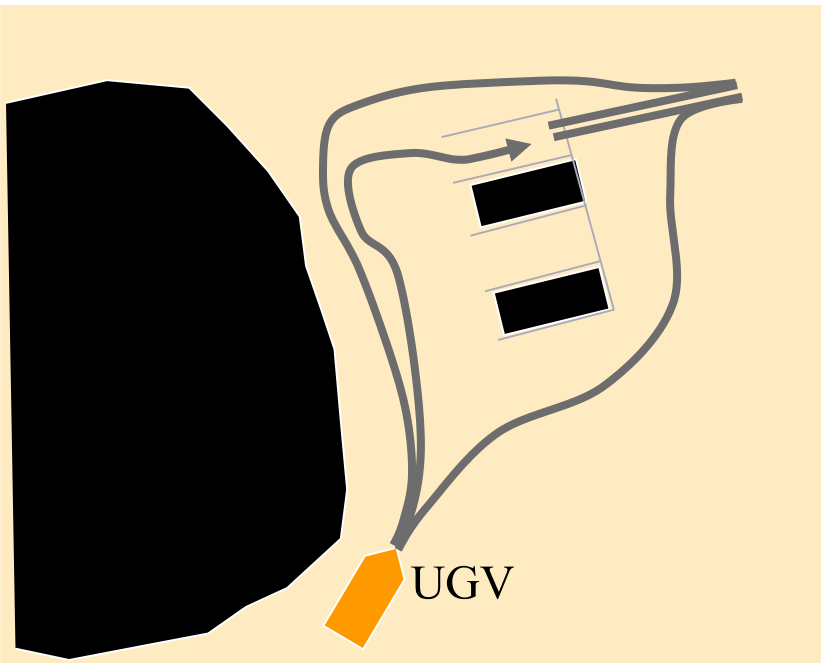
- 3D (x,y,Θ) path planning with **full body collision checking**

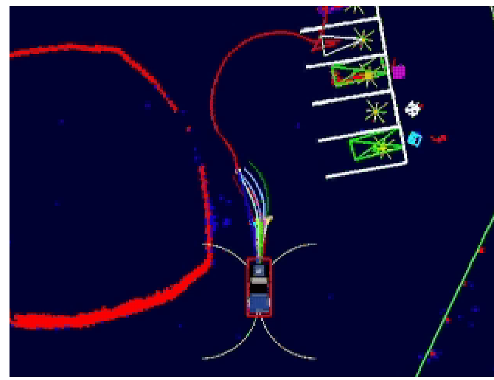




can PR2 squeeze through here?

- 4D (x,y,Θ,v) path planning for a ground vehicle

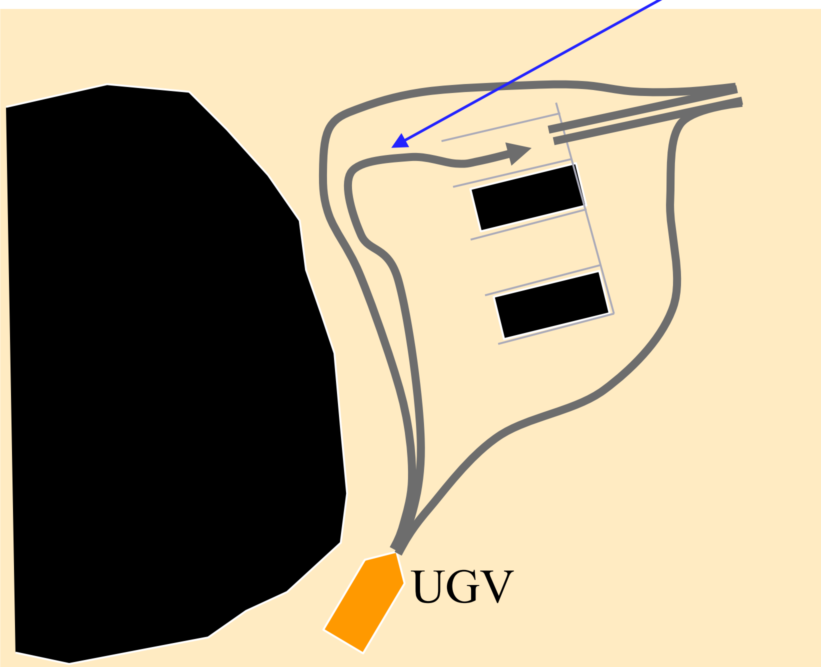


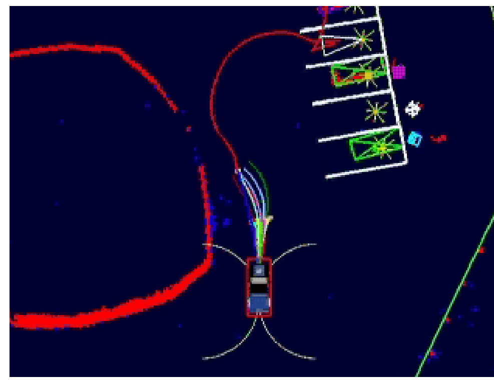


Multiple Hypotheses in Planning

- 4D (x,y,Θ,v) path planning for a ground vehicle

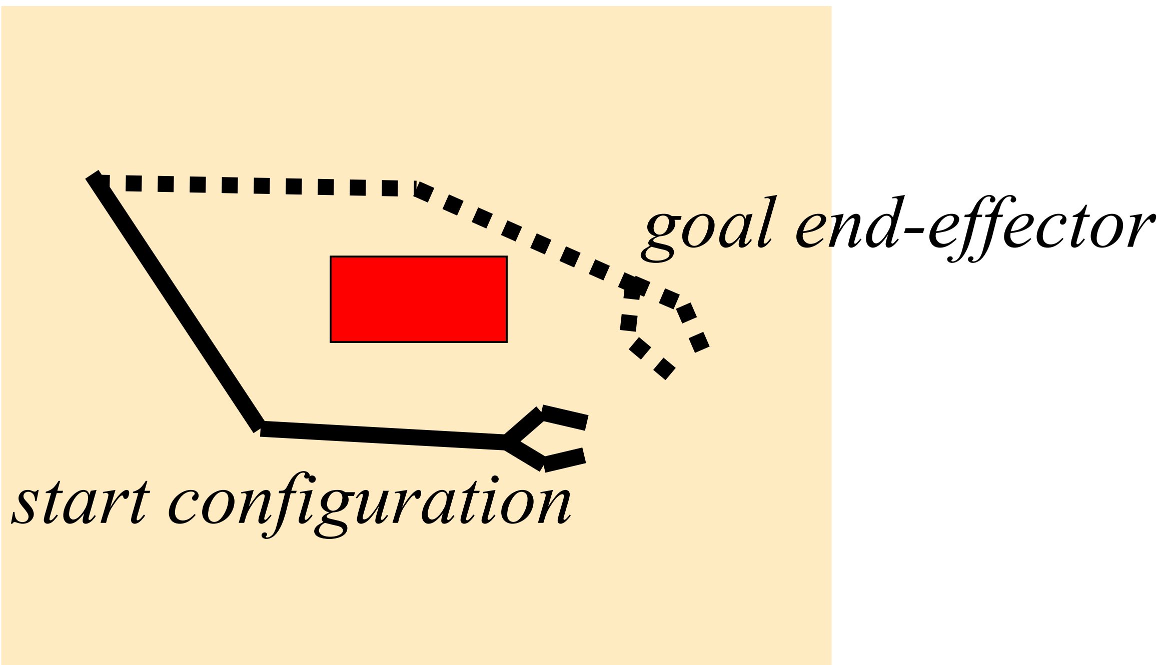
is there enough space for a turn within UGV's min. turning radius?





Multiple Hypotheses in Planning

- 7D ($q_1, q_2, \ldots, q_7$) planning for a robotic arm



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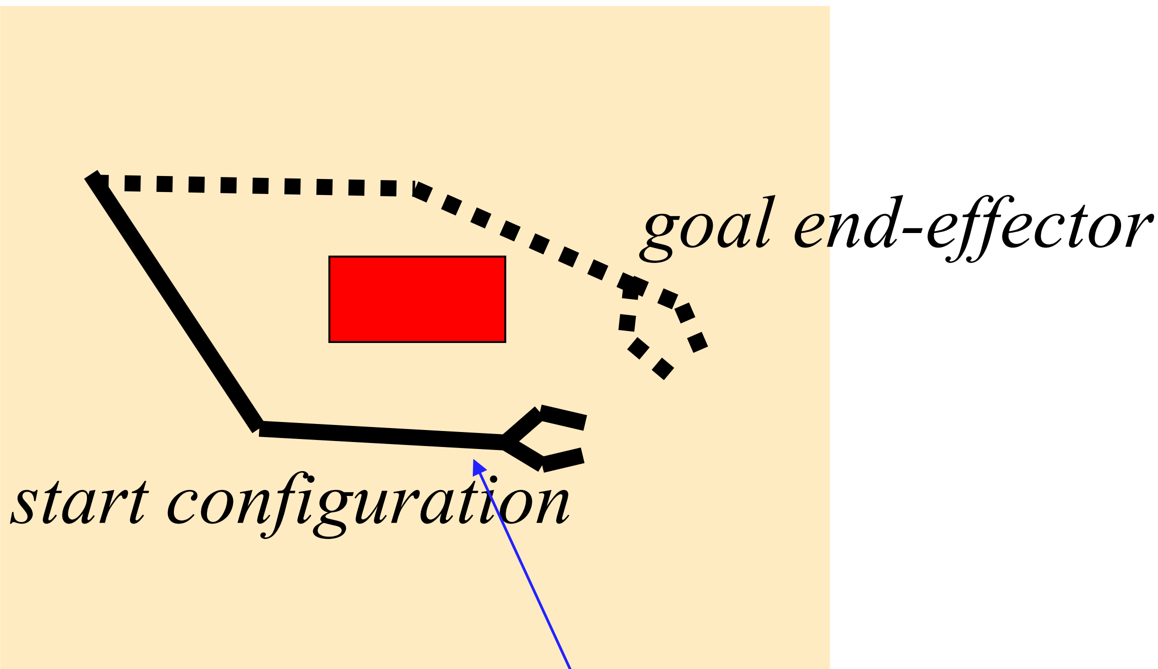
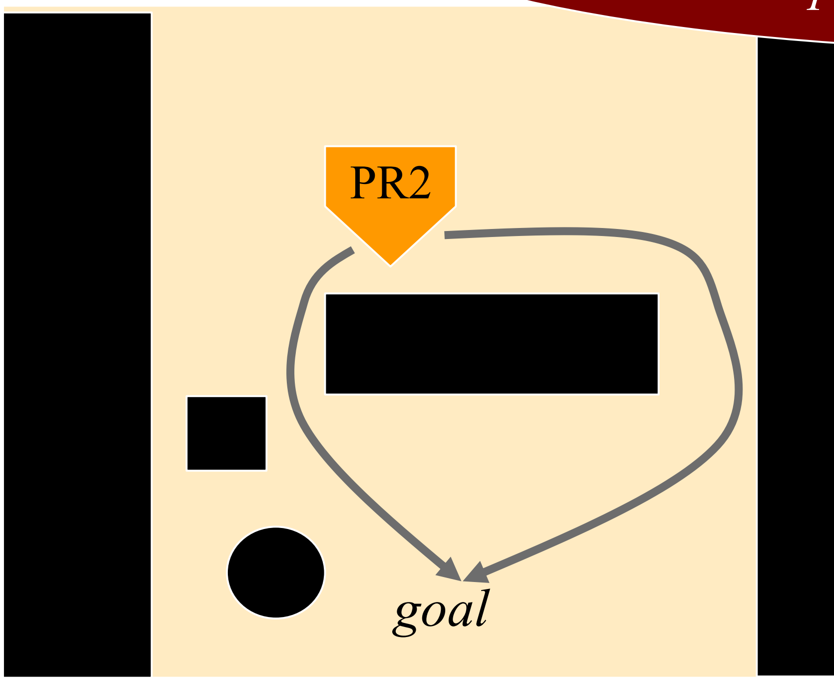


is there enough reach in the arm to go from this side?

Multiple Hypotheses = Multiple Low-D solutions

- 3D (x,y,Θ) path planning with **full body collision** checking

each hypothesis is a solution to
a low-dimensional projection of the problem:
a shortest path within its own topological class



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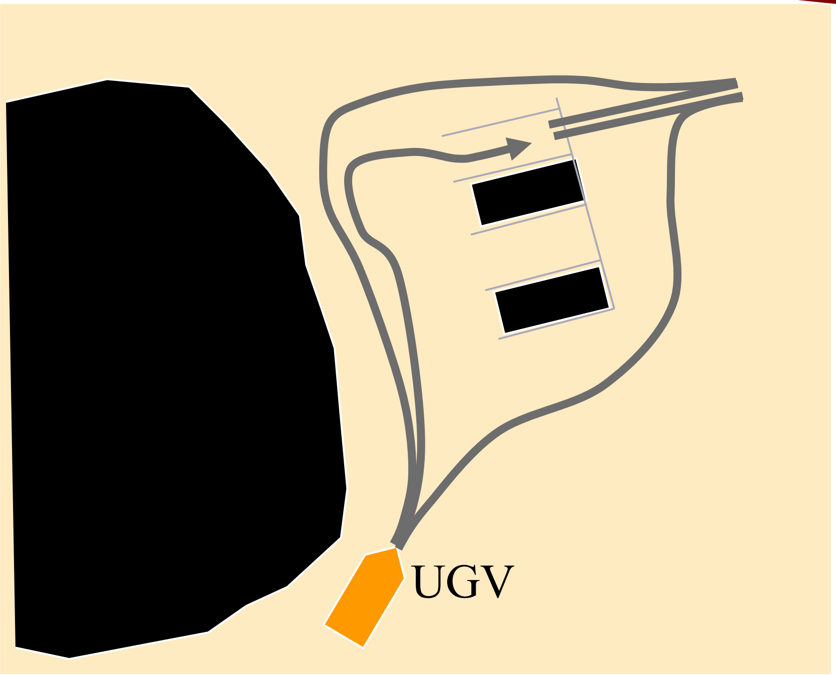

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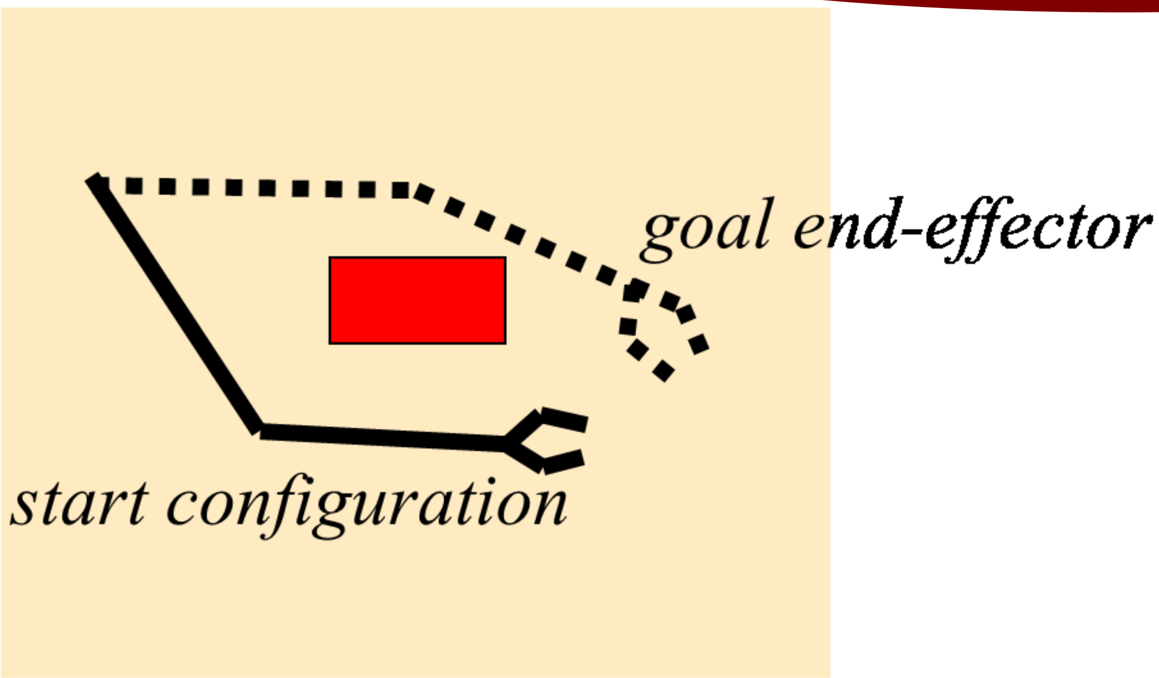
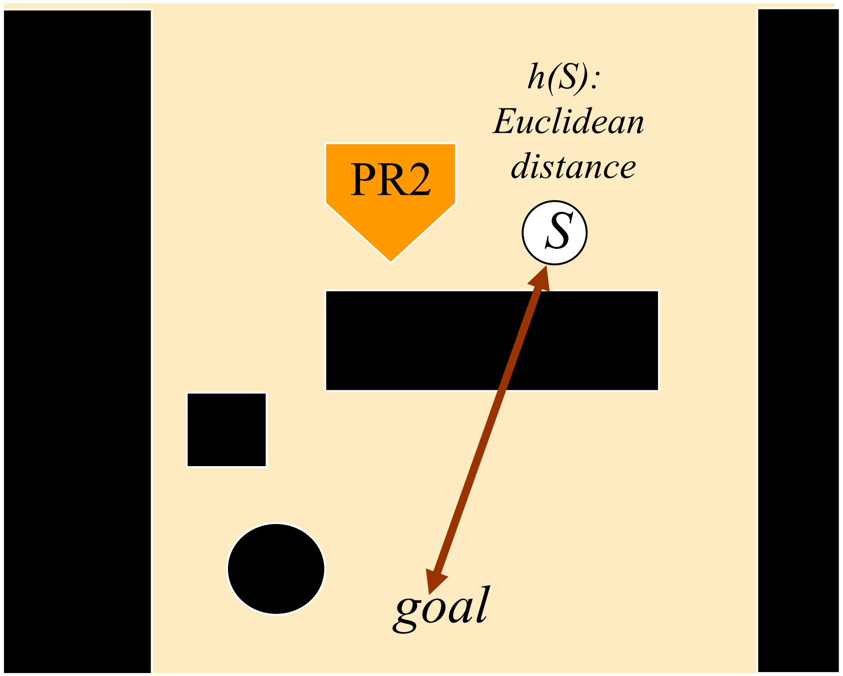


- Heuristic values in A*-like searches = estimates of the cost-to-goal



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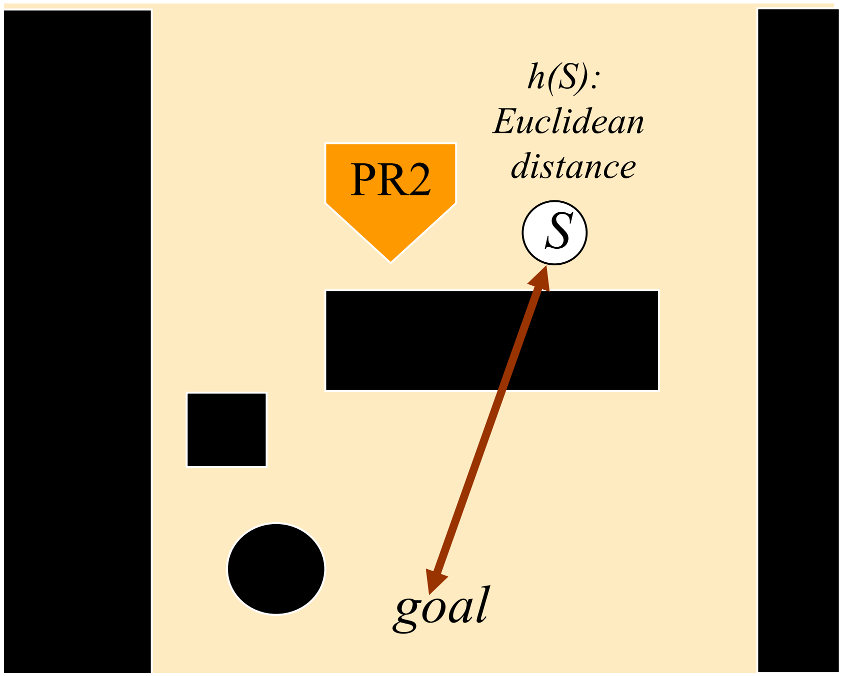

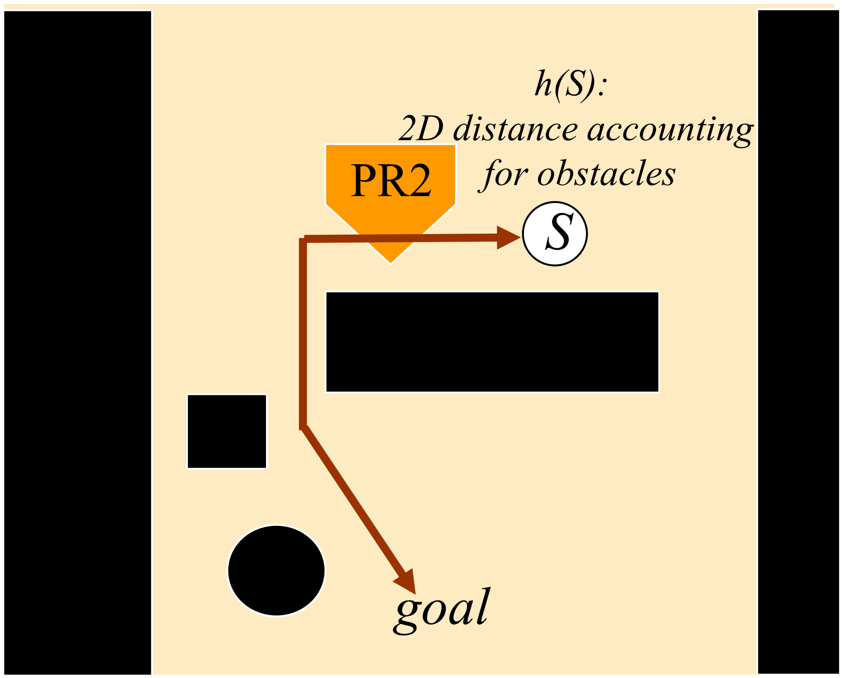

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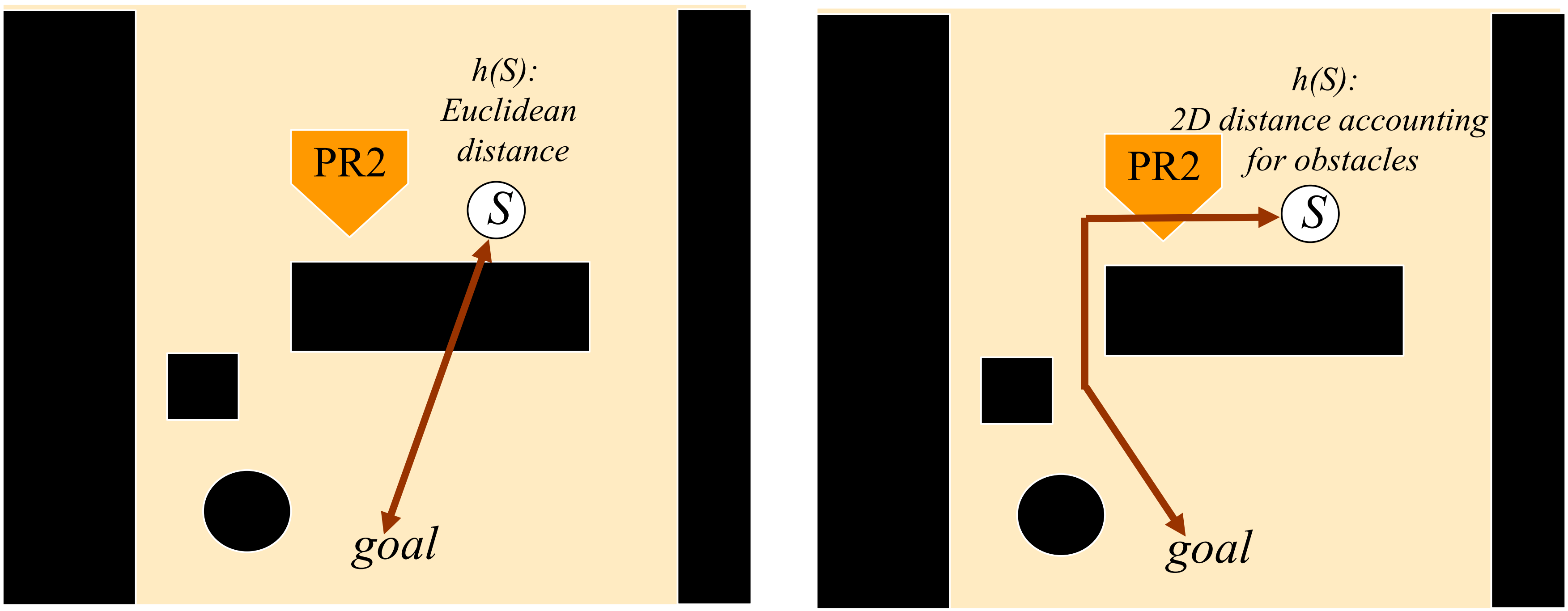

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such heuristic guides the search along the "best" hypothesis

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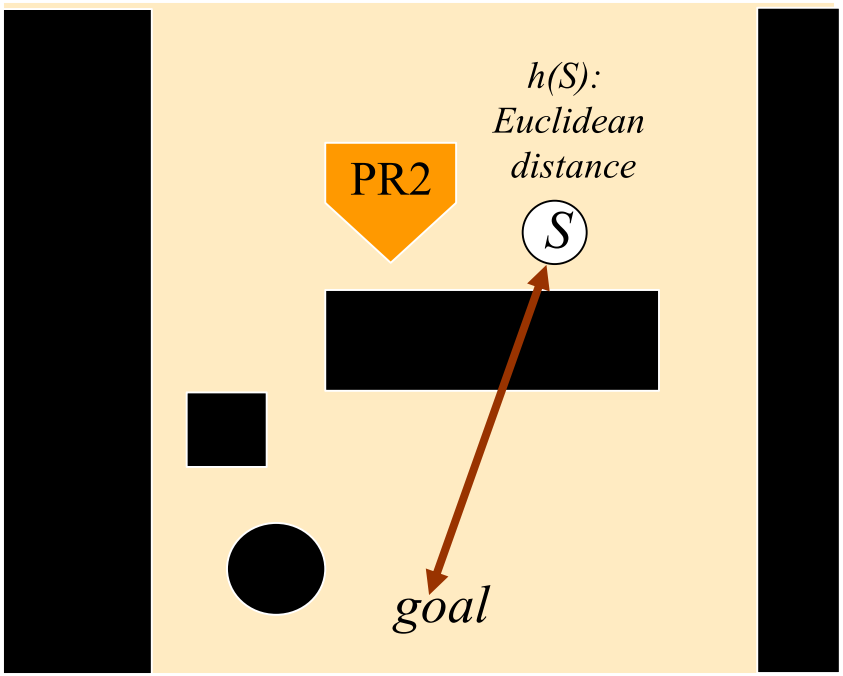

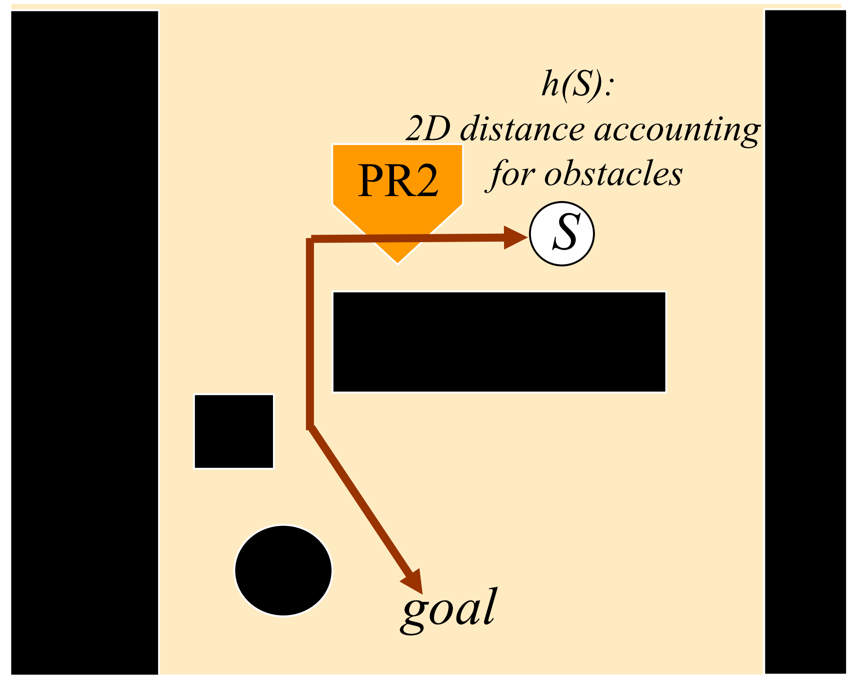

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But what if the "best" hypothesis is wrong?

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Multiple Hypotheses = Multiple Heuristics

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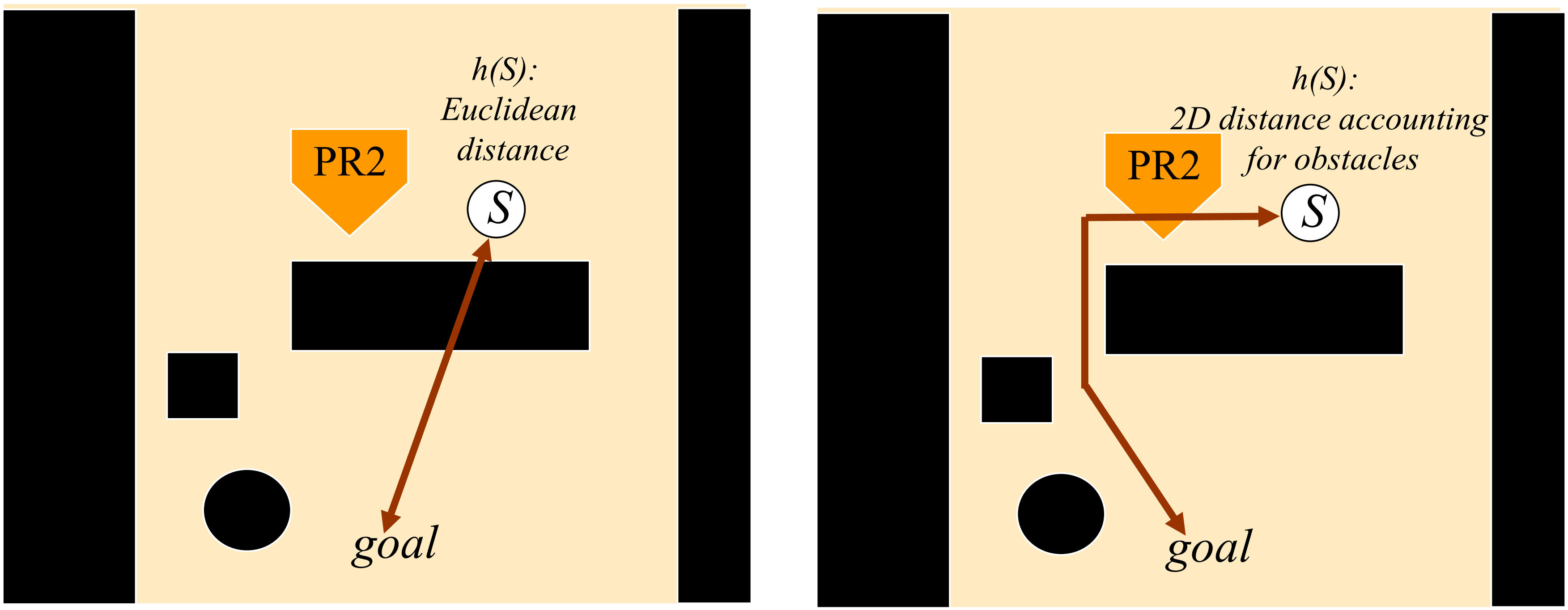

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But what if the "best" hypothesis is wrong?

*we need the search to explore **multiple** heurisics simultaneously!*

Multiple Hypotheses = Multiple Heuristics

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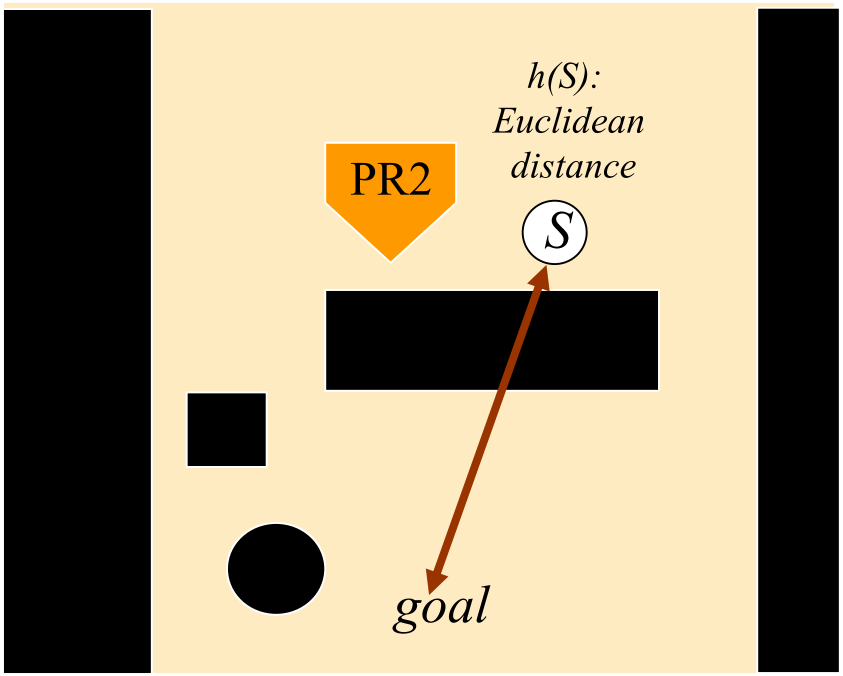

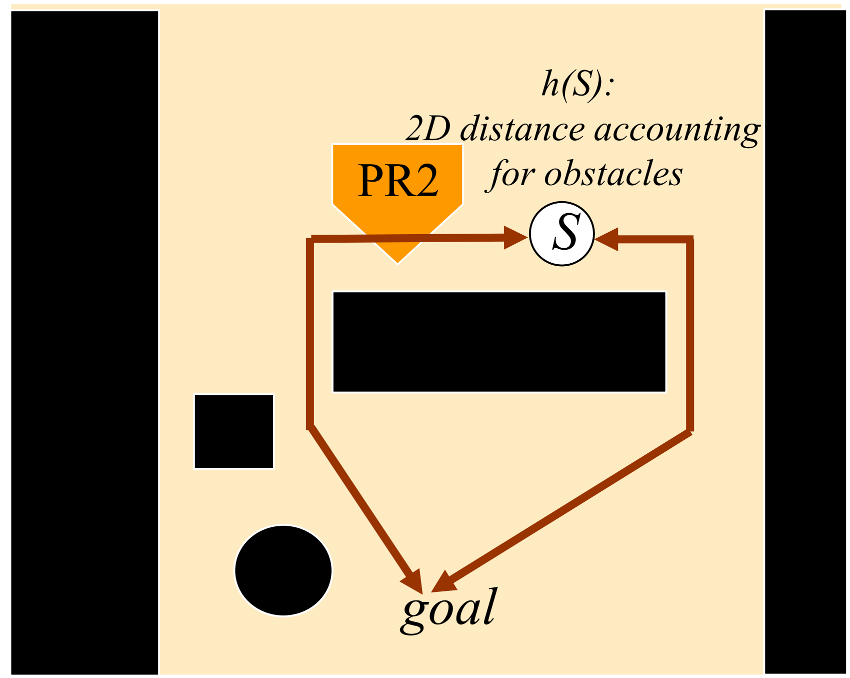

- Solutio… ften excellent

*we need the search to explore **multiple** heurisics simultaneously!*

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Utilizing Multiple Heuristics

More generally: we can often easily generate N arbitrary heuristic functions that estimate costs-to-goal

Can we utilize a bunch of inadmissible heuristics simultaneously, *leveraging their individual strengths while preserving guarantees on completeness and bounded sub-optimality?*

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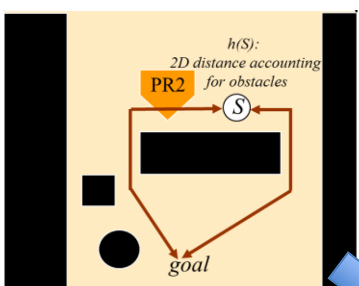
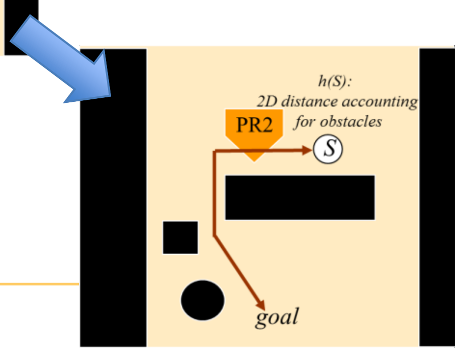
Combining multiple heuristics into one (e.g., taking max) is often inadequate





- *information is lost*
- *creates local minima*
- *requires all heuristics to be admissible*

Multi-Heuristics A* (MHA*)

Multi-Heuristic A* (MHA*) [Aine et al., IJRR'15]:

Heuristic search that does support multiple arbitrary heuristics with guarantees on completeness and bounded sub-optimality

Multi-Heuristics A*: version 1

- We have N inadmissible heuristics
- We run N independent searches
- Hope one of them reaches goal

Inad. Search 1

priority queue: $OPEN_1$
key = $g + w_1 * h_1$

Inad. Search 2

priority queue: $OPEN_2$
key = $g + w_1 * h_2$

Inad. Search 3

priority queue: $OPEN_3$
key = $g + w_1 * h_3$

- We have N inadmissible heuristics
- We run N independent searches
- Hope one of them reaches goal

Problems:

- Each search has its own local minima
- We do N times more work
- No completeness guarantees or bounds on solution quality

Inad. Search 1	Inad. Search 2	Inad. Search 3
priority queue: $OPEN_1$ key = $g + w_1*h_1$	priority queue: $OPEN_2$ key = $g + w_1*h_2$	priority queue: $OPEN_3$ key = $g + w_1*h_3$

Multi-Heuristics A*: version 2

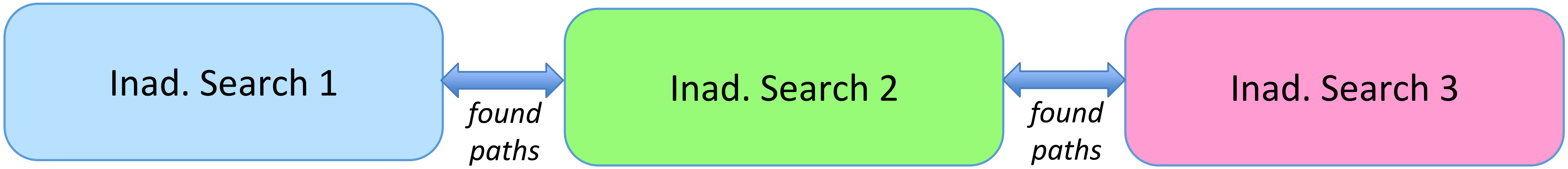
- We have N inadmissible heuristics
- We run N independent searches
- Hope one of them reaches goal
- Key Idea #1: Share information (g-values) between searches!

Benefits:

- Searches help each other to circumvent local minima
- States are expanded at most once across ALL searches

Remaining Problem:

- No completeness guarantees or bounds on solution quality



priority queue: $OPEN_1$ key = $g + w_1 * h_1$	priority queue: $OPEN_2$ key = $g + w_1 * h_2$	priority queue: $OPEN_3$ key = $g + w_1 * h_3$

Multi-Heuristics A* [Aine et al., IJRR'15]

- We have N inadmissible heuristics
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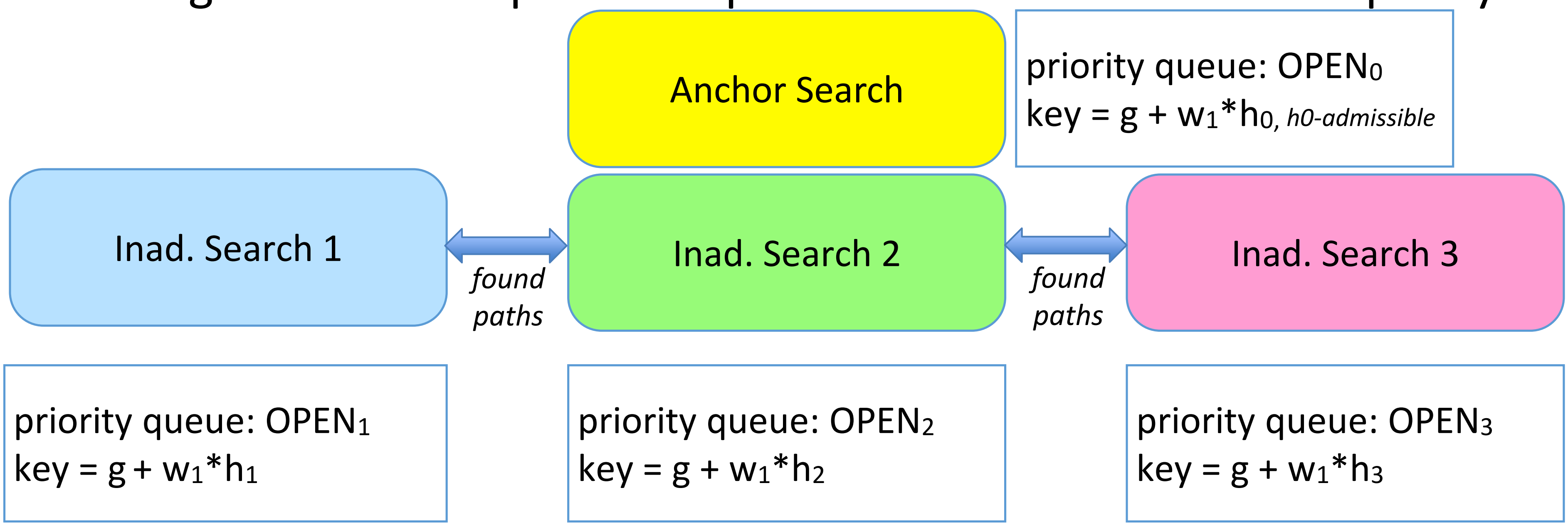
- Key Idea #1: Share information (g-values) between searches!
- Key Idea #2: Search with admissible heuristics controls expansions

Benefits:

- Algorithm is complete and provides bounds on solution quality



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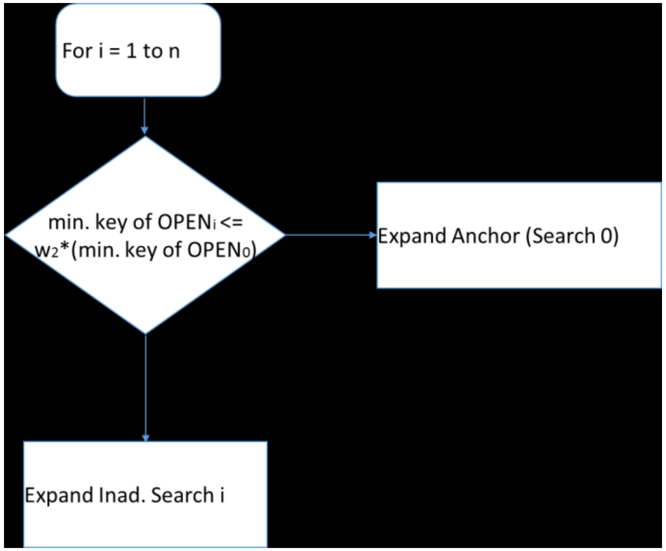

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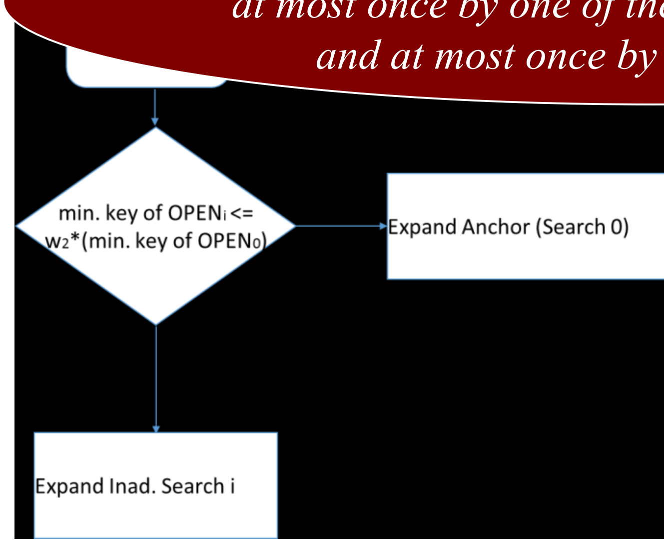
Theorem 1: *min. key of $OPEN_0$ <= w_1*optimal solution cost*

Theorem 2: *min. key of $OPEN_i$ <= w_2*w_1*optimal solution cost*

Theorem 3: *The algorithm is complete and the cost of the found solution is no more than w_2*w_1*optimal solution cost*

Theorem 4: *Each state is expanded at most twice: at most once by one of the inadmissible searches and at most once by the Anchor search*



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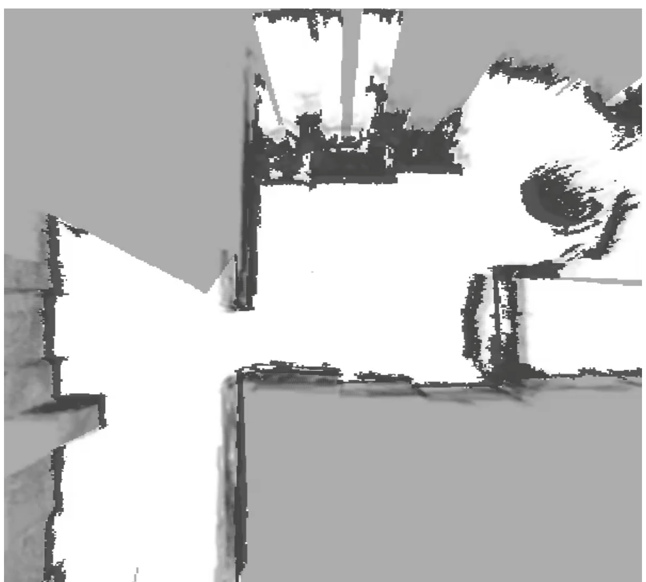

Summary

- Many planning problems in robotics have low-dimensional projects that can provide excellent estimates on cost-to-goal distances/heuristics

- Multiple topology-based heuristics correspond to different hypotheses on feasible solutions

- Multi-Heuristic A* (MHA*) can utilize multiple arbitrary heuristics with rigorous guarantees

Potential Research Directions

- Applying to different planning domains including humanoid planning, mission planning, etc.
- Figuring out what topology classes to consider
- Dynamically instantiating new topology-based heuristics (e.g., Dynamic MHA* [Islam et al., ICRA'15]

Thanks

- Students/postdocs who contributed:
  - Sandip Aine
  - Victor Hwang
  - Venkatraman Narayanan
  - Siddharth Swaminathan
- Funding:
  - ONR
  - ARL
  - NSF