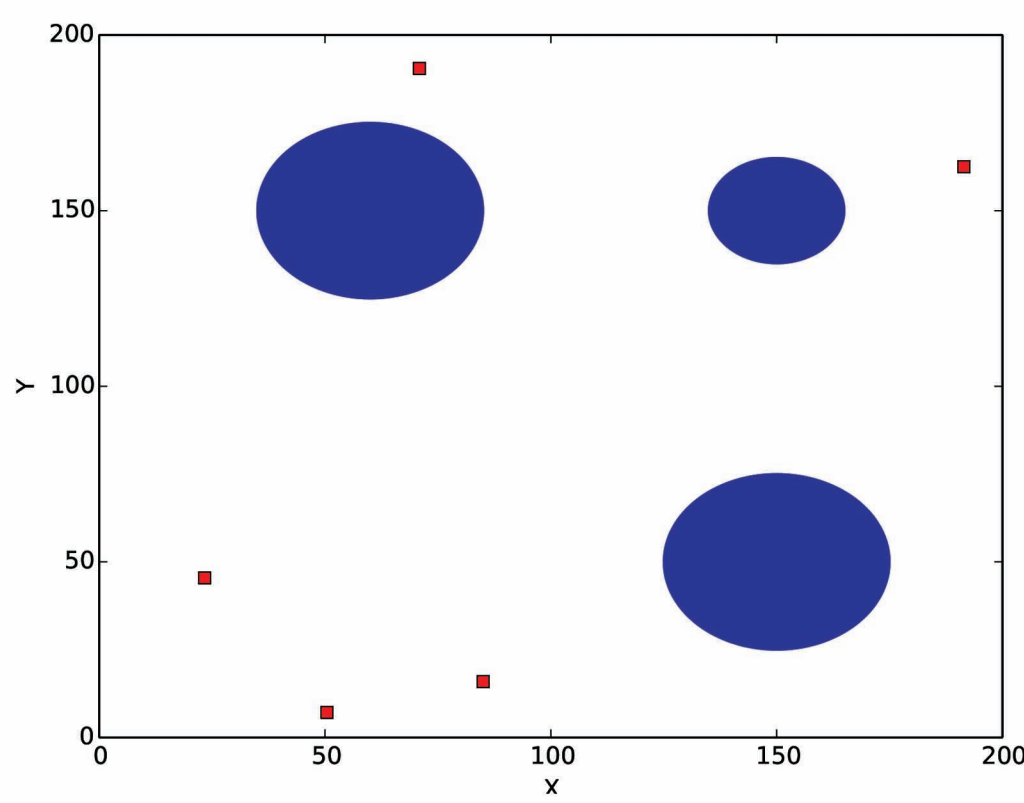


# TOPOLOGICAL MAPPING USING A ROBOTIC SWARM

RAGESH KUMAR RAMACHANDRAN AND SPRING BERMAN

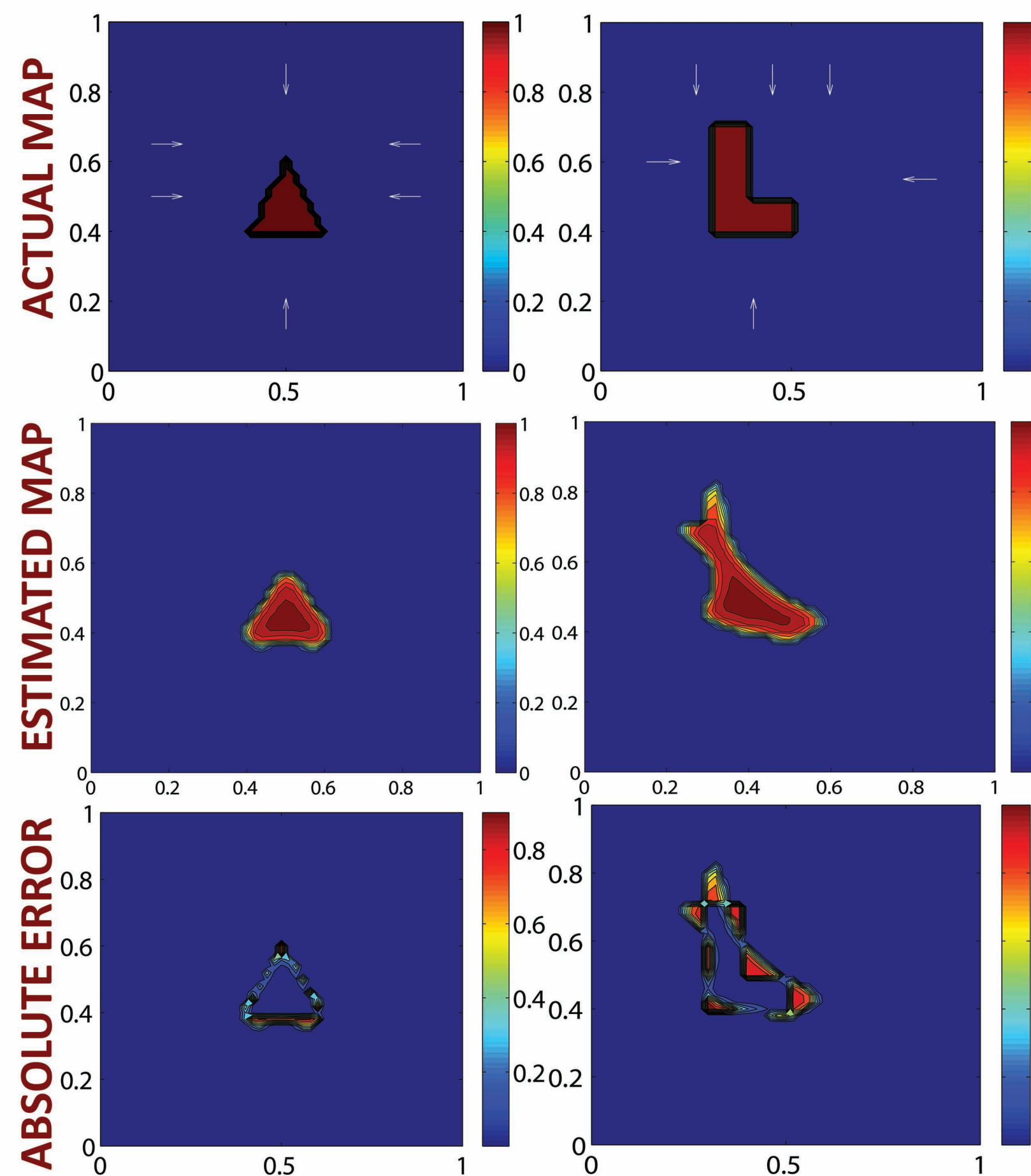


## Problem Statement



- We consider a scenario in which  $N$  ground robots without localization ability are deployed into an unknown, bounded environment to map topological features of interest.
- By topological features we mean the number of holes or obstacles in the domain.

## Previous work



- In our previous work [1] we had tried to estimate the map of a domain by modelling the swarm population dynamics with a set of advection-diffusion-reaction partial differential equations.
- Map of the environment is incorporated into this model using a spatially-dependent indicator function that marks the presence or absence of an obstacle.
- This indicator function is defined as the solution of an optimization problem in which we minimize an objective functional that is based on temporal data acquired by the robots during deployment.

## Methodologies

- We try to solve this problem using two different approaches.

### Approach I

- In this approach each robot performs a correlated random walk in the domain avoiding obstacles and other robots and estimate its position using its odometry and compass data.
- The estimated position is stored in its limited internal memory at fixed time intervals.
- After some time period  $T$  the robots are retrieved and the stored estimated position information from each robot is obtained. This gives rise to a **point cloud**.

- We construct a filtration of combinatorial objects from the point cloud, which can represent geometrical and topological features of the data set.
- Combinatorial objects called simplicial complexes are constructed from the point cloud.
- Vietoris-Rips complex is a computationally efficient way of constructing these simplicial complexes.
- A filtration is generated by varying radius  $\epsilon$  of the open ball around each point in the point cloud and constructing Vietoris-Rips complex for that particular radius  $\epsilon$ .
- **Persistent homology** is a widely used technique from **topological data analysis** to characterize the holes in a data.
- The topological structures of a complexes can be summarized using Betti-numbers. Betti-numbers are ranks of **homology groups** which are topological invariants.
- Barcode diagrams of 0 and 1 dimensions are obtained from the filtration to determine the number of connected components and holes in the domain respectively.

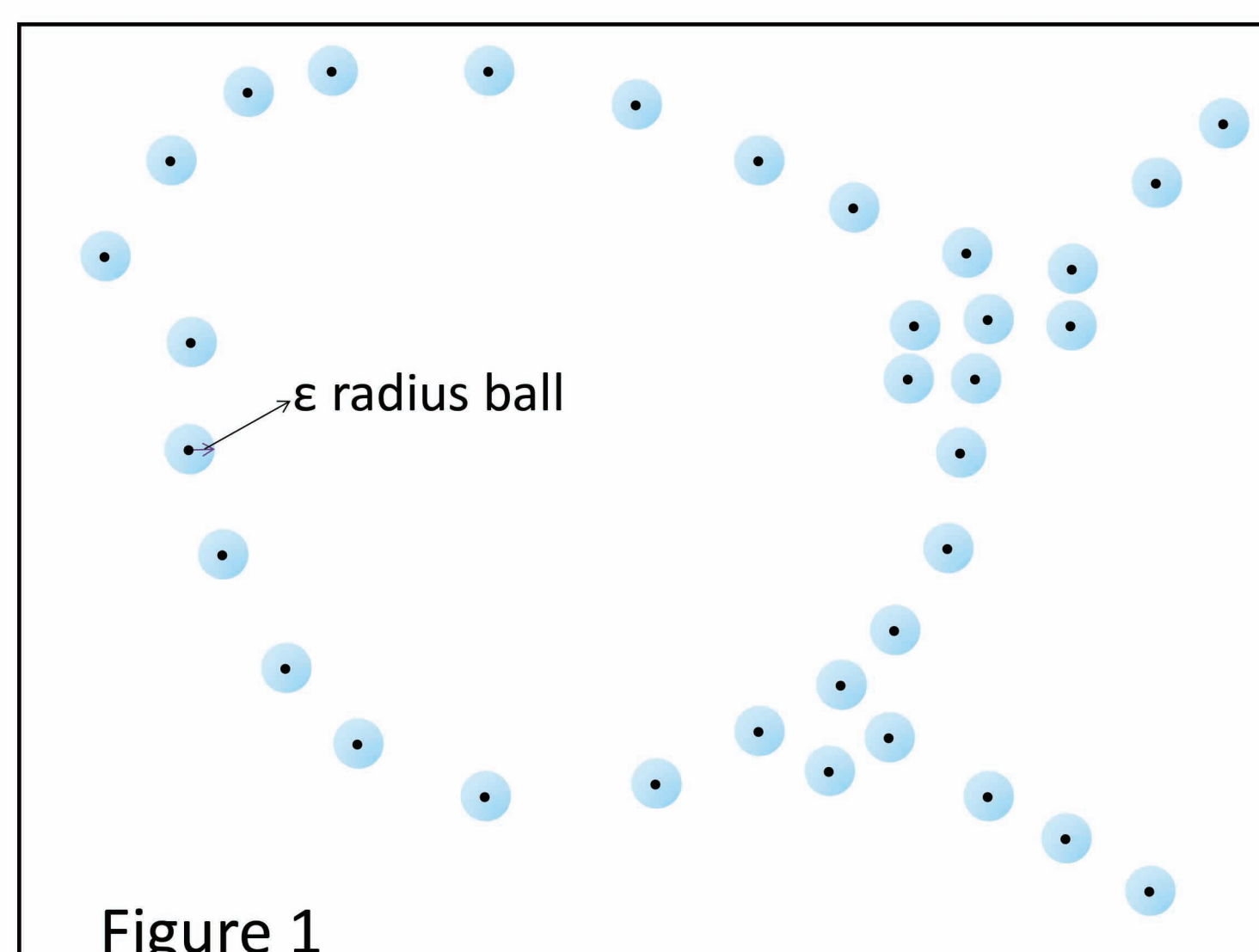


Figure 1

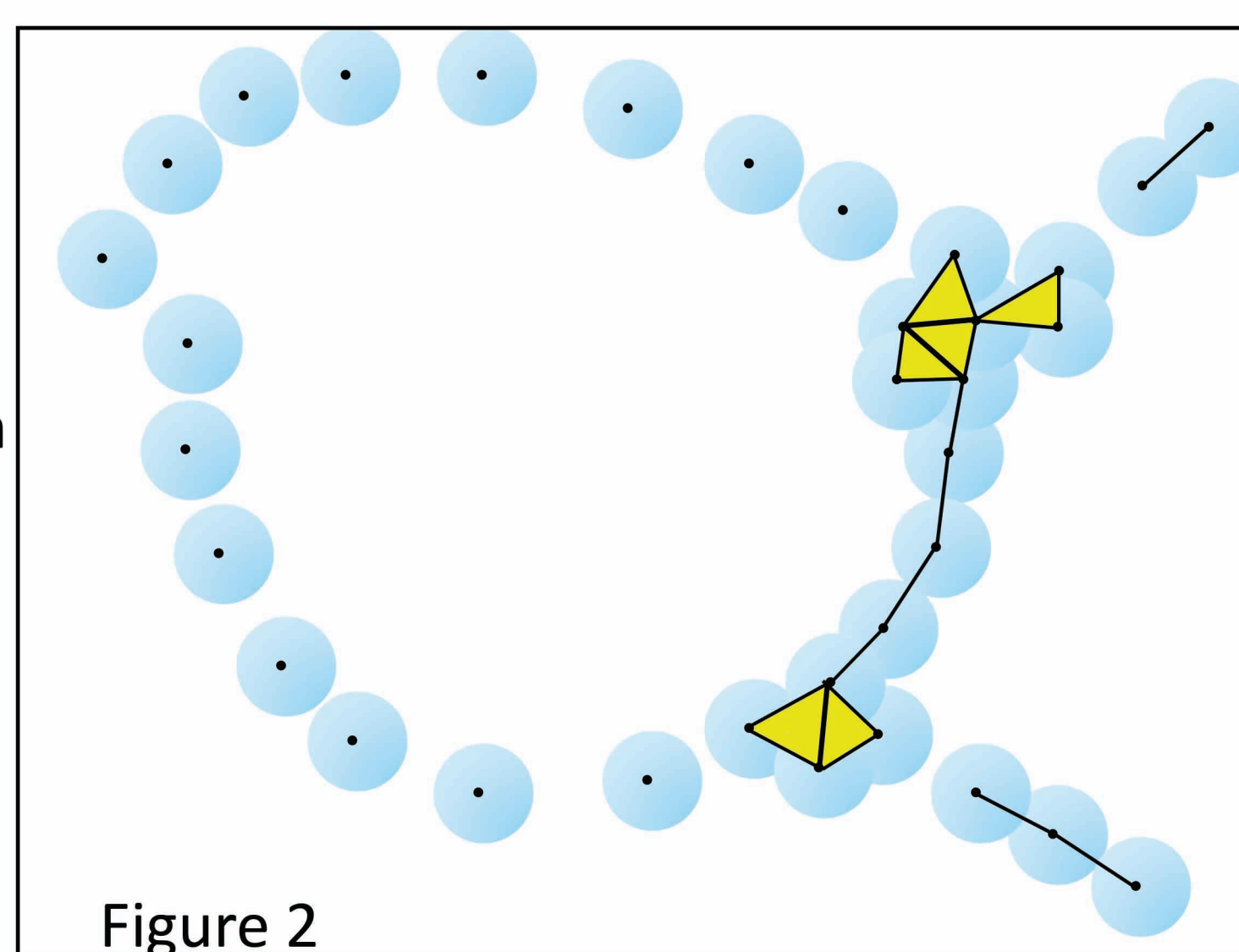


Figure 2

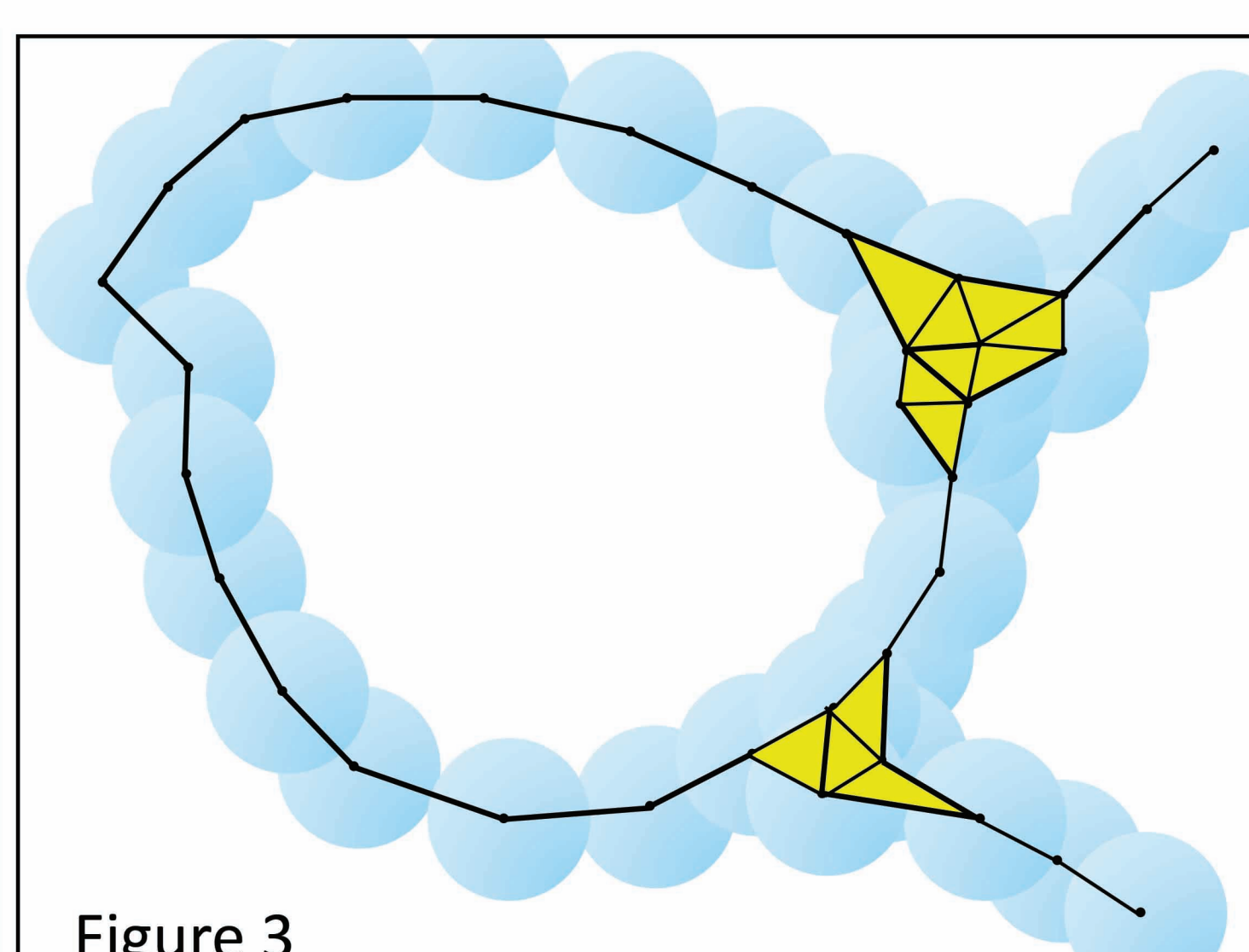


Figure 3

- Figures 1 to 3 shows how to create a filtration of simplicial complexes from a point cloud by varying the radius  $\epsilon$  of the open ball.
- The simplicial complexes constructed here are Vietoris-Rips complexes shown in figures 2 and 3.
- From figure 3 we see that the underlying topological structure of the data contains a 1-dimensional hole and has a single connected component.

## Approach II (ongoing work)

- The basic idea behind this approach is to deploy a heterogeneous swarm containing two kinds of robots, **transmitters** and **receivers**
- The transmitter robots are more expensive than the receiver robots, and thus the swarm contains many more receivers than transmitters.
- Each transmitter has a transmission coverage range that is large enough to encompass the whole domain
- The receivers perform random walks in the domain ( $D \subset \mathbb{R}^2$ ), sampling the smooth signals sent by the transmitters.
- The data recorded by the robots is a **transmission map** from the robot configuration space to a higher-dimensional manifold given by:  $T: \mathbb{R}^{2N} \rightarrow \mathbb{S}$ , where  $N$  is the number of receivers and  $\mathbb{S}$  is the signal space.
- Using the signal embedding theorem proved in [2], it can be shown that the condition for the map  $T$  to be injective is:

$$\dim \mathbb{S} > 4N \quad (1)$$

- This means that if each transmitter provides only one piece of information at a time, then the number of transmitter robots should exceed four times the number of receiver robots. This is undesirable.
- If the transmitter robots can provide multiple pieces of information, then the number of transmitters can be reduced.
- Also by examining the persistent homology of the image of transmission map we can identify the holes in the domain  $D$ .

Currently we are trying to construct a transmission map which could reduce down the number transmitter robots required for estimation.

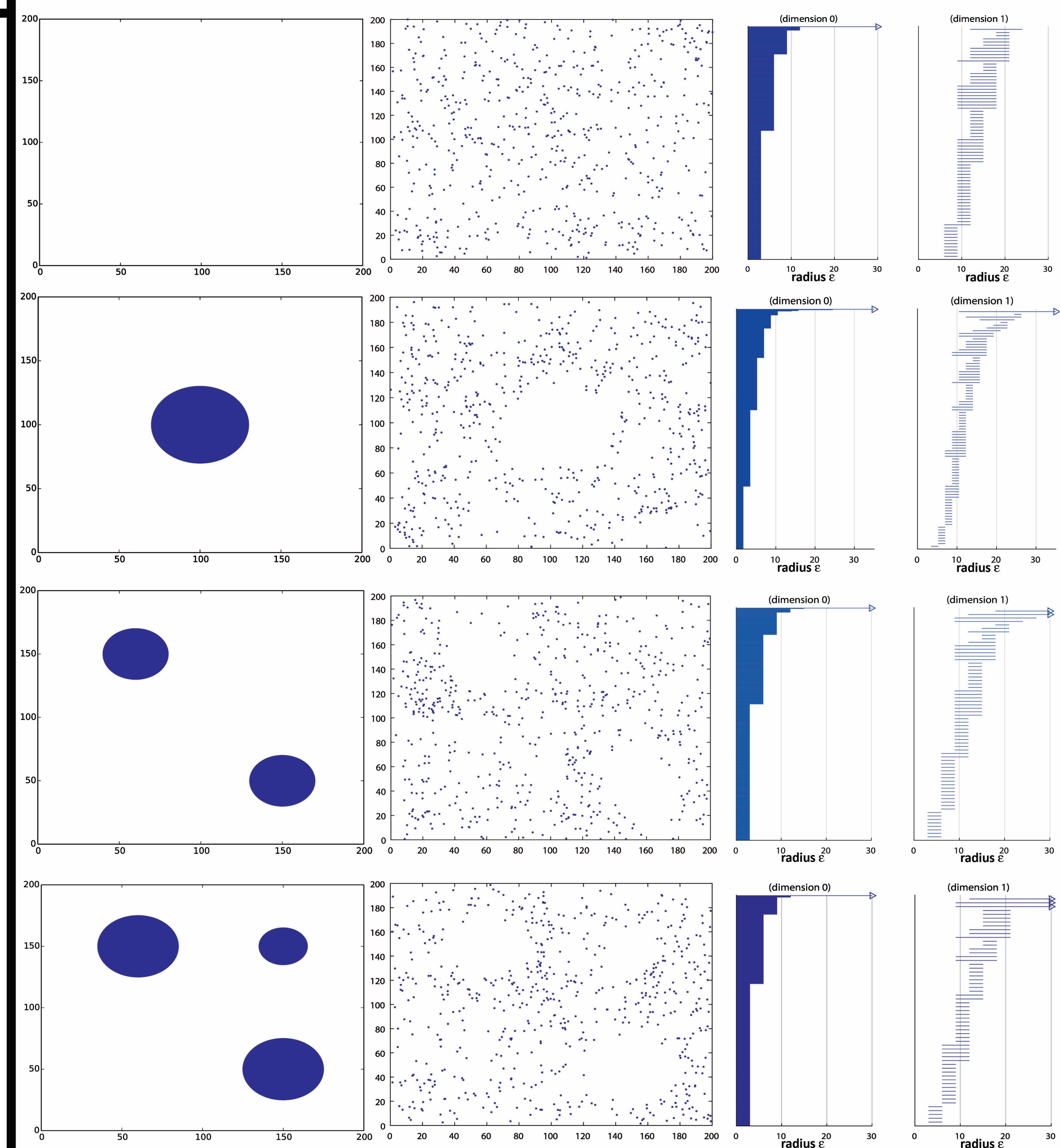
## Simulation and Results (Approach I)

Simulation was done with five robots in a bounded domain with number of obstacles ranging from zero to three.

### Domain

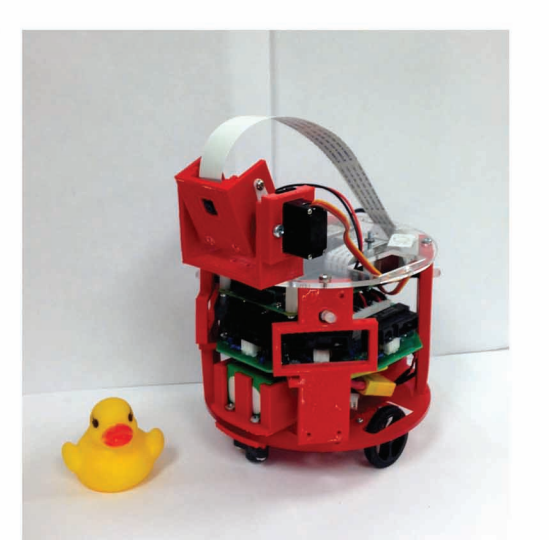
### Point Cloud

### Barcode



## Future Work

- Investigate deeper on the second approach towards topological mapping.
- Validate the first approach by experiments using the Pheeno robotic platform.
- Study how to exploit the metric information in the data using techniques from manifold learning to generate metric maps rather than topological maps.



Pheeno Robot

## References

- [1] R. K. Ramachandran, K. Elamvazhuthi, and S. Berman, "An optimal control approach to mapping GPS-denied environments using a stochastic robotic swarm," in Proceedings of the 2015 International Symposium on Robotics Research (ISRR), September 12–15, 2015.
- [2] M. Robinson and R. Ghrist, "Topological Localization Via Signals of Opportunity," Signal Processing, IEEE Transactions, on, vol. 60, no. 5, pp. 2362–2373, May 2012