Topological Motion Planning

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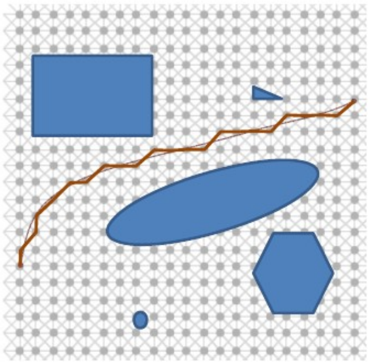
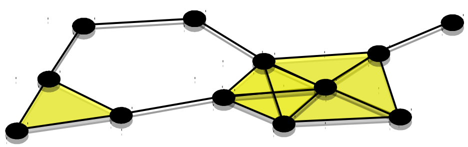
Topological Motion Planning?

- Motion planning (*planning a set of actions that achieves a specific objective*) is fundamental to autonomy:
  - An efficient, versatile and effective method in motion planning is ***graph search based motion planning***.



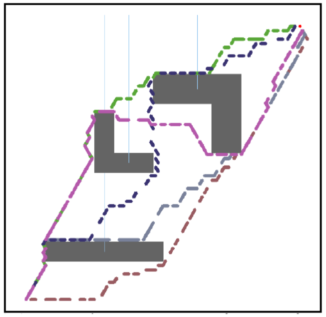
Graph search-based planning
[A: Hart, et al.; D* Stenz et al; RRT: Lavalle]*

- Topology attempts to re-incorporate those ***richer information*** that a discrete graph representations (a 1-dimensional entity) fails to capture.



Simplicial complex
[Derenick, et al.; Ghrist et al;]

- Many specifications of goal/obective in motion planning can be formally described using the language of topology.
- Locally computed topological quantities can be used (***reliably integrated***) to provide global guarantees.
- Topological methods do not rely on precise metric information (***robust to errors***).



Topological Trajectory Planning
[Bhattacharya, et al.]
“go to the left of an obstacle” vs. “go to the right of an obstacle”

Outline

- Topological Trajectory Planning and its Applications
- Dimensionality Reduction using Topological Abstraction
- Sensor Coverage of Unknown, GPS-denied Environments using Robot Swarms
- Simplicial Search Algorithms

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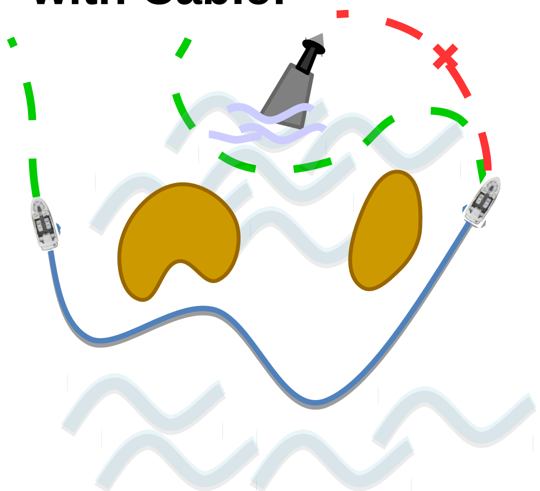
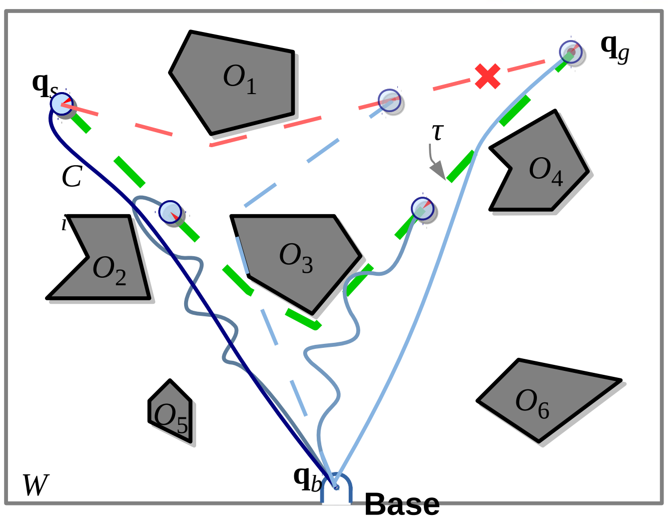
Topological Trajectory Planning: **Motivation**

Optimal Trajectory Planning for Systems with Cable:

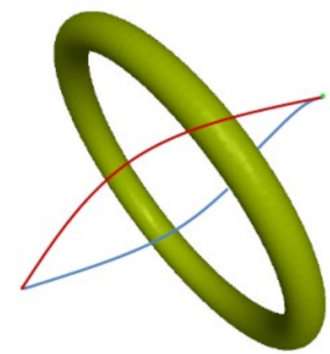


Deep Horizon Oil Spill Cleanup Operation:

Tethered robot:

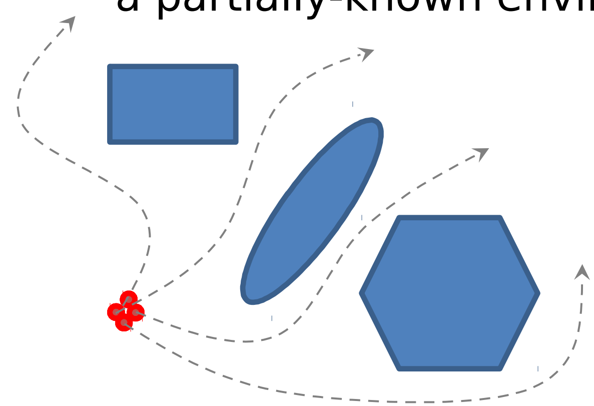


Tasks requiring Topological Reasoning and Multi-agent exploration:



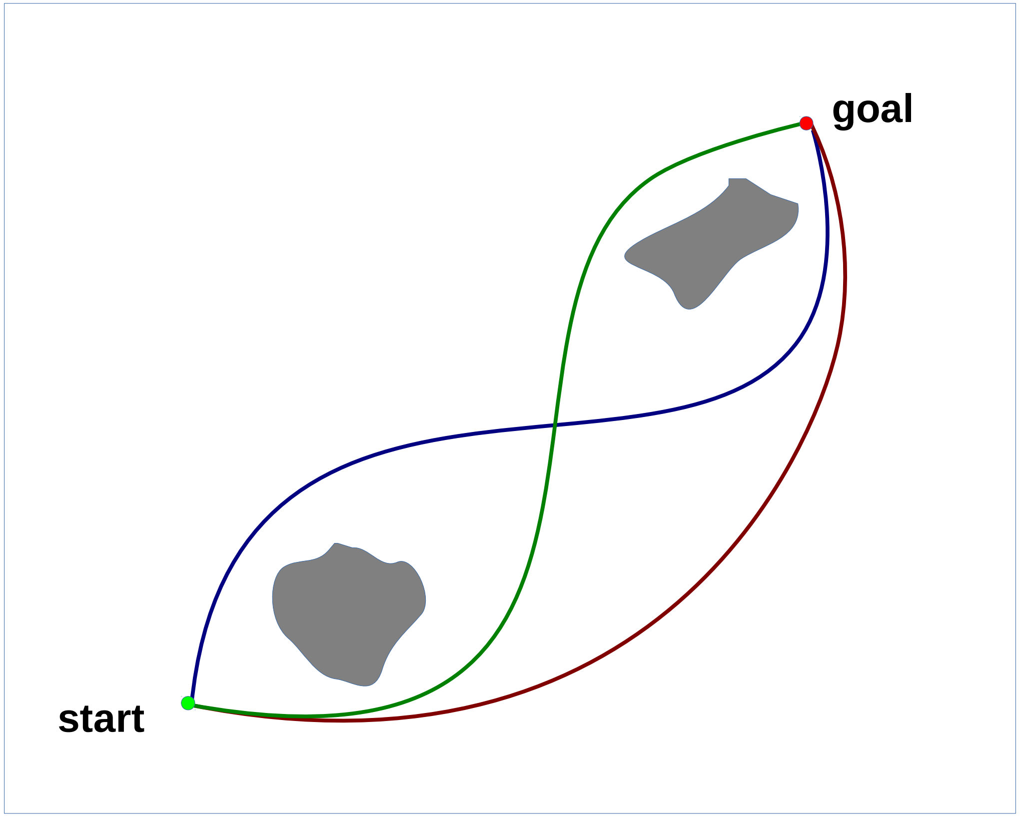
[Mellinger, Michael, Kumar, IJRR 2012]

Multi-agents search/ exploration in a partially-known environment:



Topological Classes of Trajectories

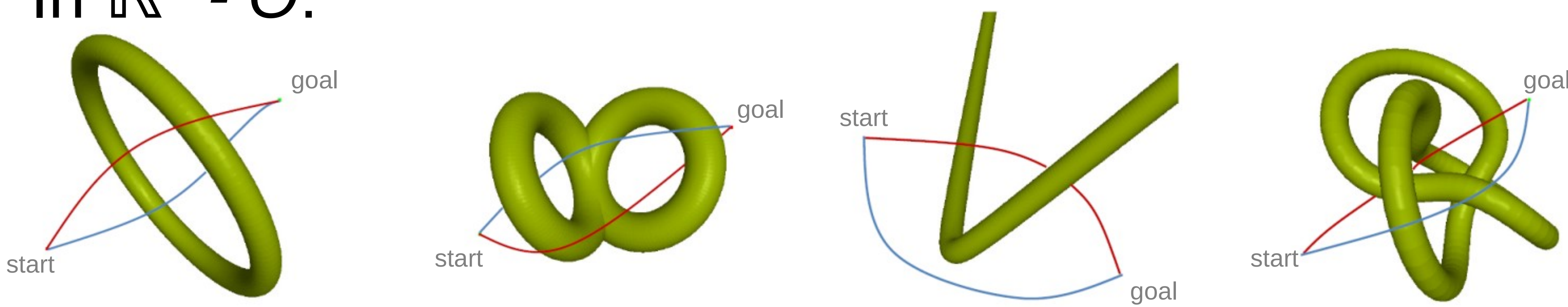
In $\mathbb{R}^2 - O$:



We would like to be able to:

1. Make distinction between the different topological classes of trajectories.
2. Exploit that information for optimal trajectory planning in different topological classes.
3. Apply that to solving real problems in robotics.

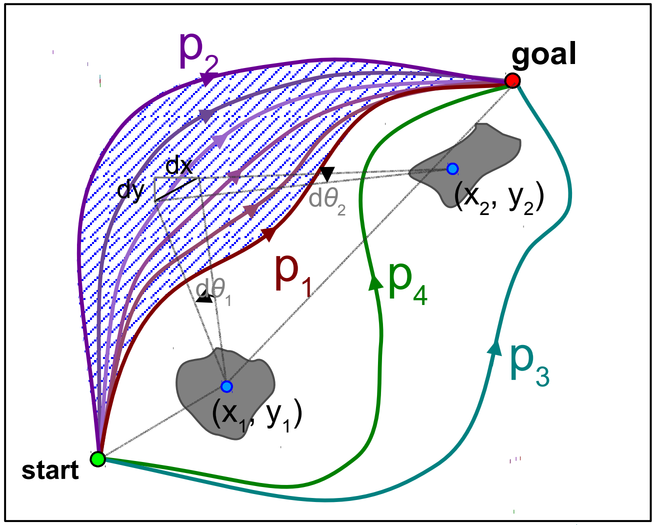
In $\mathbb{R}^3 - O$:



Related Work

- **Cell-decomposition (e.g., Voronoi decomposition, Delaunay triangulation) and Semi-algebraic Description of Environments:** [Demyen, Buro, AAAI, 2006; Hershberger, Snoeyink, JCGTA, 1991; Grigoriev, Slissenko, ISSAC, 1998; Schmitzberger, Bouchet, Dufaut, Wolf, Husson, ICIRS, 2002.]
  - Often construction is difficult / expensive, especially for a environment presented as an occupancy-grid.
  - If not carefully constructed (*e.g*., arbitrary triangulation), the classification may not be one-to-one.
  - While possible to classify given trajectories, the representation is not best suited for search-based optimal path planning.
- **Simplicial Complex Representation and Persistence Homology:** [Pokorny, Hawasly, Ramamoorthy, RSS, 2014;]
  - Requires only a simplicial description of the free space (without an embedding)
  - Well-suited for classifying given trajectories in different ***homology*** classes.
  - Recent developments in computational ***cohomology*** on simplicial complex allows construction of topological invarants [*Pokorny, et. al, RSS, 2015*].
- **Topological Invariant (*can be used in conjunction with graph search*):**
  - Simple construction
  - Ideal for graph search-based optimal motion planning for finding optimal paths in different homology classes.
  - Suitable for both ***homology*** and ***homotopy*** path planning.

Homotopy and Homology

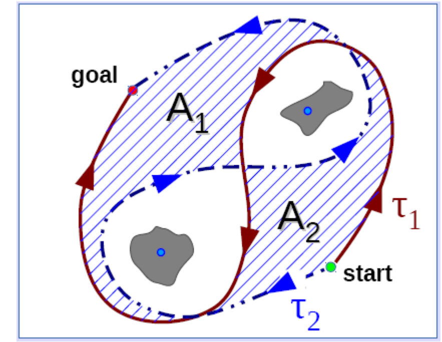


Homotopy: (computationally difficult in dim.>2)

$$p_1 \sim p_2 \nsim p_3 \nsim p_4$$

Homology: (computationally favorable)

$$p_1 \sim p_2 \nsim p_3 \nsim p_4$$



τ_1 and τ_2 homologous, but not homotopic.

p_1 and p_2 belong to same homotopy class

$\Rightarrow$ they belong to same homology class.

Converse is not necessarily true!

Homology invariant, $\mathcal{H}(p_i)$:

$$\mathcal{H}(\tau) = \int_\tau \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \end{pmatrix} \quad (e.g., \ \xi_1 = d\theta_i)$$

($\mathcal{H}$-signature of τ)

Key concept: Find a set of linearly independent ***closed, non-exact*** differential 1-forms which forms a basis for the de Rham cohomology group, $H^1_{dR}(\mathbb{R}^N - O)$: $\xi_1, \xi_2, \ldots \in \mathrm{Ker}(d^1)$ $\notin \mathrm{Img}(d^0)$

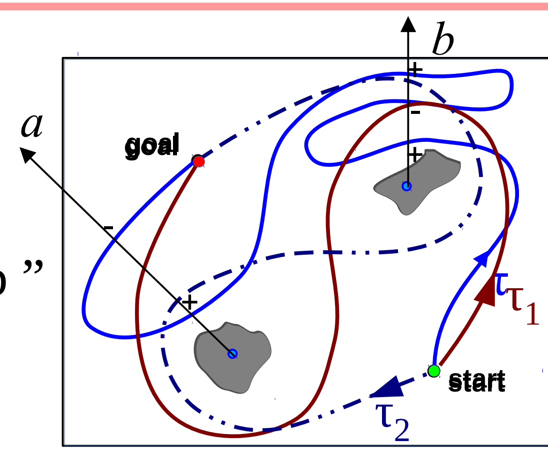
[Bhattacharya, Likhachev, Kumar. 2012, AURO 33(3)]
[Bhattacharya, Lipsky, Ghrist, Kumar. 2013, AMAI 67(3-4)]

Homotopy invariant, $h(p_i)$, in 2-dimensions:

Key concept: ***Words*** constructed by tracing a trajectory and inserting letters based on *rays* crossed.

$$h(\tau) = \text{“ } b\, b^{-1}\, b\, a\, a^{-1} \text{ ”} = \text{“ } b \text{ ”}$$

$$h(\tau_1) = \text{“ } b\, a^{-1} \text{ ”} \neq h(\tau_2) = \text{“ } a^{-1}\, b \text{ ”}$$

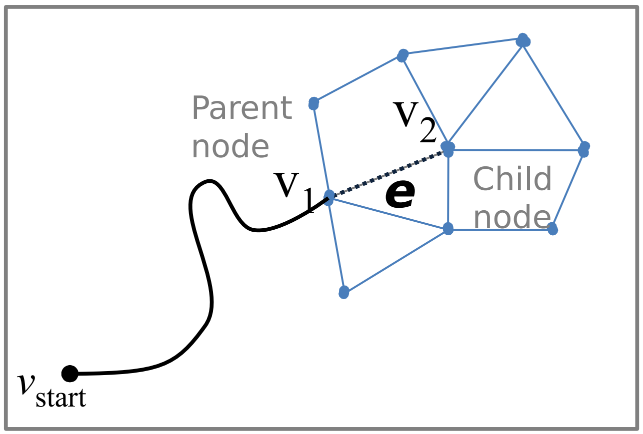


[Tovar, Cohen, LaValle, WAFR, 2008]
[Narayanan, Vernaza, Likhachev, LaValle, ICRA, 2013]
[Bhattacharya, Kim, Heidarsson, Sukhatme, Kumar. IJRR, 2014]
[Bhattacharya, Ghrist, IMAMR, 2015.]

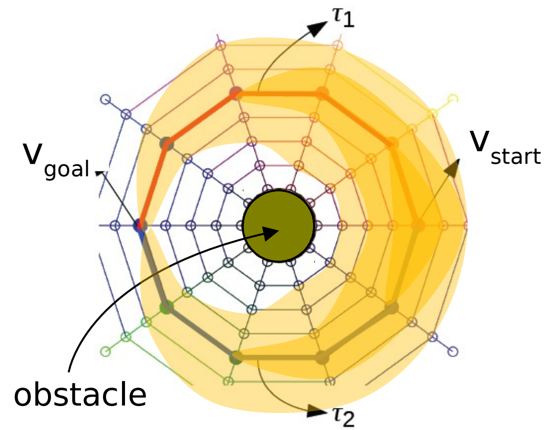
Use in Graph Search

H-augmented graph construction (Illustration in cyllindrically discretized 2-D env.):

$$H(v_{\text{start}} \rightarrow \mathrm{v}_2) = H(v_{\text{start}} \rightarrow \mathrm{v}_1) + H(\boldsymbol{e})$$

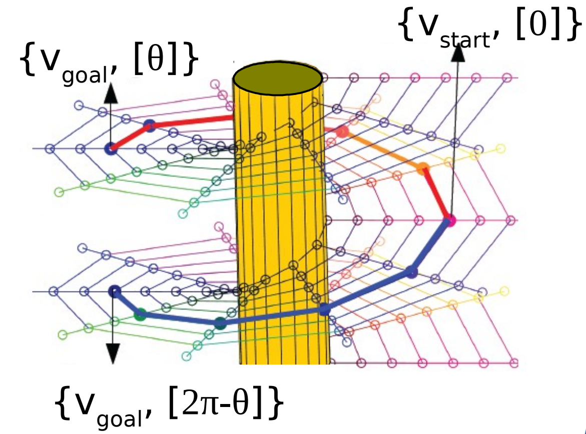


Original graph, G
Vertices: v

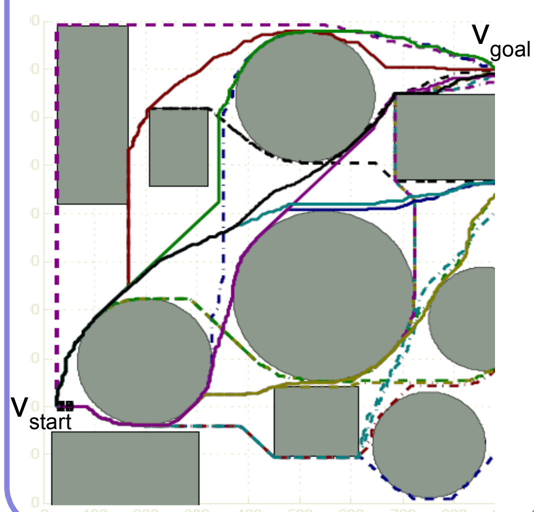


Graph search algorithm

H-augmented graph, G_H
Vertices: {v, [θ]}

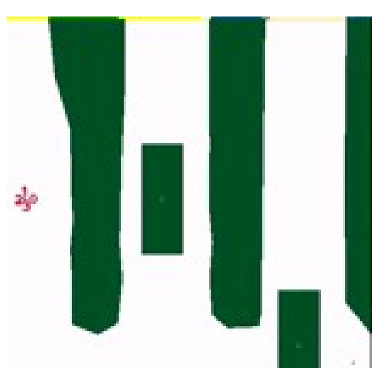


In 2D *(homology & homotopy)*:

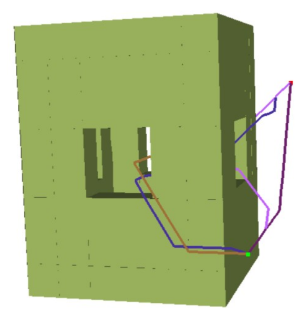


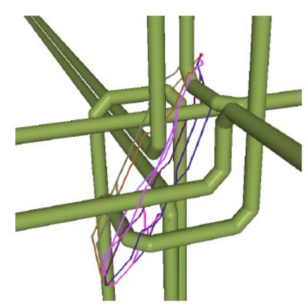
In 3D: *(homology)*

X-Y-Time config space:



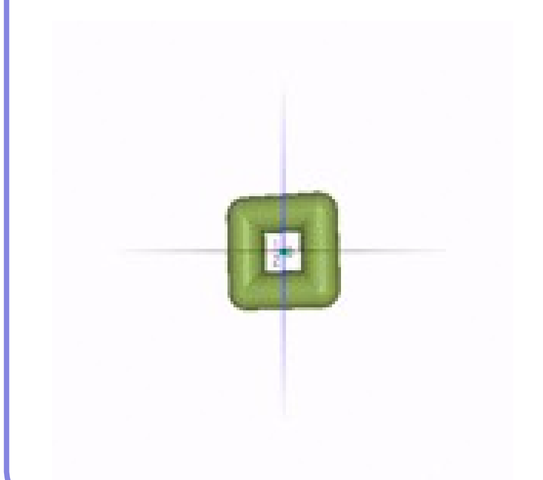
X-Y-Z config space:





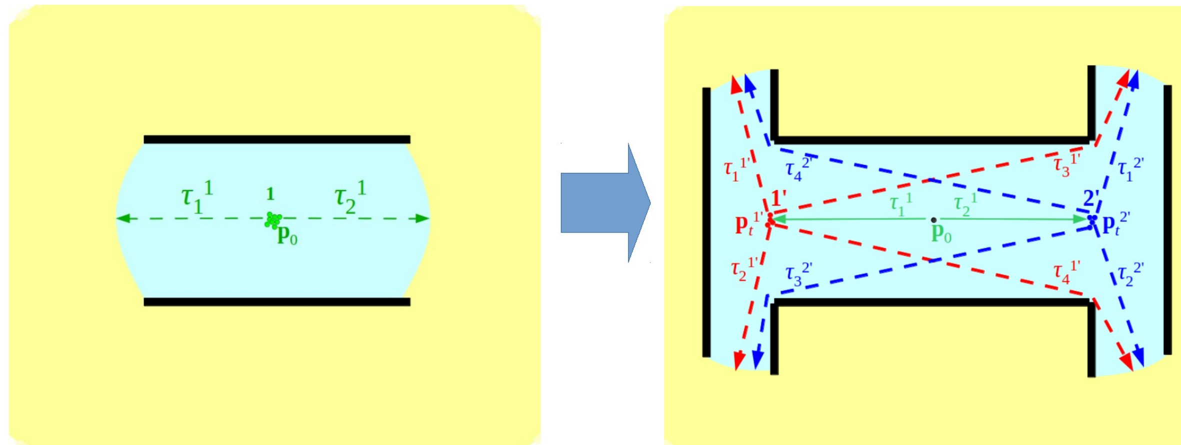
In 4D: *(homology)*

X-Y-Z-Time config space:

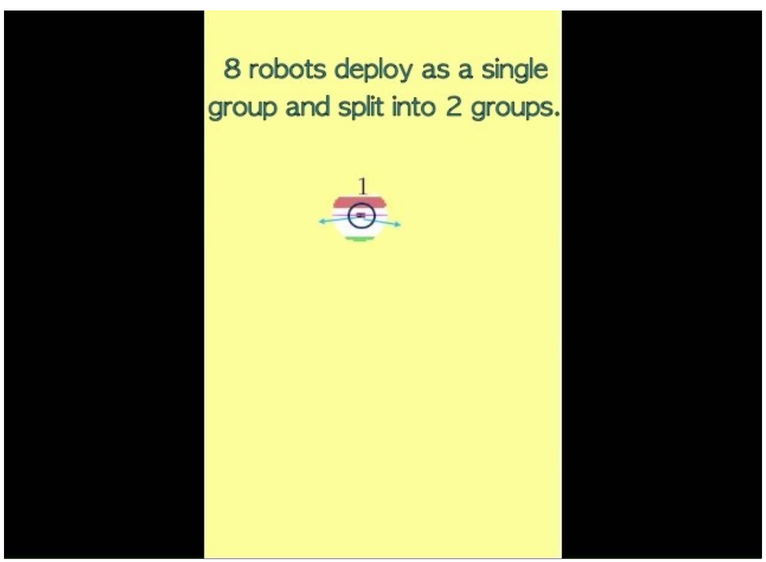


Application 1
Topological Exploration

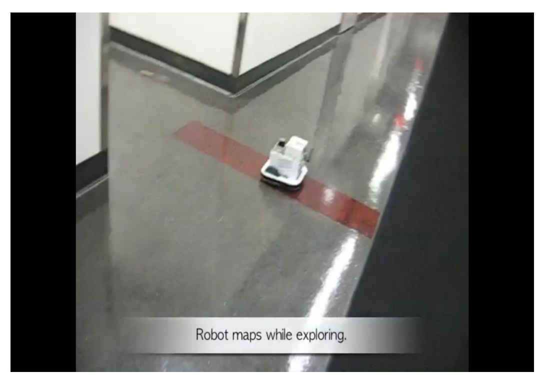
Group of robot splitting based on the available topological classes in the environment:

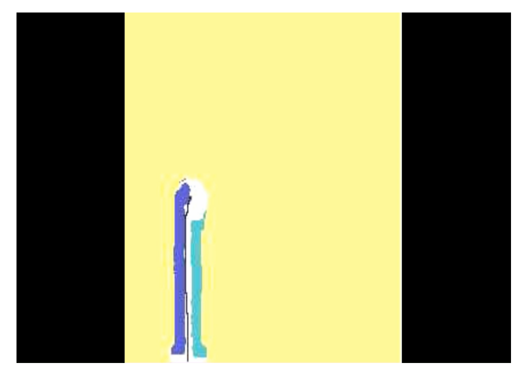


ROS simulation of topological exploration of an unknown environment using 8 robots.



Single-robot experiment
- Scarab mobile robot platform (differential drive, laser range sensors**)**
- Visual odometry localization module

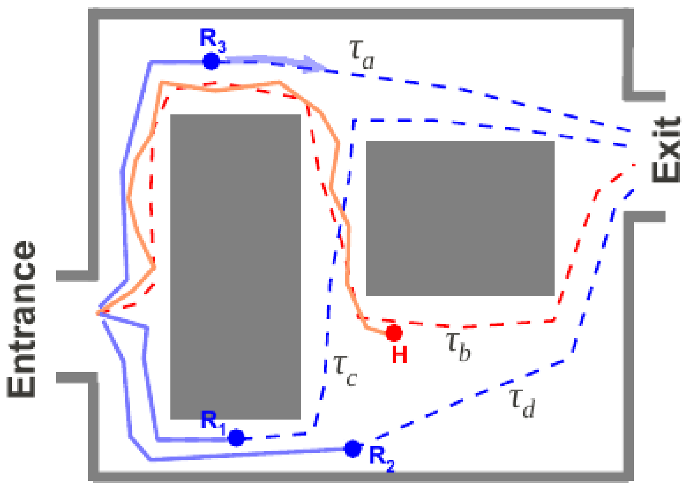




[*Kim, Bhattacharya, Ghrist, Kumar. IROS, 2013*]

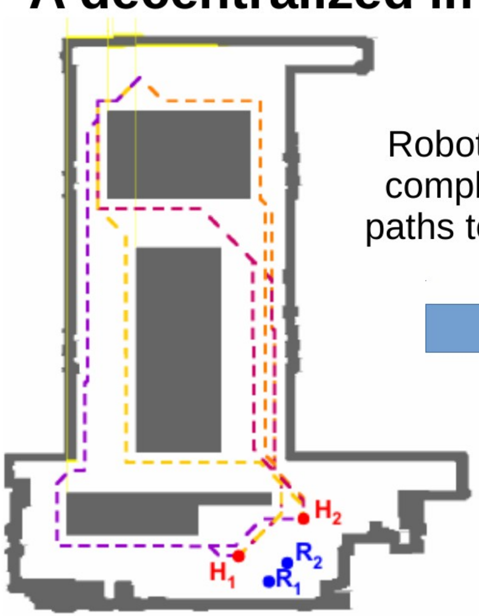
Application 2

Human-Robot Collaborative Topological Exploration for Search and Rescue Mission

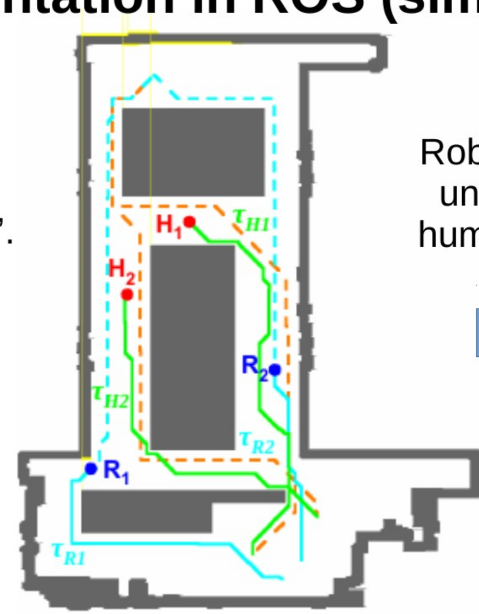


- Heterogeneous team of humans and robots need to explore an environment for search & rescue missions.
- Human(s) chooses trajectories at their discretion.
- Robots need to adapt and choose complementary topological classes to maximize exploration / clearing.

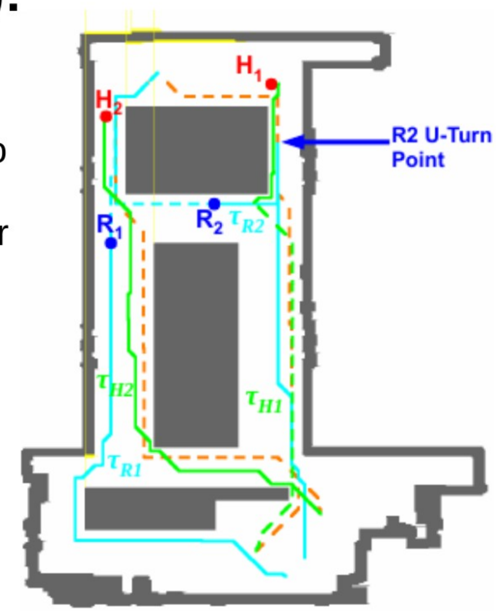
A decentralized implementation in ROS (simulation):



Robots choose complementary paths to humans'.



Robots adapt to unpredictable human behavior

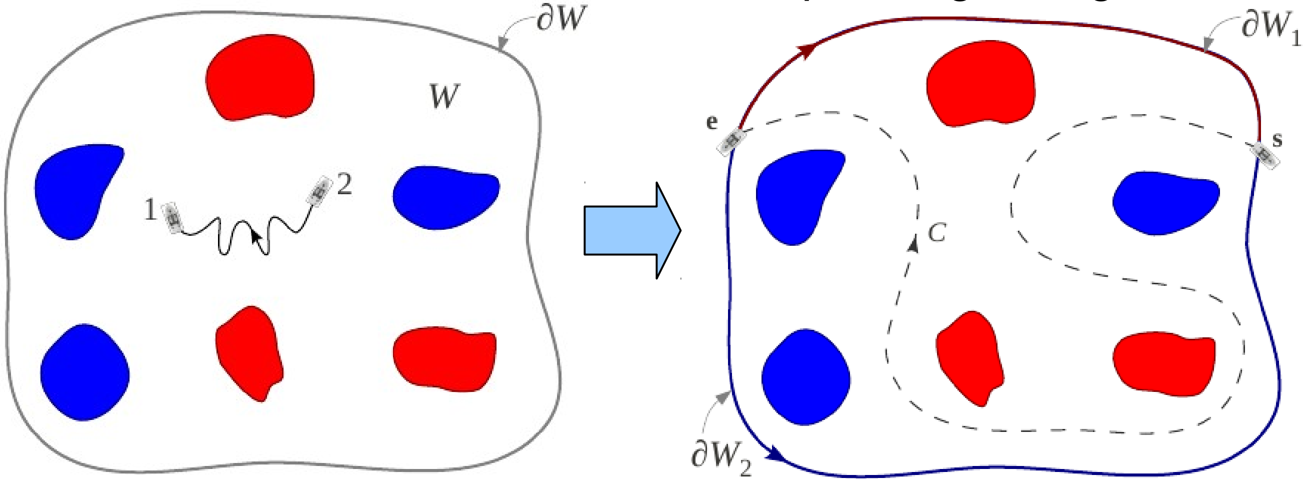


[***Govindarajan***, *Bhattacharya, Kumar, DARS'14,* ***Best paper award nomination!***]

Application 3: Object Separation Using Cable

Problem definition:

Separating configuration

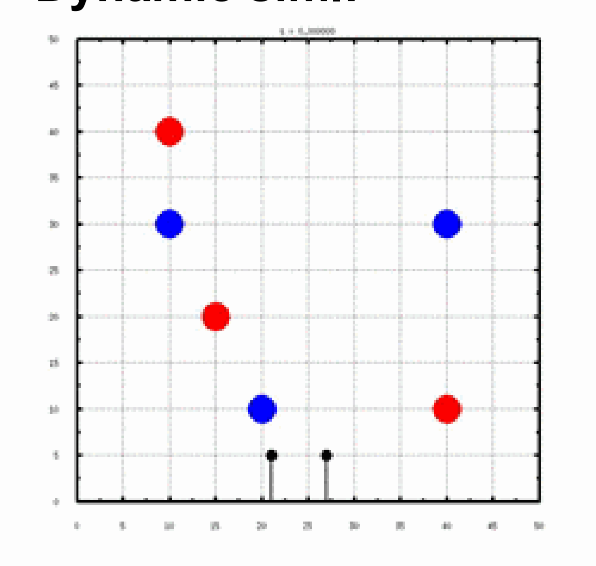


Motivation:

Basic idea:

1. Mathematically describe a "separating configuration" (identified by its homology class).
2. Find optimal trajectories in the right homotopy classes leading to a separating configuration.

Dynamic sim.:

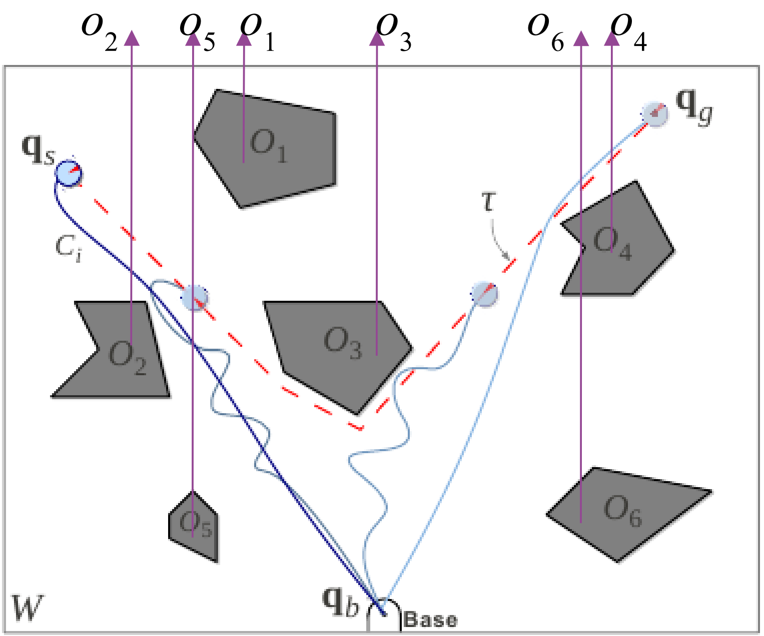


Field experiment in collaboration with USC.:

[Bhattacharya, Kim, Heidarsson, Sukhatme, Kumar. IJRR, 2014]

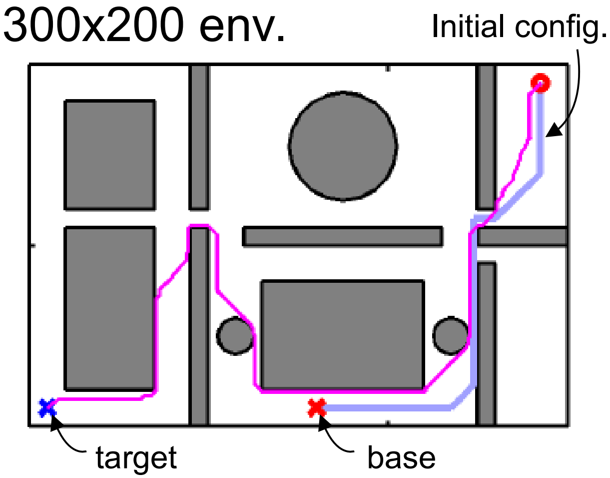
Application 4: Planning for a Tethered Robot

Problem definition:

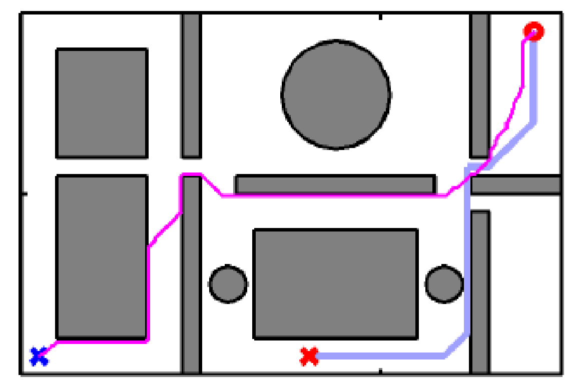


Method: Perform search in h-augmented graph

Results:
300x200 env.

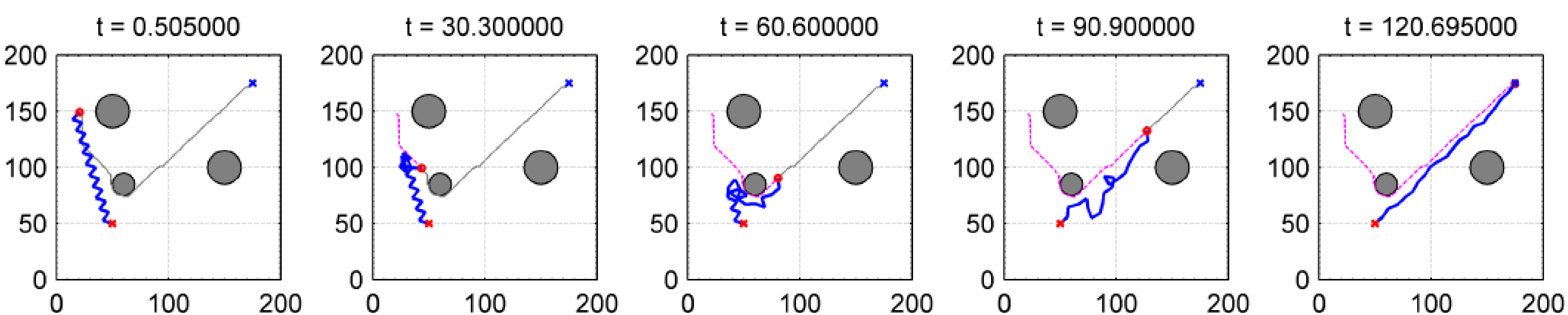


Cable length:350 disc. units



Cable length: 450 disc. units

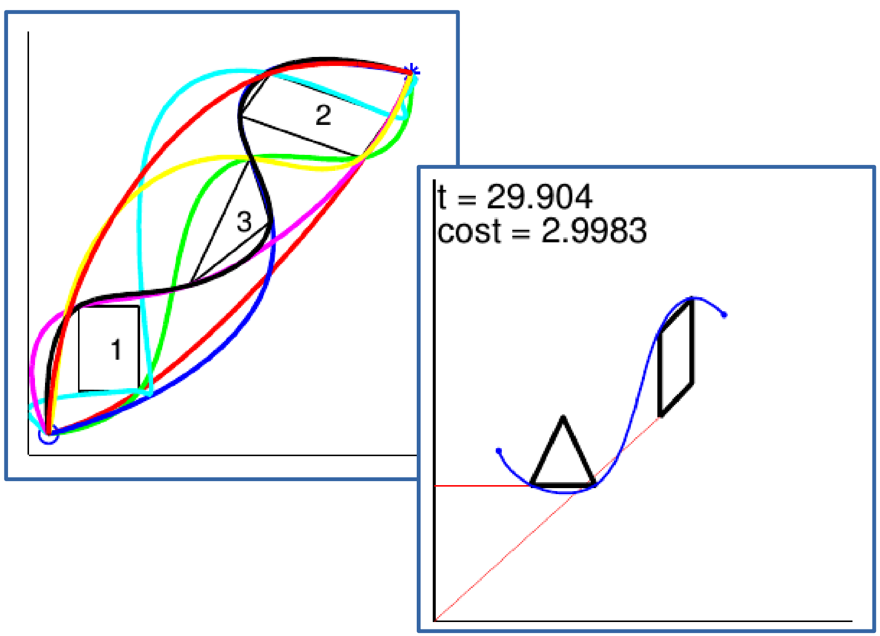
Dynamic simulation:

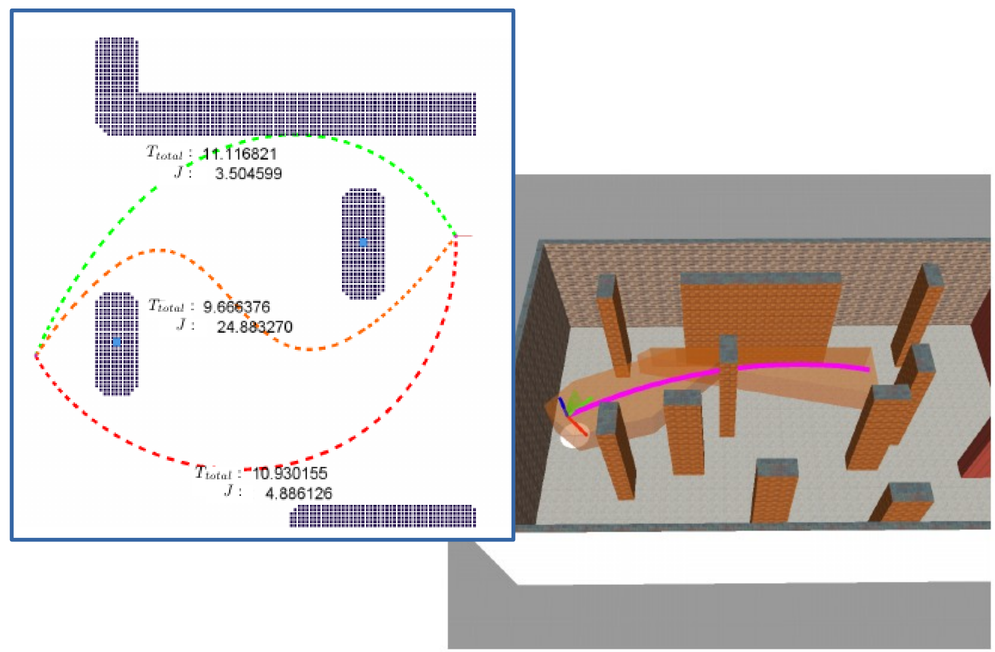


[Kim, Bhattacharya, Kumar, ICRA'14]

Other Applications

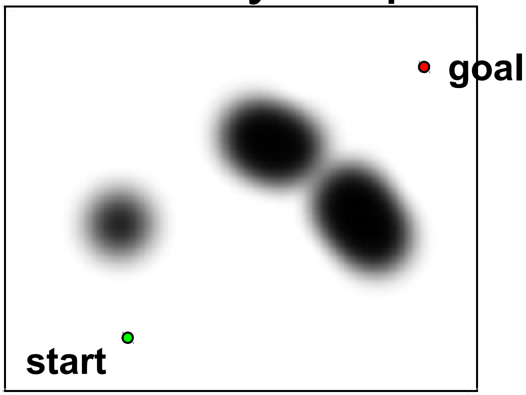
- Highway Vehicle Navigation (homotopic consideration in changing lanes and passing vehicles).
  [Park, Karumanchi, Iagnemma, T-RO, 2015.]
- Conflict minimization in multi-robot motion planning.
  [Kimmel, Bekris, 2012]
- Smooth optimal trajectory planning in different topological classes using QP and MIQP frameworks
  [Kim, Sreenath, Bhattacharya, Kumar, CDC, 2012;
  Kim, Sreenath, Bhattacharya, Kumar, ARK, 2012;
  Sikang Liu, Watterson, Bhattacharya, Kumar (under preparation).]



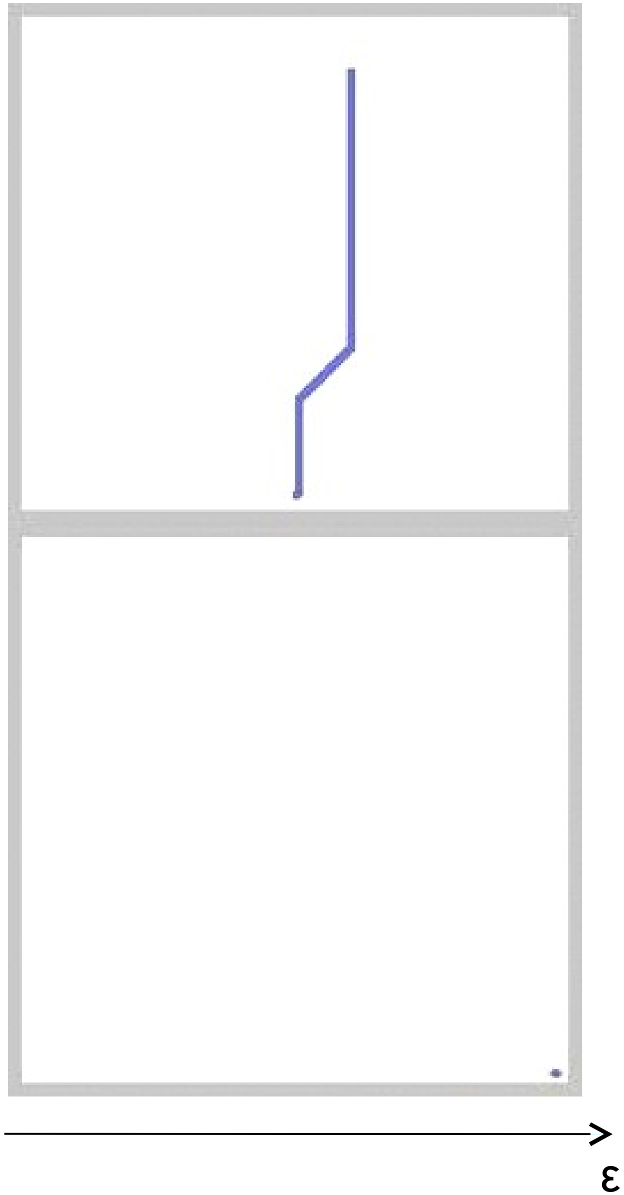


A Persistent Homology Approach to Topological Path Planning in Uncertainties

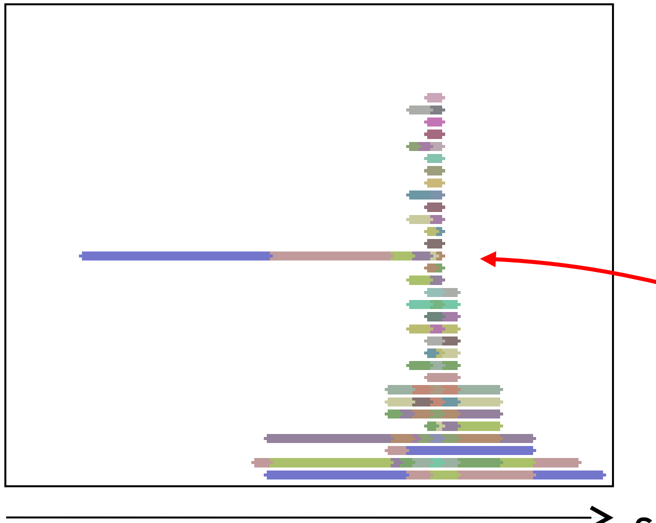
Probability map, P

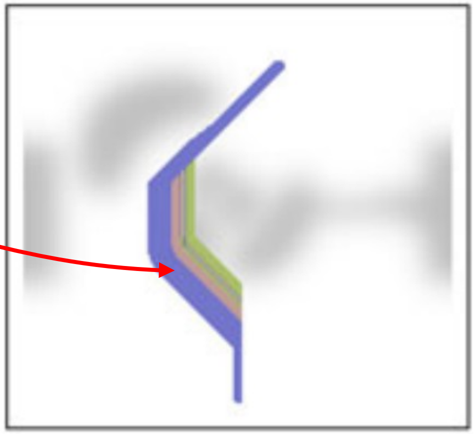


How to do path planning given a probability map?
- threshold?
At what value?



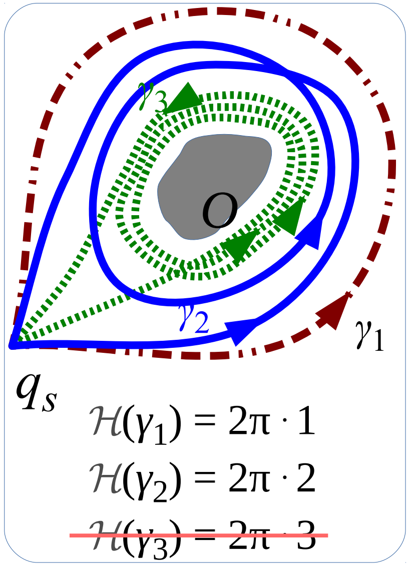
We consider $U^{\epsilon} = \{q \in W \mid P(q) \leq \epsilon\}$
for different value of ε, and how the homology classes of trajectories join and split.

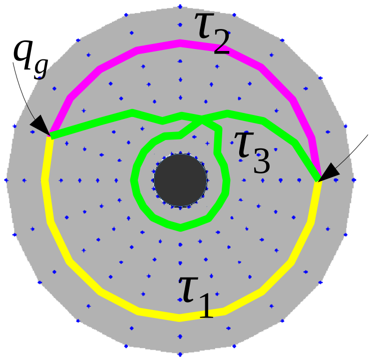




[Bhattacharya, Ghrist, Kumar. T-RO, 31(3), 578-590, 2014]

$\mathbb{Z}_2$ coefficients (homology)



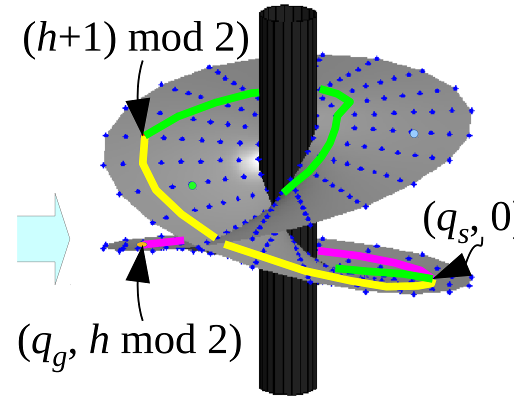


$\mathcal{V}_{\mathcal{G}}$

Advantage in graph search- based planning:

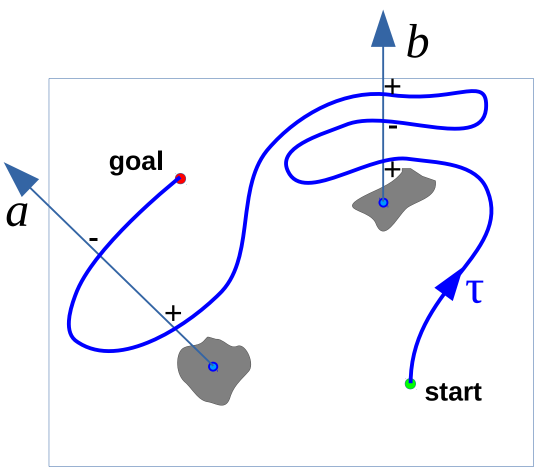
$[\tau_1] \sim [\tau_3] \nsim [\tau_2]$

Eliminate trajectories that "loop" around obstacles.



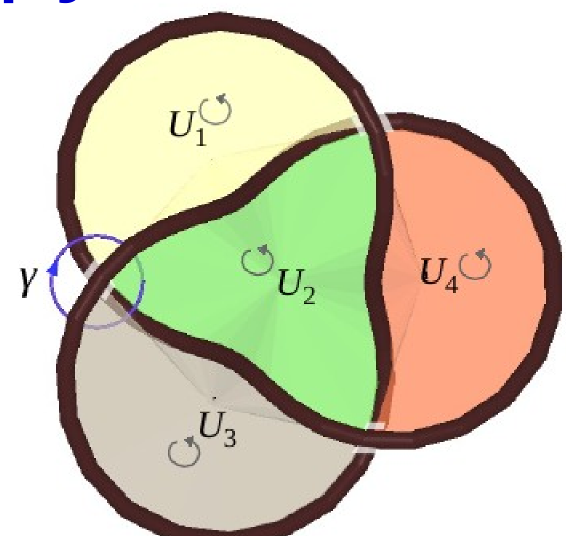
$\mathcal{V}_{\mathcal{G}_{\mathcal{H}2}}$

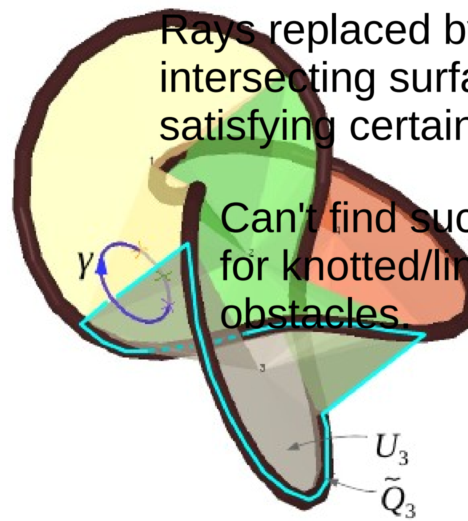
Homotopy Invariants in 3D



Recall: Homotopy invariants in 2D

$h(\tau_1) =$ " $b\, b^{-1}\, b\, a\, a^{-1}$ " $=$ " b "





Rays replaced by non-intersecting surfaces satisfying certain properties.

Can't find such surfaces for knotted/linked obstacles.

Trivial loops can have non-empty words: $h(\gamma) =$ "$u_1^{-1} u_2\, u_3^{-1}$"

Need to map these words to identity (empty word)

– Quotient group $\pi_1(X) \simeq \pi_1(X_0) * \pi_1(X_1) * \cdots * \pi_1(X_n) \;/\; N$

[Bhattacharya, Ghrist, IMAMR, 2015]

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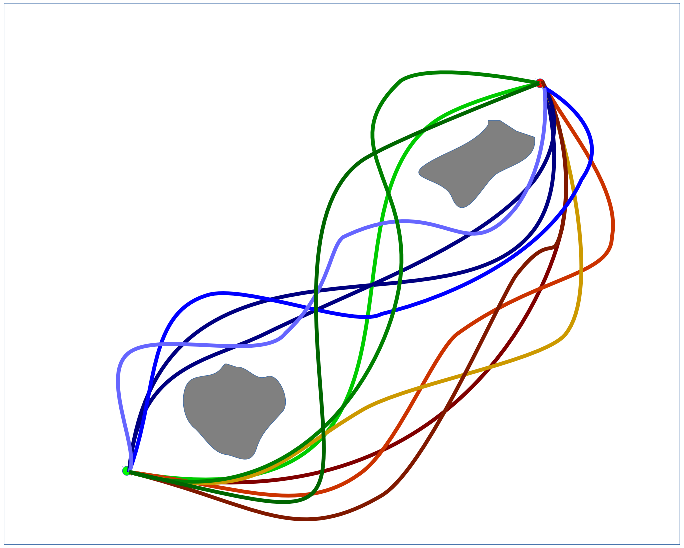
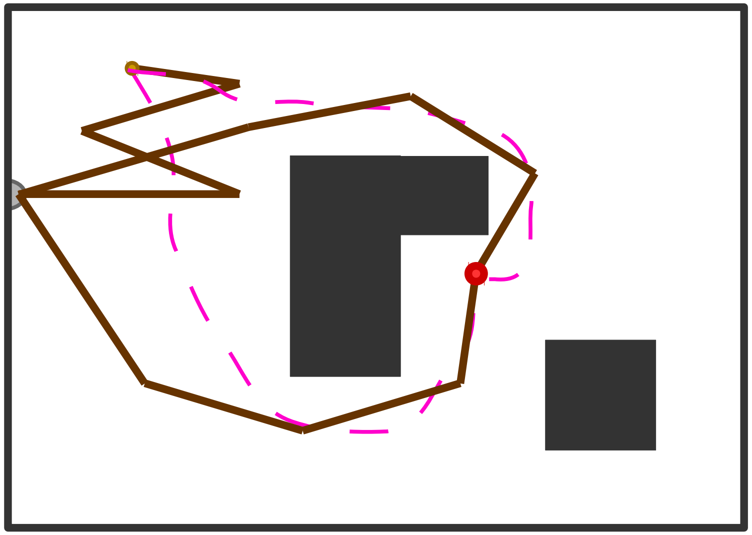
Abstraction / Dimensionality Reduction using Topology



What we have done so far is topological abstraction

- We reduced the infinite dimensional & continuous ***path/curve space*** into a finite-dimensional, searchable space
  - Involved classification of (the high-dimensional configuration space) paths based on homotopy/homology classes (topological invariants).

Configuration Spaces of Robot Arms *(finite, but high dimensional)*



Objectives

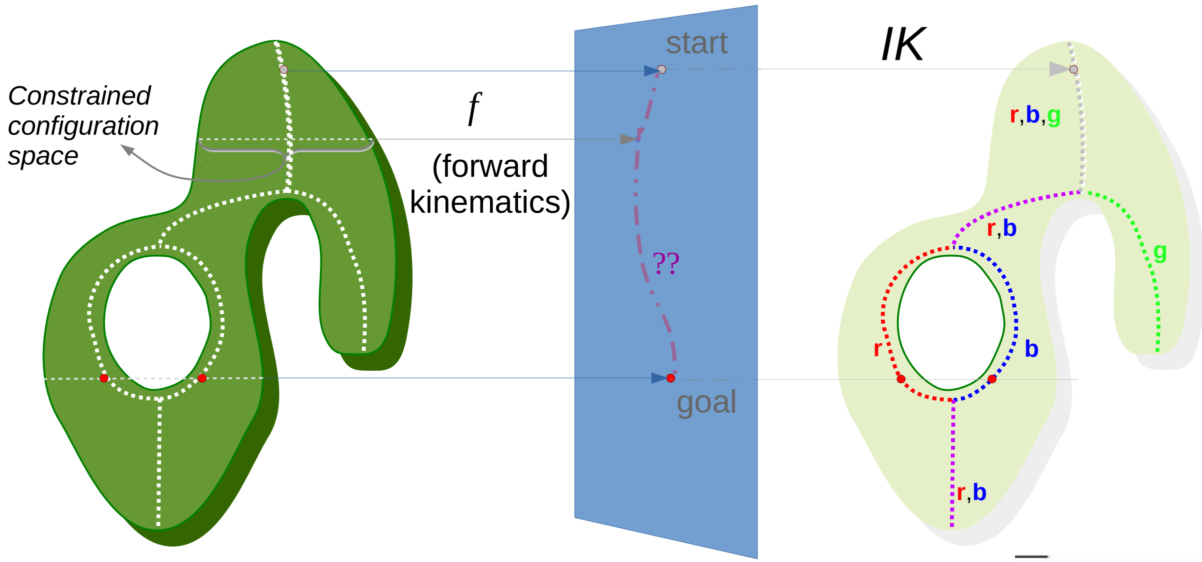
- Take end-effector to a desired target location
- Optimization of trajectory of end-effector (*e.g.*, its length)
  (We do not care where the rest of the arm is, as long as it does not intersect an obstacle!)

Challenges: (High-dimensional configuration space)

- Randomized search in configuration space gives suboptimal solution.
- Planning trajectory in end-effector space does not guarantee traversability / algorithmic completeness.
- Not sufficient to consider only the homotopy classes of arm configuration in the end-effector space (e.g., 4-bar linkage violating Grashof criterion).

Low-dimensional Sub-sampling of Configuration Space

Schematic:



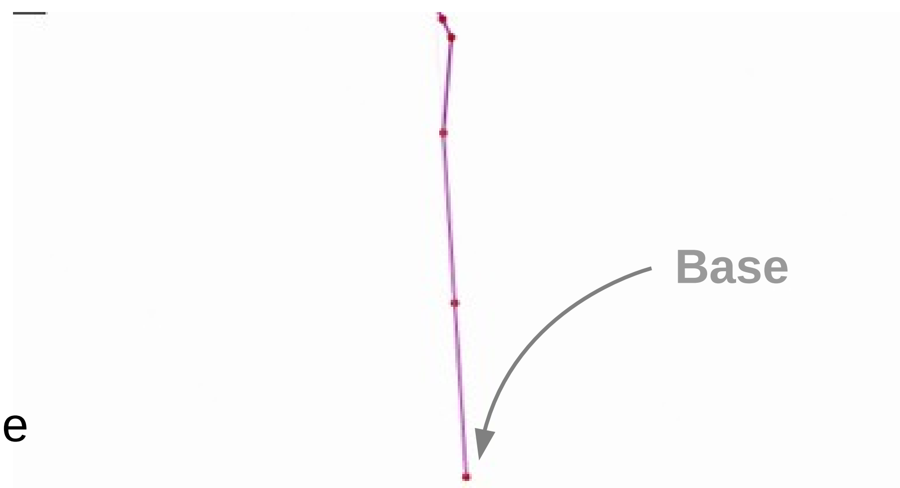
Full *n*-D configuration space (joint angle space – *assume path-connected*)

End-effector space

Approach:
Construct an explicit description of the Reeb graph of the FK function as *k*-tuple of inverse kinematics (IK) functions.

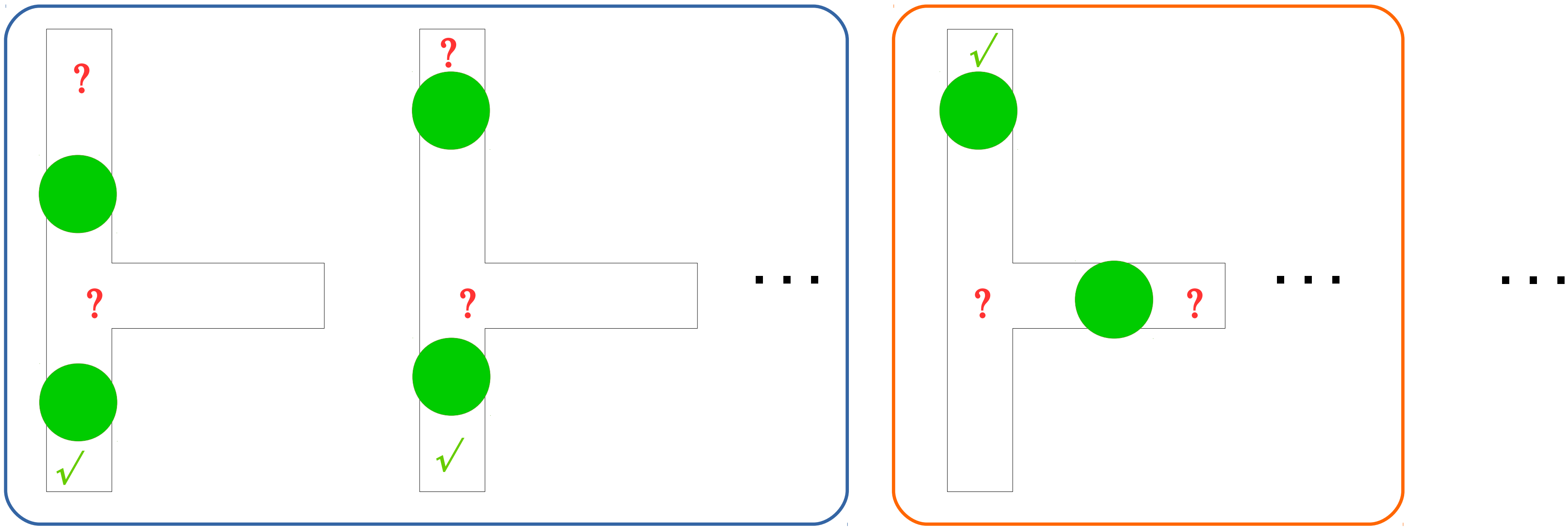
(closed-form solution for planar arm in absence of obstacles)

- Construct the Reeb graph of the FK function
  (given a fixed end-effector pos. sample a configuration from each connected component of preimage)
- Find path from the start configuration to a preimage of the goal end-effector pos. in the Reeb graph.
  *(**Guarantee:** A path in the Reeb graph exists if and only if a path exists in the configuration space between the start configuration and the pre-image of goal end-effector positions (and there is a natural projection map).)*



[Bhattacharya, Pivtoraiko, Acta Applicandae Mathematicae, 139(1):133-166, 2015.]

Topological Abstraction for Motion Planning in Pursuit-Evasion Problems



Contamination state remains the same
(maps to the same abstract state)

Topological invariant:

Connected components of the evader space and their contamination state (can be formulated as zero-th (co)homology of a ***sheaf***).

Sheaf theory allows us to place/attach additional data on a topological space.

[Ramaithitima (Tee), Srivastava, Bhattacharya, Speranzon, Kumar, (under preparation)]

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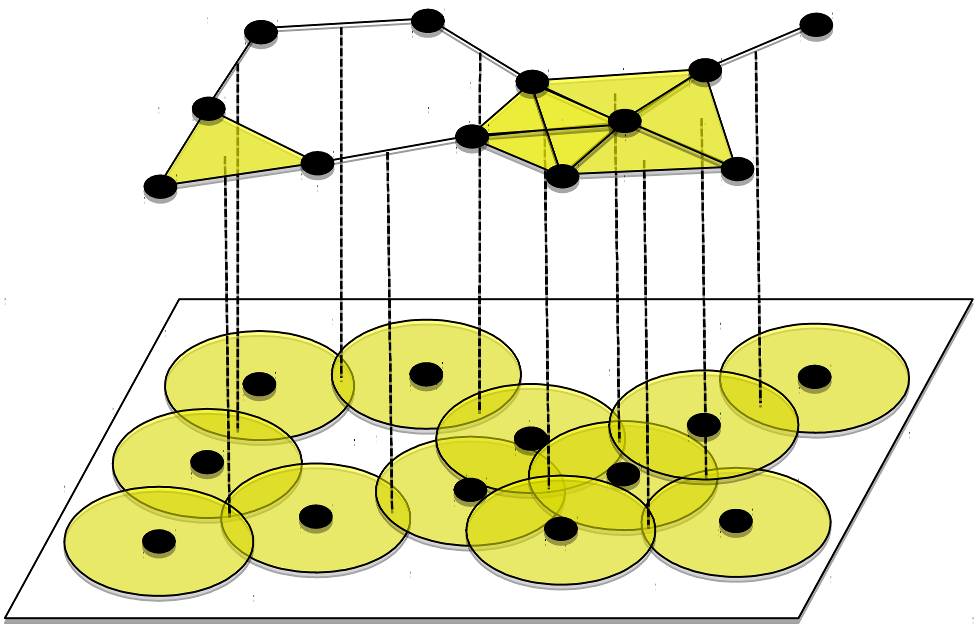
Topological Representation

An n-simplex for every (n+1)-tuple of sensors that are pair-wise neighbors.

The Rips Complex

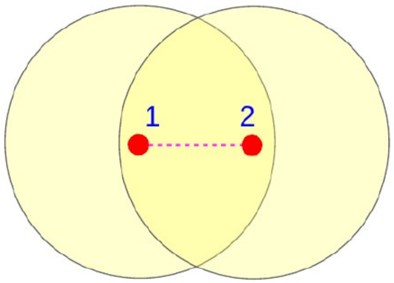


[Derenick, Kumar, Jadbabaie, ICRA, 2010]

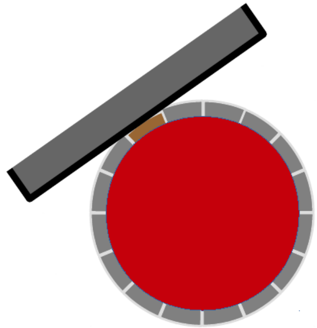
[de Silva, Ghrist, IJRR, 2006]

Rips Complex has been used to detect holes, but little research in controlling mobile sensors to attain coverage.

- Requires only local connectivity data for construction.
  - Sensor model:



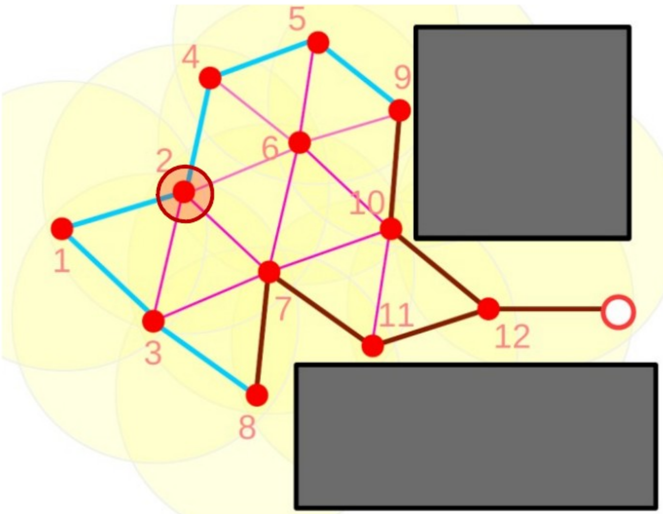
Local connectivity and noisy bearing measurement



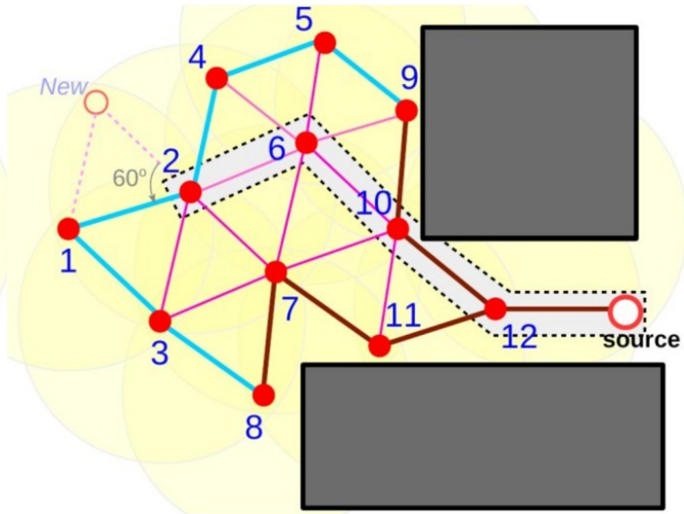
Touch sensor with coarse directionality

- Gives a faithful representation of sensor coverage
  - Can be used to ***detect holes*** in sensor coverage.
  - Very limited work in literature on actually controlling the mobile sensors.

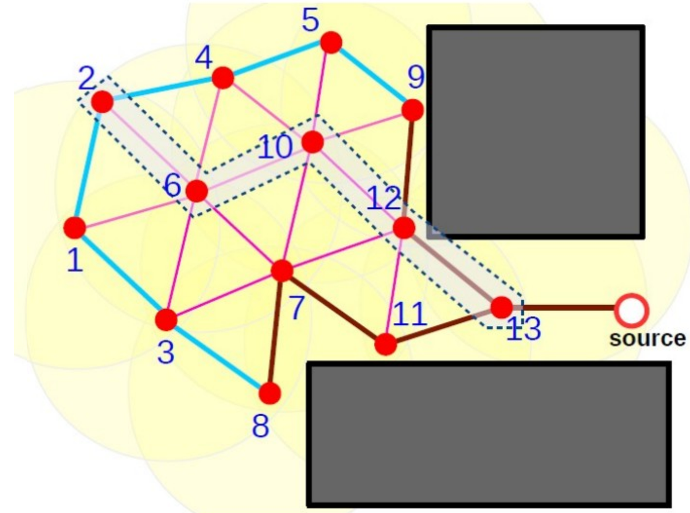
Overall Algorithm



Step 1: Identify ***a robot on the frontier*** subcomplex (closest to source in hop counts) for next deployment.

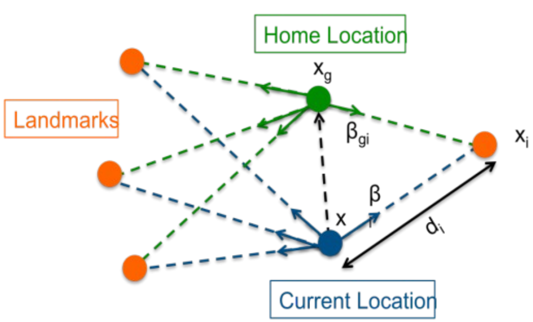


Step 2: Find a new location outside the frontier (in the local coordinate of the frontier robot), and identify ***shortest path*** through graph for robot deployment.



Step 3: "***Push***" robots along the path using bearing-only controller using other robots as landmarks.

Visual Homing (Bearing-only) Control for Robot Navigation



[R. Tron and K. Daniilidis. Technical report on Optimization-Based Bearing-Only Visual Homing with Applications to a 2-D Unicycle Model. ArXiv e-prints, February 2014.]

- Control velocity computed using
  - Bearing to landmarks (neighbors),
  - Desired home/goal location in local coordinates,
  - Landmarks can be moving.

$$c_i(x) = \beta_{gi}^T \beta_i(x)\ ,\ \varphi = \sum_{i=1}^{N} \varphi_i,\ \ \varphi_i = r_i f(c_i)$$

$$u = -\mathrm{grad}_x \varphi$$

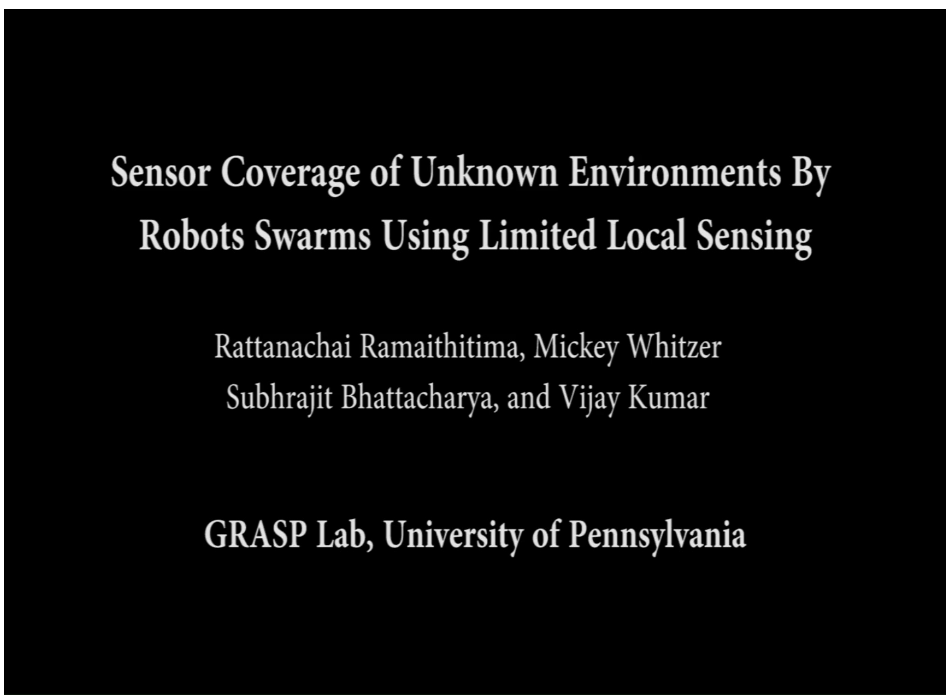
Simulation and Experiment

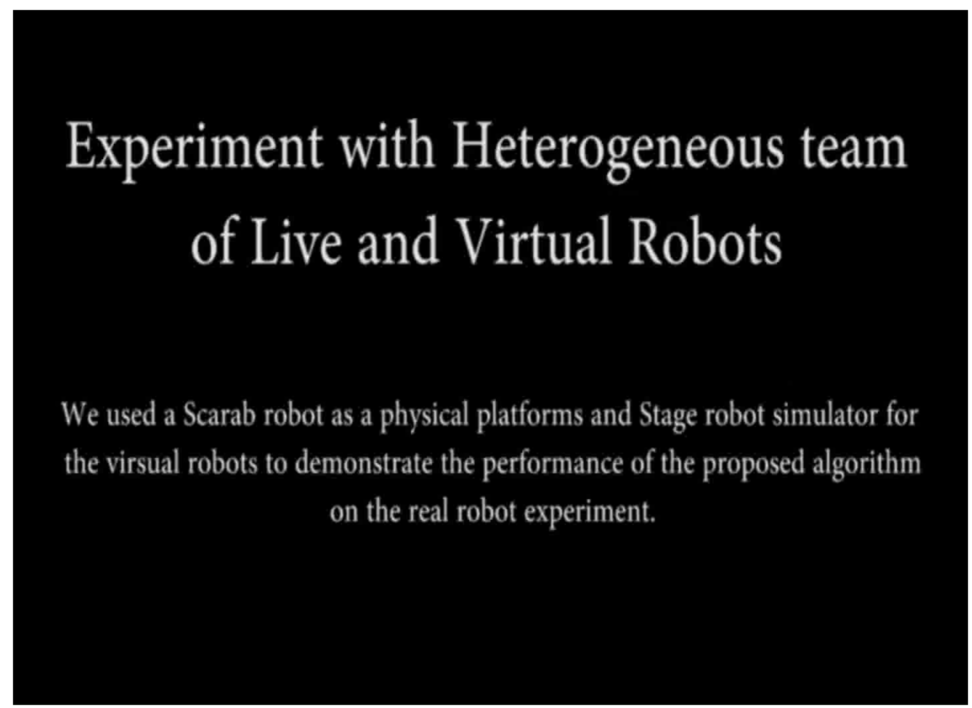
ROS Simulation:

- ROS + Stage simulation – running on a 8-core Intel processor
- Non-holonomic robots
- Single source (at the entrance to the environment), unending supply of robots.

Experiment with Real-Virtual Robots:

- Heterogeneous team of live (green) and virtual (red) robots.
- New paradigm in demonstrating swarm algorithms using limited number of live robots.
- Feedback loop between simulated robots, live robots and simulated version of live robots for coherent working of real & virtual robots.

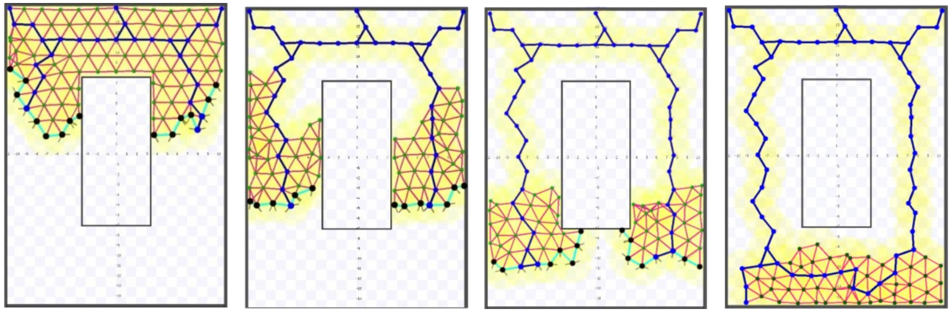




[Ramaithitima (Tee), Whitzer (Mickey), Bhattacharya, Kumar, ICRA 2015]

Applications

In unknown, GPS-denied environments, with limited sensing:



- Coarse Topological Mapping
- Topological Localization and Capture of Evaders
- Persistent Surveillance
- Establishment of Landmarks for Topological Landmark-based Navigation.

[R. Ghrist, D. Lipsky, J. Derenick, and A. Speranzon, "Topological landmark-based navigation and mapping", electronic pre-print, 2012.].

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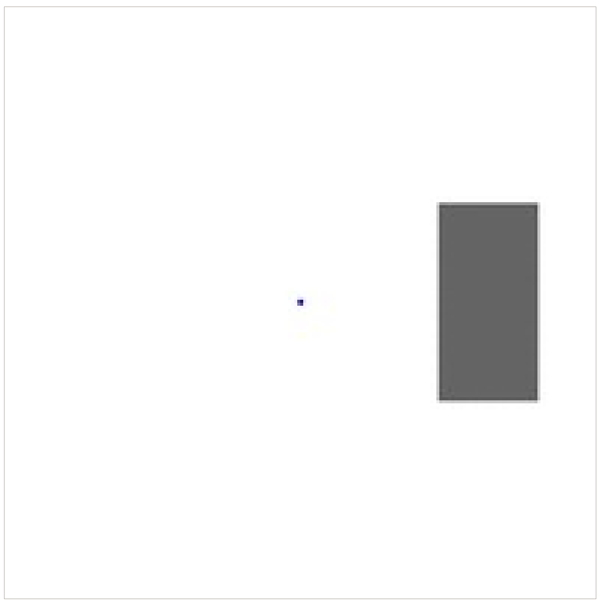
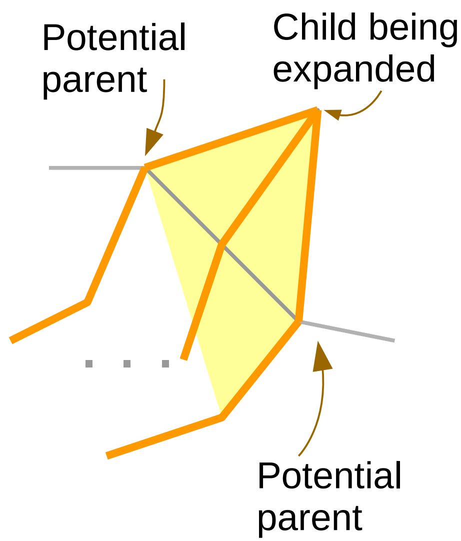
Search Algorithm For Simplicial Complexes

Dijkstra's search:



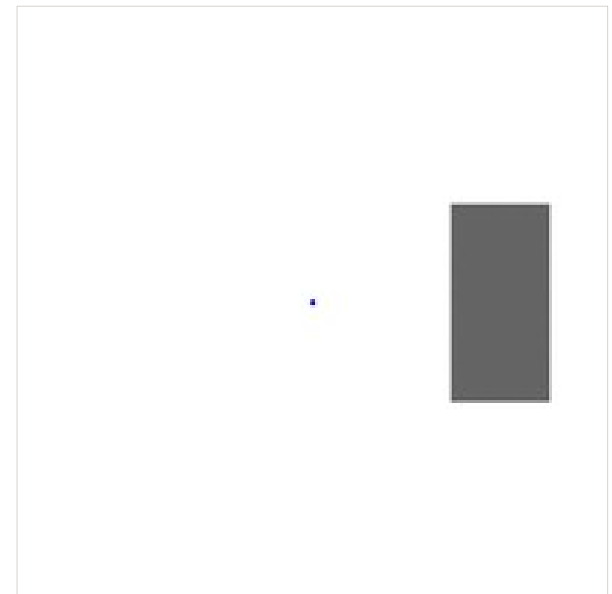
Paths are restricted to graph

Vertex "expansion":



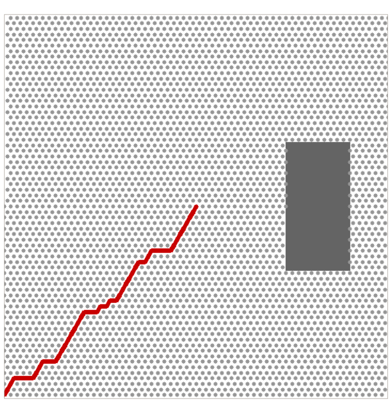
Included higher-dimensional simplices

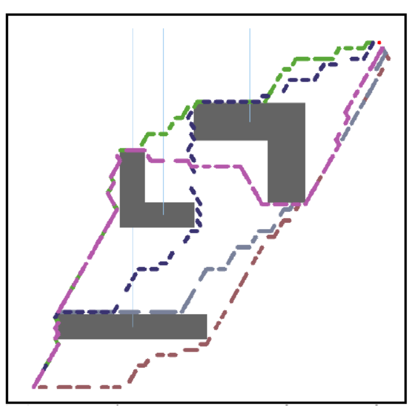
S* search:



Paths can lie in a simplicial complex (a Rips complex of the given graph)

Consequence:





Path reconstruction:
Under progress.

Conclusion

Topology helps capture **richer (and meaningful/relevant) information** about a system/configuration space (using topological invariants and representations), while **keeping the problem tractable**. Purely graph-based approaches alone fail to achieve this.

Thank you!

Questions?

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