Algorithms and Complexity
2014
Extra Mästarprov 2: Complexity

This test is given to students who failed on the ordinary Mästarprov 2. It consists of four problems. If at least two problems are solved correctly the test gives grade E. Your solutions should be handed in latest May 26th.

1. A variant of Subset Sum

In SUBSET SUM we are given integers $a_1, a_2, ..., a_n$ and an integer M. The goal is to decide if there is a subset with sum M.

a. If we ask if there is a subset with sum $\leq M$, is the problem still NP-Complete?

b. If we ask if there is a subset with sum $\geq M$, is the problem still NP-Complete?

Motivate your answer.

2. Longest Path

The problem LONGEST PATH is the problem of, given an undirected graph G, finding the length of the longest path in G. (The length of the path is the number of edges in the path.). Let us formulate a decision variant of this problem by asking if, given G and a number K as input, there is a path of length $\geq K$. Show that this problem is NP-hard be reducing the problem HAMILTONIAN PATH to LONGEST PATH (the decision variant).

3. Competent teachers

Let us assume that we have a set of n teachers and a set of m courses. All teachers have competence for teaching certain of the courses. Let us assume that we have list L_i which tells us what courses teacher i can teach. Let us formulate the problem COMPETENT TEACHERS as the problem of, given n, m, n lists $L_1, ..., L_n$ giving the teachers competences and a number K, to decide if there is a group of K teacher such that every course can be taught by at least one teacher in the group. Show that this problem is NP-hard by reducing VERTEX COVER to this problem.

4. K-Coloring

We know that the problem K-coloring of graphs is NP-Complete. Show that The problem K-coloring of graphs (for fixed K) is NP-Complete if $K \geq 3$.