Lab Manual - DD2427 Image Based Classification and Recognition

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Face Detection

Real-time face detection in multi-scale images with a boosted classifiers.

In this project you will explore the machine learning method called Adaboost by implementing it for the computer vision task of real-time face detection in images. The final classifier/detector should be capable of detecting upright frontal faces observed in reasonable lighting conditions.

Face detection is an important problem in image processing. It could potentially be the first step in many applications – marking areas of interest for better quality encoding for television broadcasting, content-based representation (in MPEG-4), face recognition, face tracking and gender recognition. In fact for this latter task computer-based algorithms out-perform humans.

During the past decade, many methods and techniques have been gradually developed and applied to solve the problem. These include vector quantization with multiple codebooks, face templates and Principal Component Analysis (PCA). The latter technique is directly related to Eigenfaces and Fisherfaces. Here we will develop a face detection system based on the well-known work of Paul Viola and Michael Jones Viola and Jones [2001a]. This basically involves the interpretation of Haar-like features in a boosted cascade, see paper on the course homepage.

The lab project

Getting started

Your task is to code up, from scratch, a face detector. This set of notes guides you through the solution you should initially follow. Fairly explicit instructions are given. To ensure the code you've written is correct, there are many debugging checks set up along the way. Please use them. If you want to follow your own design I suggest, first implement the solution described here and then incrementally change things to your own design. In fact, this is expected if you hope to get a high grade for the lab. It is important to note that the implementation described here is computationally quite efficient, however, due to the just-in-time compilation of *Matlab* it is slow to run on lots of data. But it is an easily understood implementation and lends itself to easy debugging. Also it is not hard to convert the implementation to a more *Matlab* efficient one and this document will explain how to do this once you have a version up and running.

Material to download

Training images: To get started download **TrainingImages.tar.gz**, the database of face and non-face images, from the course website. Under the directory **TrainingImages** you will find the images of face (FACES) and non-face (NFACES) images.

Test images: When you have a face detector up and running, you will want to use it to detect faces of all sizes in large images. Initially, you will perform these tasks on the set of images contained in TestImages.tar.gz; available at the course website.

Debugging information: Throughout this lab you will verify the correctness of your code by checking your output against previously computed results DebugInfo.tar.gz; once again available at the course website.

Task I - Integral image and feature computation

Initially, you will write the functions to compute the Haar-like features on which the face detector is based, see figure 2. Let I(x, y) represent the intensity of a pixel at (x, y) and B(x, y, w, h) the sum of the pixel intensities in a rectangular region. This rectangular region has width w and height h

pixels and (x, y) are the coordinates of its top left coordinate. So formally

$$B(x, y, w, h) = \sum_{x'=x}^{x+w-1} \sum_{y'=y}^{y+h-1} I(x', y')$$
(1)

Each Haar-like feature is formed from adding and subtracting sums from



Figure 1: The features used are constructed from the sum of pixel intensities within rectangular regions. In this work a rectangular region is parametrized by the coordinates of its top left hand corner and its width and height.

different rectangular regions. So for instance features of type I and type II respectively have the form

$$B(x, y, w, h) - B(x, y + h, w, h)$$
 and $B(x + w, y, w, h) - B(x, y, w, h)$

These Haar-like features can be computed quickly from the image's integral



Figure 2: The four type of features used in the *Viola & Jones* system and consequently this lab. The figure shows how they are parametrized by the four postive integers (x, y, w, h). The sum of pixels in white rectangles are subtracted from those in the black rectangles.

image. To refresh your memory, the integral image is defined as

$$I'(x,y) = \sum_{y'=1}^{y} \sum_{x'=1}^{x} I(x',y')$$
(2)

Without further ado then let's get started with the lab. As a word of advice, you should create a separate directory (subdirectories) to contain your code.

2.1 Initial image processing

To begin you will write a function LoadImage.m. It will take as input the filename of an image and return two arrays. The first corresponds to a normalized version of the pixel data of the image and the second to its integral image. This function will contain three parts.

Program 1: function [im, ii_im] = LoadIm(im_fname)

Read in image: The Matlab function imread can be used to do this. *Remember to cast the loaded data to* double.

Image normalization: You will want your face detector to have some invariance to illumination changes. Thus you have to normalize the pixel values by applying this transformation to your image:

$$I(x,y) = \frac{I(x,y) - \mu}{\sigma}$$
(3)

where μ is the average intensity value of the image and σ is the standard deviation of the pixel intensity values. (Matlab functions mean and std can compute these values.) Note you will run into problems if σ equals zero. In this case you can either decide not to divide by σ or add a small value to σ .

Compute the integral image: This can be done efficiently using the Matlab function cumsum.

Sanity Check:

Is data in im normalized? Check that the average intensity of im is 0 and that its standard deviation is 1.

Does ii_im contain the correct values? For instance check that ii_im(y, x) equals sum(sum(im(1:y, 1:x))) for different values of x and y.

Debug Point: Run your function LoadImage.m on the the image face00001.bmp from the FACES directory. The matrices you calculate should match those in DebugInfo/debuginfo1.mat. Check this with

```
>> dinfo1 = load('DebugInfo/debuginfo1.mat');
>> eps = 1e-6;
>> s1 = sum(abs(dinfo1.im(:) - im(:)) > eps)
>> s2 = sum(abs(dinfo1.ii_im(:) - ii_im(:)) > eps)
```

If everything is correct then s1 and s2 should both be equal to zero. When you display im and ii_im, they should look as in figure 3.



image integral image

Figure 3: The original image and its integral image when computed from the original image normalized.

2.2 Computation of the Haar-like features

You now have the code to load an image, normalize it and compute its integral image. The next stage is to *efficiently* compute the Haar-like features. The steps required to write this code are presented here.

2.2.1 Sum of pixel values within a rectangular region

The first step is to write a function ComputeBoxSum.m that computes the sum of the pixel values within a rectangular area using the integral image. A rectangular region is defined by 4 numbers (x, y, w, h) as shown in figure 1. Note this function is a short one.

Program 2: function A = ComputeBoxSum(ii_im, x, y, w, h)

Use the integral image, ii_im , to compute the sum of the pixel values in the original im in the rectangular region defined by (x, y, w, h), as in equation (1).

Sanity Check: Compute B(x, y, w, h) using sum(sum(im(y:y+h-1, x:x+w-1))). Check your function ComputeBoxSum(ii_im, x, y, w, h) produces the same output.

The function ComputeBoxSum.m forms the basis for the evaluation of the four features types used in this lab. And these are what you will now compute.

2.2.2 Feature computations

Each feature is defined by its type and four numbers (x, y, w, h). (x,y) is the coordinate of the upper left corner of the feature. w is the width and h is the height of the sub rectangular regions from which the feature is constructed. So in this case type I features have total width w and total height 2h while type IV features have total width 2w and total height 2h, see figure 2. Your task is to write the four different functions to compute these four different features which take the integral image and (x, y, w, h) as input.

Program 3: function f = FeatureTypeI(ii_im, x, y, w, h)

Use the integral image and the function ComputeBoxSum to compute

 $F_1(x, y, w, h) = B(x, y, w, h) - B(x, y + h, w, h)$

Sanity Check:

Compute $F_1(x, y, w, h)$ from im using the function sum and check your function FeatureTypeI(ii_im, x, y, w, h) produces the same results.

Program 4: function f = FeatureTypeII(ii_im, x, y, w, h)

Use the integral image and the function ComputeBoxSum to compute $F_2(x, y, w, h)$.

Sanity Check:

Compute $F_2(x, y, w, h)$ from im using the function sum and check your function FeatureTypeII(ii_im, x, y, w, h) produces the same results.

Program 5: function f = FeatureTypeIII(ii_im, x, y, w, h)

Use the integral image and the function ComputeBoxSum to compute $F_3(x, y, w, h)$.

Sanity Check:

Compute $F_3(x, y, w, h)$ from im using the function sum and check your function FeatureTypeIII(ii_im, x, y, w, h) produces the same results.

Program 6: function $f = \text{FeatureTypeIV}(\text{ii_im}, x, y, w, h)$ Use the integral image and the function ComputeBoxSum to compute $F_4(x, y, w, h)$. **Sanity Check:** Compute $F_4(x, y, w, h)$ from im using the function sum and check your function FeatureTypeIV(ii_im, x, y, w, h) produces the same results.

Debug Point: Using the integral image, ii_im, computed from the image face00001.bmp, check your newly written functions with the following code. Note that you are checking the ouput of your function with values previously calculated.

```
>> dinfo2 = load('DebugInfo/debuginfo2.mat');
>> x = dinfo2.x; y = dinfo2.y; w = dinfo2.w; h = dinfo2.h;
>> abs(dinfo2.f1 - FeatureTypeI(ii_im, x, y, w, h)) > eps
>> abs(dinfo2.f2 - FeatureTypeII(ii_im, x, y, w, h)) > eps
>> abs(dinfo2.f3 - FeatureTypeIII(ii_im, x, y, w, h)) > eps
>> abs(dinfo2.f4 - FeatureTypeIV(ii_im, x, y, w, h)) > eps
```

2.3 Enumerate all features

At this point you have written code to calculate each feature type of a certain size and at a certain position. Now you have to enumerate all the different possible position and sizes of a feature type that can be computed within the 19×19 image patch. The latter sentence means the support of the entire feature must be included entirely within the image. Thus, for example, features of type II can have starting positions and sizes enumerated by:

for w = 1:floor(W/2)-2
for h = 1:H-2
for x = 2:W-2*w
for y = 2:H-h
....

Figure 4 displays a small subset of all the possible type II features. Now write a function EnumAllFeatures.m which enumerates all the features given the width W and height H of the image.

Program 7: function all_ftypes = EnumAllFeatures(W, H)

Write a function that enumerates all the features. Keep a record of these features in the matrix all_ftypes. all_ftypes will have size $nf \times 5$ where nf is the number of features. Each row is a description of the feature and has the form (type, x, y, w, h) where type is either 1, 2, 3, 4 corresponding to the feature type. While the rest of the numbers are the starting position and size of the feature. Tip allocate the memory for all_ftypes at the start. Note you can declare an array that has too many rows and trim it at the end when you know the exact number of features you have declared.

Sanity Check:

Check the limits of the for loops used to define all the different features and that all the features have support within the 19×19 image. Do this by checking that for every feature defined $x+w-1 \leq W$ and $y+h-1 \leq H$. nf should have a value around 32,746.



Figure 4: A small subset of different possible features of type II.

Program 8: function fs = ComputeFeature(ii_ims, ftype)

The inputs of this function are a cell array ii_ims and the parameters of a feature ftype. Each element of ii_ims is an integral image. Now write the code to extract the feature defined by ftype from each integral image defined in ii_ims. The output is stored in an array fs of size 1×length(ii_ims).

Debug Point: Load and compute the integral image for the first 100 images in the directory 'FACES', (i.e. face00001.bmp, ..., face00100.bmp). Store the integral images in a cell array ii_ims. Check your newly written function produces the same output as a version we have written.

```
>> dinfo3 = load('DebugInfo/debuginfo3.mat');
>> ftype = dinfo3.ftype;
>> sum(abs(dinfo3.fs - ComputeFeature(ii_ims, ftype)) > eps)
```

2.4 Vectorize your code

You have now written debugged code that computes all the features. However, in its current form the code is not very efficient computationally (w.r.t. *Matlab*). Therefore, in this subsection you will write code to vectorize the operations. You are aiming to perform the feature computation as a matrix multiplication. That is if given an integral image *ii_im* then the computation of a feature can be written as

ii_im(:)' * ftype_vec

where ftype_vec is a column vector. To achieve this first write a function VecBoxSum.m that computes the correct b_vec to compute the sum of the pixel intensities in a rectangle defined by (x, y, w, h) in the form just described.

Program 9: function b_vec = VecBoxSum(x, y, w, h, W, H)

If W and H are the dimensions of the integral image then this function returns the column vector b_vec which will be zeros except for 4 elements such that

```
ii_im(:)'*b_vec equals ComputeBoxSum(ii_im, x, y, w, h)
```

Sanity Check:

Given the integral image ii_im of face00001.bmp, calculate

```
>> b_vec = VecBoxSum(x, y, w, h, 19, 19);
>> A1 = ii_im(:)' * b_vec
>> A2 = ComputeBoxSum(ii_im, x, y, w, h)
for differing values of x,y,w and h. Check each A1 equals A2.
```

Given the ability to compute the correct **b_vec** to apply to an integral image to compute the sum of the pixels in rectangular image, it is then easy to compute the **b_vec** needed to be applied to an integral image to compute features defined by **ftype**. This is what you should do in the function **VecFeature.m** that you will write next. Program 10: function ftype_vec = VecFeature(ftype, W, H)

In this function, calculate the column vector required to compute the response for a feature defined by ftype. So for instance features of type I can be calculated with

```
ftype_vec=VecBoxSum(x, y, w, h, W, H)-VecBoxSum(x, y+h, w,
h, W, H)
```

Notice how this is analogous to the functions you wrote in previous subsections. Note that you can use the *Matlab* switch function to check which feature type is defined by ftype(1).

Sanity Check:

Given the integral image ii_im of face00001.bmp, calculate

```
>> ftype_vec = VecFeature([1, x, y, w, h], 19, 19);
>> A1 = ii_im(:)' * ftype_vec
>> A2 = FeatureTypeI(ii_im, x, y, w, h)
```

for differing values of x,y,w and h and check A1 equals A2. Try similar calculations for the different feature types 2, 3 and 4.

You can now generate ftype_vec for any feature type. The next task is to compute a feature matrix fmat. The columns of this matrix are the ftype_vec for each feature defined by all_ftypes. Write the function VecAllFeatures.m to compute this.

```
Program 11: function fmat = VecAllFeatures(all_ftypes, W, H)
```

This function will generate the column vectors used to generate each feature defined in all_ftypes. It will return an array fmat of size $W*H \times nf$ where each column corresponds to a feature. Note all you need to do is call VecFeature the appropriate number of times and store the output in fmat.

Your final task in this section is to replicate the function ComputeFeature.m with a new function VecComputeFeature.m. This time round the inputs are an array ii_ims containing the integral images you want to process and ftype_vec the column vector defining your feature. Program 12: function fs = VecComputeFeature(ii_ims, ftype_vec)

The inputs of this function are an array ii_ims of size ni \times W*H and ftype_vec is a column vector containing the description of a feature. Each row of ii_ims is an integral image. Now write the code to extract the feature defined by ftype_vec from each integral image defined in ii_ims. The output is stored in an array fs of size size(ii_ims, 1)×1.

Sanity Check:

Given the integral image ii_im of the first 100 face images face00001.bmp, face00002.bmp, ... and check that if you calculate

```
>> fs1 = VecComputeFeature(ii_ims, fmat(:, 1));
>> fs2 = ComputeFeature(ii_ims1, all_ftypes(1, :));
```

then fs1 and fs2 are equal. Note that ii_ims1 contains the same data as ii_ims but just in a cell array and the images are stored as 2d arrays. Try similar calculations for different features.

Note you will probably not use this function again as it is only one line of code, it will be better to directly write this line into any function.

2.5 Extract features and training data

A list of ni numbers randomly chosen images in the directory 'FACES' can be obtained with the following code:

```
face_fnames = dir('FACES');
aa = 3:length(face_fnames);
a = randperm(length(aa));
fnums = aa(a(1:ni));
```

These numbers correspond to the images you will use for training so for instance the first image will be:

im_fname = ['FACES/', face_fnames(fnums(1)).name];

The next function you will write is to load all the images you will use for training and also to compute fmat for the features you have defined.

Program 13: function LoadSaveImData(dirname, ni, im_sfn)

Choose images: Randomly pick **ni** images in the directory **dirname** as described in the text.

Load data: For each image use the function LoadIm to load it and compute its integral image. Then store each integral image as a row in an array called *ii_ims*.

Save the image data: Save the details of the training image data in the file im_sfn:

save(im_sfn, 'dirname', 'fnums', 'ii_ims');

Sanity Check:

Run your newly written function on the 'FACES' directory with ni = 100. Check that the code runs smoothly.

Program 14: function ComputeSaveFData(all_ftypes, f_sfn)

Compute feature vectors: Use VecAllFeatures to compute fmat from all_ftypes and set the values of W and H to 19.

Save the feature info: Save the details of the features you will use in the file **f_sfn**:

save(f_sfn, 'fmat', 'all_ftypes', 'W', 'H');

Sanity Check:

Check that the code runs smoothly.

Debug Point: Set dirname to the FACES directory. Read in the file debuginfo4.mat and follow the instructions below to check the output of your newly written functions.

```
>> dinfo4 = load('DebugInfo/debuginfo4.mat');
>> ni = dinfo4.ni;
>> all_ftypes = dinfo4.all_ftypes;
>> im_sfn = 'FaceData.mat';
>> f_sfn = 'FeaturesToMat.mat';
>> rng(dinfo4.jseed);
>> LoadSaveImData(dirname, ni, im_sfn);
>> ComputeSaveFData(all_ftypes, f_sfn);
```

Then check that dinfo4.fmat equals the fmat you calculated and similarly for dinfo4.ii_ims.

You are now ready to write the final function in this section. Basically, in this function you compute and save the training data extracted from both the face images and the non-face images.

Program 15: function GetTrainingData(all_ftypes, np, nn)

Write a function which calls LoadSaveImData twice - once for data extracted from the face images and the second time round for the non-face images. np is the number of face images used and nn is the number of non-face images used. In the two calls to LoadSaveImData, set im_sfn to FaceData.mat and NonFaceData.mat respectively. The function should also call ComputeSaveFData and set f_sfn to FeaturesToUse.mat.

Now to create the data required for training the face detector run the following code:

```
>> dinfo5 = load('DebugInfo/debuginfo5.mat');
>> np = dinfo5.np;
>> nn = dinfo5.nn;
>> all_ftypes = dinfo5.all_ftypes;
>> rng(dinfo5.jseed);
>> GetTrainingData(all_ftypes, np, nn);
```

Note that it may take up to several minutes for the function GetTrainingData to complete, depending on the speed of your machine and the effeciency of your code. Once it has completed then load your saved files into *Matlab* with the following commands:

```
>> Fdata = load('FaceData.mat');
>> NFdata = load('NonFaceData.mat');
>> FTdata = load('FeaturesToUse.mat');
```

Fdata, NFdata and FTdata are structures and contain the data you have just saved. So, for instance, the name of the directory containing the face training images is accessed with Fdata.dirname, while the integral image data is accessed with Fdata.ii_ims.

If you have successfully reached this point then you are ready to get this part of the lab, **Check I**, signed off by one of the Teaching Assistants (TA). See the accompanying form to see what the TA will expect you to demonstrate and answer questions on.

Task II - Boosting to learn a strong classifier

Boosting is a process of forming a strong strong classifier through the linear combination of weak ones. In the context of Viola-Jones face detection, a binary classification task, the weak classifiers are derived from the extracted set of features.

The details of the *AdaBoost* algorithm are given in algorithm 1. The core idea behind the use of AdaBoost is the application of a weight distribution to the training examples and the modification of the distribution during each iteration of the algorithm. At the beginning the weight distribution is flat, but after each iteration of the algorithm each of the weak classifiers returns a classification on each of the sample-images. If the classification is correct the weight on that image is reduced (seen as an easier sample), otherwise there is no change to its weight. Therefore, weak classifiers that manage to classify difficult sample-images (i.e. with high weights) are given higher weighting in the final strong classifier.

Now let's go and implement the *AdaBoost* algorithm to build a face detector.

3.1 Defining & learning a weak classifier

At this moment you have extracted many Haar-like features from many training images. How can these simple features be used to build weak classifiers from which we will build the strong classifier? We choose the weak classifiers to have a very simple form. In the mathematical description of the algorithm we denote the feature vector extracted when all the Haarlike filters are applied to an image \mathbf{x} as $\mathbf{f}_{\mathbf{x}} = (f_1(\mathbf{x}), \dots, f_N(\mathbf{x}))$ where N is the total number of features extracted. Then one weak classifier $h(\cdot)$ with parameters $\mathbf{\Theta} = (j, p, \theta)$

$$h(\mathbf{f}_{\mathbf{x}}; \boldsymbol{\Theta}) = h(\mathbf{f}_{\mathbf{x}}; j, p, \theta) = \begin{cases} 1 & \text{if } p f_j(\mathbf{x}) (8)$$

where $f_j(\mathbf{x})$ is the response of the *j*th feature when applied to image \mathbf{x} . This is the type of weak classifier you will use in the lab. However, you are free to define another form of weak classifier when you make changes to the default detector. In this subsection you will write code to automatically set the parameters p, θ associated with the weak classifiers of this form when there is a weight associated with misclassifying each training algorithm. Algorithm 2 describes a very simple way to do this.

Before finding the parameters of a weak classifier we will ensure that you understand what is meant when referring to feature responses. Remember, the structure FTdata contains the matrix fmat. Each column of this matrix

- **Input:** A set of feature vectors $\{\mathbf{f}_{\mathbf{x}_1}, \ldots, \mathbf{f}_{\mathbf{x}_n}\}$ extracted from each example image \mathbf{x}_i and associated labels $\{y_1, \ldots, y_n\}$ where $y_i \in \{0, 1\}$. $y_i = 0$ denotes a negative example and $y_i = 1$ a positive one. m is the number of negative examples. A postive integer T which defines the number of weak classifiers used in the final strong classifier.
- **Output:** A set of parameters $\{\Theta_1, \ldots, \Theta_T\}$ associated with the weak classifier $h(\cdot)$ and a set of weights $\alpha_1, \ldots, \alpha_T$ which define a stong classifier of the form:

$$H(\mathbf{f}_{\mathbf{x}}) = \begin{cases} 1 & \text{if } \left(\sum_{t=1}^{T} \alpha_t h(\mathbf{f}_{\mathbf{x}}; \boldsymbol{\Theta}_t) \right) \ge \frac{1}{2} \sum_{t=1}^{T} \alpha_t \\ 0 & \text{otherwise} \end{cases}$$
(4)

Steps of Algorithm:

Initialize the n weights to:

$$w_i^{(1)} = \begin{cases} (2m)^{-1} & \text{if } y_i = 0\\ (2(n-m))^{-1} & \text{otherwise} \end{cases}$$
(5)

for $t = 1, \ldots, T$ do

- Normalize the weights so they sum to one: $w_i^{(t)} = \frac{w_i^{(t)}}{\sum_j w_j^{(t)}}$.
- For each feature j train a weak classifier h restricted to using this feature that tries to minimize the error

$$\epsilon_j = \sum_i w_i^{(t)} \left| h(\mathbf{f}_{\mathbf{x}_i}; j, \boldsymbol{\theta}_j) - y_i \right| \tag{6}$$

- Choose the weak classifier with the lowest error: $j^* = \arg\min \epsilon_j$
- Set $\boldsymbol{\Theta}_t = (j^*, \boldsymbol{\theta}_{j^*})$ and $\epsilon_t = \epsilon_{j^*}$.
- Update the weights:

$$w_{i}^{(t+1)} = w_{i}^{(t)} \beta_{t}^{1-|h(\mathbf{f}_{\mathbf{x}_{i}}; \Theta_{t}) - y_{i}|}, \quad \text{with } \beta_{t} = \frac{\epsilon_{t}}{1 - \epsilon_{t}}.$$
 (7)

• Set $\alpha_t = \log \frac{1}{\beta_t}$.

end for

corresponds to a column vector which when multiplied with the integral image of an image (represented as a row vector) produces the value of applying a particular Haar-like feature to the original image. The structures Fdata and NFdata contain the integral images extracted from the face and nonface training images. Using these integral images and one column of fmat, say fmat(:, 12028), one can compute the feature responses for all the images for this feature type with a simple matrix multiplication - remember VecComputeFeatures.m. From this data create a vector of responses fs for the positive and for the negative examples. Next use hist to compute the histogram of the feature responses from the face images and from the nonface images. Display the histograms on the same figure. You should plot curves that look like those in figure 5.

Now your task is to write the function LearnWeakClassifier that implements algorithm 2. It takes as input the vector of weights associated with each training image, a vector containing the value of a particular feature extracted from each training image and a vector of the labels associated with each training image. The outputs are then the learnt parameters of the weak classifier and its associated error.

Program 16: function [theta, p, err] =
 LearnWeakClassifier(ws, fs, ys)

Compute the threshold and parity as described in algorithm 2.

Sanity Check:

As stated before the structure FTdata contains the feature array fmat while the structures Fdata and NFdata contain the integral images extracted from the face and non-face training images. Using these integral images and one column of fmat, say fmat(:, 12028), compute the feature responses for one ftype. Use this data to create fs and ys. Then set the weights ws as they are initialized in algorithm 2. Using this input run your newly written function to compute theta and p. You should get values similar to theta = -3.6453and p = 1.

Next use **hist** to compute the histogram of the feature responses from the face images and from the non-face images. Display the histograms on the same figure as well as the line $x = \theta$, see figure 5. You can repeat this process for different features and check that your function produces sensible results.

3.2 Display functions

Before proceeding to write a program to implement the boosting algorithm, you will write a couple of functions used for display purposes. These will

Algorithm 2 Simple weak classifier

- **Input:** A set of feature responses $\{f_j(\mathbf{x}_1), \ldots, f_j(\mathbf{x}_n)\}$ extracted by applying the feature f_j to each training image \mathbf{x}_i and associated labels $\{y_1, \ldots, y_n\}$ where $y_i \in \{0, 1\}$. A set of non-negative weights $\{w_1, \ldots, w_n\}$ associated with each image that sum to one.
- **Output:** $\theta = (p, \theta)$ and $\epsilon > 0$. θ is a threshold value and $p \in \{-1, 1\}$ is a parity value. Together they define a weak classifier of the form:

$$g(f_j(\mathbf{x}); p, \theta) = \begin{cases} 1 & \text{if } p f_j(\mathbf{x}) (9)$$

 ϵ is the value of the error associated with this classifier when applied to the training data.

(The parameters $\boldsymbol{\theta} = (p, \theta)$ along with j will then be the parameters of the weak classifier defined in equation (8).)

Steps of Algorithm:

• Compute the weighted mean of the postive examples and negative examples

$$\mu_{\rm P} = \frac{\sum_{i=1}^{n} w_i f_j(\mathbf{x}_i) y_i}{\sum_{i=1}^{n} w_i y_i}, \quad \mu_{\rm N} = \frac{\sum_{i=1}^{n} w_i f_j(\mathbf{x}_i) (1-y_i)}{\sum_{i=1}^{n} w_i (1-y_i)}$$
(10)

- Set the threshold to $\theta = \frac{1}{2}(\mu_{\rm P} + \mu_{\rm N})$.
- Compute the error associated with the two possible values of the parity

$$\epsilon_{-1} = \sum_{i=1}^{n} w_i |y_i - g(f_j(\mathbf{x}_i); -1, \theta)|, \qquad (11)$$

$$\epsilon_1 = \sum_{i=1}^n w_i |y_i - g(f_j(\mathbf{x}_i); 1, \theta)|$$
(12)

• Set $p^* = \underset{p \in \{-1,1\}}{\operatorname{arg min}} \epsilon_p$ and then $\epsilon = \epsilon_{p^*}$.



Figure 5: A simple weak classifier. The blue curve is the histogram of the feature responses for the non-face training images. The red curve those for the face images. The black line is the threshold value chosen using algorithm 2. Marked on the figure are the regions, defined by the threshold, where feature responses will be classified as face /non-face.

be extremely useful when debugging your boosting implementation and interpreting its output. The first function is to make an image representing a feature, as in figure 2, defined by the vector ftype.

Program 17: function fpic = MakeFeaturePic(ftype, W, H)

Create a matrix, fpic, of zeros of size (H, W). From the information in ftype, set the appropriate pixels to 1 and to -1.

Sanity Check: Run

fpic = MakeFeaturePic([4, 5, 5, 5, 5], 19, 19);

and then display fpic via imagesc. The feature should appear as in figure 6(a).

The strong classifier, though, consists of a weighted sum of the weak classifiers. Thus the second display function you have to write takes as input the array defining each feature, a vector **chosen_f** of postive integers that correspond to the features used in the classifier and the weights **alphas** associated with each feature/weak classifier.



Figure 6: Example output of the feature and classifier display.

3.3 Implement the Boosting algorithm

should appear as in figure 6(b).

You are now ready to write the code to implement the AdaBoost algorithm to produce a face detector. Before we start we introduce the concept of a structure (struct) in Matlab as the function BoostingAlg returns a structure, Cparams, containing the parameters of the strong classifier and those of the associated weak classifiers. The structure Cparams contains the following fields:

Cparams.alphas, Cparams.Thetas, Cparams.fmat, Cparams.all_ftypes

where alphas represents the α_t 's in equation (4) and Thetas represents the Θ_t 's in algorithm 1 which are the parameters of the weak classifiers. Thus alphas is vector of length T and Thetas is an array of size T×3 where the first column represents the features chosen, the second column the thresholds of the weak classifiers and the third column the associated parities. The other fields have already been introduced.

Program 19: function Cparams = BoostingAlg(Fdata, NFdata, FTdata, T)

Implement the boosting algorithm as described in algorithm 1. The inputs to this function are the training data obtained from the positive and negative images and the number of weak classifiers T to include in the final strong classifier. The output is then the structure representing the final classifier. Remember during training you have to learn the parameters for each weak classifier (which takes the weight of each training example into account) and then choose the one with lowest error. So you have to use the array of the integral images and the appropriate column of fmat to generate the feature responses for each feature. This whole process is repeated T times.

Sanity Check:

While debugging and writing this function only use a fraction of the features defined in FTdata.fmat as otherwise things will run very slowly. I suggest just use the first 1000 features defined in FTdata.fmat to begin with and run the command

```
Cparams = BoostingAlg(Fdata, NFdata, FTdata, 3);
```

Then use the function MakeFeaturePic to display the 3 different features selected and MakeClassifierPic to display the learned classifier. I got the results in figure 7.

Once you have written this command and think you have passed the sanity check then you should do a more exact check. Remember this is just using the first 1000 features defined in FTdata.fmat.

Debug Point: To check the output of your code, run the following commands

```
>> dinfo6 = load('DebugInfo/debuginfo6.mat');
>> T = dinfo6.T;
>> Cparams = BoostingAlg(Fdata, NFdata, FTdata, T);
>> sum(abs(dinfo6.alphas - Cparams.alphas)>eps)
>> sum(abs(dinfo6.Thetas(:) - Cparams.Thetas(:))>eps)
```

If you have successfully passed this latest check then update BoostingAlg to use all the features defined in FTdata.fmat. Before you run this function on all this data try and optimize your code slightly so that it runs relatively efficiently in the inner most loop. Now just run BoostingAlg with T set to 1. The feature my code selected is shown in figure 8. If this seems to be working then run this debug check and go get yourself a cup of coffee! It may take from 10-30 minutes to run depending on your machine and the efficiency of your code.



Figure 7: The initial 3 features chosen by boosting (left to right) and the final strong classifier. The final strong classifier in this example consists of 3 features and is the rightmost figure. The first feature chosen by boosting is the leftmost one. These are the features chosen when you use a very small pool features for training.

```
>> dinfo7 = load('DebugInfo/debuginfo7.mat');
>> T = dinfo7.T;
>> Cparams = BoostingAlg(Fdata, NFdata, FTdata, T);
>> sum(abs(dinfo7.alphas - Cparams.alphas)>eps)
>> sum(abs(dinfo7.Thetas(:) - Cparams.Thetas(:))>eps)
```

Once you have computed Cparams, save it using the command save.



Figure 8: The initial features chosen by boosting (left to right) and the final strong classifier. The final strong classifier in this example consists of 10 features and is the rightmost figure. The first feature chosen by boosting is the leftmost one.

If you have successfully reached this point then you are ready to get this part of the lab, **Check II**, signed off by one of the Teaching Assistants (TA). See the accompanying form to see what the TA will expect you to demonstrate and answer questions on.

Task III - Classifier evaluation

Congratulations you have constructed your first boosted face detector! But is it any good? In this part of the lab you will investigate how good it is. You will do this via the ROC-curve (Receiver Operator Characteristic).

However, before computing the ROC-curve you have to write a function that can apply your strong classifier.

Program 20: function sc = ApplyDetector(Cparams, ii_im)

This function applies your strong classifier to a test image of size 19×19 . It takes as input the parameters of your classifier **Cparams** and the integral image, ii_im, computed from a normalized version of your test image. It extracts each feature used in the strong classifier from the test image and then computes a weighted sum of the weak classifier outputs. That is the function returns the score

$$\sum_{t=1}^{T} \alpha_t \, h(\mathbf{f}; \boldsymbol{\Theta}_t)$$

Sanity Check:

Run your new function on the image ${\tt face00001.bmp}.$ I got a score of around ${\tt 9.1409}.$

Now we introduce some concepts which are used in the definition of the ROC-curve. Look at table 1 to review the definitions of true-positive, false-positive etc. From these definitions the definition of *false positive rate* and

Label	Predicted Class	True Class
true-positive (tp)	Positive	Postive
false-positive (fp)	Positive	Negative
true-negative (tn)	Negative	Negative
false-negative (fn)	Negative	Positive

Table 1: A classifier predicts the class of a test example. If the true class is known then the test example can be labelled according to the above table.

true positive rate are based

true positive rate = tpr =
$$\frac{n_{\rm tp}}{n_{\rm tp} + n_{\rm fn}}$$
 (13)

false positive rate = fpr =
$$\frac{n_{\rm fp}}{n_{\rm tn} + n_{\rm fp}}$$
 (14)

where n_{tp} is the number of true-positives etc. The number of true-positives and false-positives will vary depending on the threshold applied to the final strong classifier. The ROC-curve is a way to summarize this variation. It is a curve that plots **fpr Vs tpr** as the threshold varies from $-\infty$ to $+\infty$. (NOTE the default AdaBoost threshold is designed to yield a low error rate on the training data.) From this curve you can ascertain what loss in classifier specifity you will have to endure for a required accuracy. With this knowledge you can write the function to compute the ROC curve on the training data you didn't use when learning your classifier.



Figure 9: **ROC curve computed from the images omitted from training**. The classifier usesd consisted of 10 weak classifiers.

Program 21: function ComputeROC(Cparams, Fdata, NFdata)

Get test images: From Fdata and NFdata list the images that were used for training. Given this information then you can choose the face and non-face images that were not used for training as those to be used for testing. The command setdiff may help.

Apply detector to each test image: Run the learnt detector, using ApplyDetector, on each test image. Keep a record of the score of each image and its label (face/non-face).

Compute *true* and *false* **positive rates:** Choose a threshold to apply to the recorded scores. This predicts a labelling for each image. Check how this corresponds with the ground truth and from this compute the *true positive rate* and the *false positive rate*.

Plot the ROC curve: Let the threshold vary at fixed intervals from the minimum score value to the largest. For each threshold value keep a record of the *true positive rate* and the *false positive rate*. Then plot the *false positive rate* values Vs the *true positive rate* values.

Sanity Check: Values of fpr and tpr for large threshold values ?

Values of fpr and tpr for small threshold values ?

Check the shape of the ROC curve. Run your function. The ROC curve you plot should look something like the one in figure 9.

Debug Point: You have created the ROC-curve for your detector. Now choose the threshold of your detector such that you get a true positive rate of above 70% on the test examples. This may seem like a low number but this is to ensure a relatively low false positive rate. This threshold value should be around 6.5. Add an extra field **Cparams.thresh** to the **Cparams** structure to retain the value of the overall threshold.

If you have successfully reached this point then you are ready to get this part of the lab, **Check III**, signed off by one of the Teaching Assistants (TA). See the accompanying form to see what the TA will expect you to demonstrate and answer questions on.

Task IV - Face detection in an image

You have now learnt a classifier via boosting that detects faces. The next step is to apply this classifier to an image and see if it detects the faces it may or may not contain. Now the function ApplyDetector.m will only classify faces occupying subwindows of size 19×19 whose pixel data has been normalized to have mean 0 and standard deviation 1. However, even if an image contains a face of size 19×19 you will have to try every, or almost every, possible subwindow of the image to detect the face, see figure 10. You now have to write such a function ScanImageFixedSize.m whose inputs are the parameters of the detector and the pixel data of the image to be processed. The output will be the parameters of the bounding boxes (sub-windows) classified as faces. This will be an array of size nd×4 where nd will be the number of face detections.



Figure 10: The sliding window of ScanImageFixedSize.m will traverse different locations in the large image.

It is important to note that the variance and the mean pixel intensity of the sub-window defined by (x, y, L, L) can be computed quickly using a pair of

integral images as

$$\mu = \frac{1}{L^2} \sum_{x'=x}^{x+L-1} \sum_{y'=y}^{y+L-1} I(x',y'), \quad \sigma^2 = \frac{1}{L^2 - 1} \left(\sum_{x'=x}^{x+L-1} \sum_{y'=y}^{y+L-1} I^2(x',y') - L^2 \mu^2 \right)$$

So the mean of the sub-window can be computed from the integral image of im while the sum of squared pixel values can be computed using the integral image of the image squared (i.e. im .* im). Remember if you calculate the sum of pixel intensities in an rectangular region where the pixel values have not been normalized then

$$B(x, y, w, h) = \sum_{x'=x}^{x+w-1} \sum_{y'=y}^{y+h-1} I(x', y')$$

while this sum if the pixel data has been normalized is

$$B^{n}(x, y, w, h) = \sum_{x'=x}^{x+w-1} \sum_{y'=y}^{y+h-1} \left[\frac{I(x', y') - \mu}{\sigma} \right] = \frac{1}{\sigma} B(x, y, w, h) - \frac{wh}{\sigma} \mu$$

We introduce the superscript n to signify a quantity has been computed from normalized data. Thus for features of type I

$$F_1^n(x, y, w, h) = B^n(x, y, w, h) - B^n(x, y + h, w, h)$$

= $\frac{1}{\sigma}(B(x, y, w, h) - B(x, y + h, w, h)) = \frac{1}{\sigma}F_1(x, y, w, h)$

Repeat these calculations for the other feature types and write down the expression for each one. With this in mind you can adjust you features extracted from a non-normalized image very easily. You must do this or the weak classifiers you learnt cannot be applied sensibly to your image.

Program 22: function dets = ScanImageFixedSize(Cparams, im)

Do image processing: If necessary convert im to grayscale. Compute its square and compute the two necessary integral images.

Adapt ApplyDetector.m: We want to apply the detector to an arbitrary sub-window of size 19×19 of the large image. Remember the pixel data in the sub-window is not necessarily normalized thus you have to compensate for this fact as described in the text. Also remember that the top-left corner of the sub-window is now not necessarily at (1, 1).

Search the image Write nested for loops to vary the top-left corner of the sub-window to be classified and keep a record of the sub-windows classified as faces in the array dets which has size $nd \times 4$. Each row contains the parameters of the face sub-window.

Sanity Checks: Is the normalization correct?

Does this function replicate previous performance? If you run this function on one of the small training images you should get the same result as when you run LoadIm and then the original ApplyDetector.

5.1 Display the detection results

From the TestImages subdirectory load the image one_chris.png and scan the image for faces of size 19×19 using your newly written function.

>> im = 'TestImages/one_chris.png';
>> dets = ScanImageFixedSize(Cparams, im);

Now, of course, you would like to see the output of your detector. Thus you have to write a function that takes the bounding box information contained in dets and displays the rectangles on top of the image. The *Matlab* function rect can be used for this purpose.

Program 23: function DisplayDetections(im, dets)

Use *Matlab*'s plotting and image display functions to show the bounding boxes of the face detections.

Sanity Check:

Running this function after ScanImageFixedSize on the image one_chris.png you should get something similar the results in figure 11 (a).



(a) search over one scale (b) search over multiple scales

Figure 11: Results of face detection using the learnt strong classifier The left image shows the results of the original strong classifier applied to an image. Every 19×19 patch is examined labelled as face or non-face. A threshold of 6.5 was used. While the right image, which is bigger than the left one, is searched over a range of scales. A threshold of 8 was applied.

5.2 Integration of multiple detections (*Optional*)

As you've probably noticed your detector is insensitive to small changes in translation and thus multiple detections occur around each face and false positive. However, you would probably like to have only one final detection per face (and per false positive). Therefore, it is useful to prune the detected sub-windows so that overlapping detections are summarized by one detection.

This can be achieved in many different ways and there is no obvious one correct solution. One solution is to partition the detections into disjoint subsets. Two detections are in the same subset if their regions intersect *significantly*. Frequently, what significantly means is the following. Let A and B correspond to the bounding boxes of two face detections then they are considered as being generated by the same face if

$$\frac{\operatorname{area}(A \cap B)}{\operatorname{area}(A \cup B)} > \rho \tag{15}$$

where $0 < \rho < 1$ and usually ρ is set relatively high. After computing these partitions each partition yields a single final detection. The corners of the final bounding regions are the average of the corners of all detections in the set or it is the corners of the bounding box with the highest response.

Program 24: function fdets = PruneDetections(dets)

Find overlapping detections: Let nd be the number of detections. Create a matrix D of zeros of size $nd \times nd$. Then set D(i,j) = 1 if the ith detection overlaps the jth detection according to equation (15) (the function rectint may be of use).

Find the connected components: Use the *Matlab* function graphconncomp to partition the detections into those that overlap. With this information it is possible, as described in the text, to reduce overlapping detections into one detection.

Sanity Check:

Check before and after pictures: Display the detections before you run this function and then afterwards. Visually inspect if the function has performed the expected task.

5.3 Face detection over multiple scales

Obviously, not all faces in images are of size 19×19 . Thus the detector needs to be scanned across an image at multiple scales in addition to multiple locations. Scaling can be achieved in two ways. Option one is to scale the image and look for 19×19 faces in the re-scaled image, see figure 12(a). While the second option is to scale the detector, rather than scaling the image, see figure 12(b). Features for the latter detector can be computed with the same cost at any scale. Remember though, in this case, you have to normalize the feature value calculated with respect to the scale change so the learned thresholds of the weak detectors are meaningful. Write the function ScanImageOverScale.m which takes as input the parameters of the detector, the image, the minimum and maximum value of the scale factor and the step size defining which intermediary scale factors will be searched over. The output will be the bounding boxes corresponding to the final face detections.



(a) Multi-scale search option one



(b) Multi-scale search option two

Figure 12: Options for performing the multi-scale search. (a) Keep the size of the detector constant and apply it to scaled versions of the image. (b) Keep the size of the image constant and scale the classifiers's window.

```
Program 25: function dets =
    ScanImageOverScale(Cparams, im, min_s, max_s, step_s)
```

Implement a multi-scale search: Decide how you'll implement the multi-scale and then write the appropriate code. I would suggest you resize the image using imresize for each scale you check as then you can re-use the function ScanImageFixedSize on each of these rescaled images as the size of your classifier window remains constant. The other option would probably require more work to implement. Also remember when you find a detection, at a certain location and scale, record and save what this bounding would correspond to in the original size of the input image.

Combine detections: If necessary adapt the function PruneDetections.m to ensure that overlapping detections are combined into one detection.

Sanity Check:

Create images with large faces: Use the *Matlab* function immessize to upscale the image one_chris.png by a factor of 1.2. Then run your new function and check if you can still detect the face it contains.

Debug Point: Run your function ScanImageOverScale on the image big_one_chris.png and plot the detections. I used the following settings min_s=.6, max_s = 1.3 and step_s = .06. Your results should be similar to those in figure 11(b).

If you have successfully reached this point then you are ready to get this part of the lab, **Check IV**, signed off by one of the Teaching Assistants (TA). See the accompanying form to see what the TA will expect you to

demonstrate and answer questions on.

Task V

Now you should build an accurate strong classifier. This task involves involve running the function BoostingAlg with T set to say around 100. As you know BoostingAlg can be slow to run. Thus before calling it with $T \approx 100$ you may need to optimize your code. You can you use the *Matlab* command profile to analyse how much time is spent by *Matlab* on each part of your code.

Some ideas for speeding up your boosting algorithm

- There is an overhead associated with function calls to user-defined functions. Thus for instance when calling FeatureTypeI 1,000,000 times, the majority of the time will be spent on function call overheads as opposed to the calculations executed in FeatureTypeI. Thus you can remove function calls and paste them into the main function. This is, in general, not good programming practice, but in the world of *Matlab....*
- You can turn fmat into a sparse matrix. Using this representation will speed up the matrix multiplication you have to perform when computing the feature responses.

Once you have built an accurate classifier the next task will be to run it on the images contained in the directory **TestImages**.

If you have successfully reached this point then you are ready to get this part of the lab, **Check V**, signed off by one of the Teaching Assistants (TA). See the accompanying form to see what the TA will expect you to demonstrate and answer questions on.

Task VI - Train and Test for Real

Now it's over to you! You now have an implementation which you have debugged pretty thoroughly. And you have passed the lab. However, you have not thrown a lot of data at the training process and it is pretty slow to run due to the just-in-time compilation of Matlab. You ran a detector that was trained using ≈ 4000 positive examples and ≈ 8000 negative examples using the very simple classifier described in the notes. The list of possible issues to be investigated or improved are endless. Here are some suggestions:

- Use all the images in the database for training. If you do this, do you get better performance on images with lots of clutter?
- Continuing in the same theme, you could potentially artificially generate more training examples by perturbing the existing training examples with random rotations, scalings and translations. If you do this and then train your classifier including these *new* images, what happens to the performance? Also it is very easy to extend the set of non-face training images. Find pictures with no faces and take random 19×19 patches from these images.
- There are many databases of faces publicly available. You could exploit these for training and testing. It is also very easy to collect a very large of negative training examples. A common approach is to apply your face detector to lots of images that do not contain faces. Record all the false positive hits and add these patches to your negative set, then retrain your classifier using this new training data.
- A better weak classifier Gambs et al. [2007]; Rasolzadeh et al. [2006]. You could use a decision tree of depth > 1 as the weak classifier, see the reference for more ideas Benenson et al. [2013].
- Speed up the training process Pham and Cham [2007].
- There is a great correlation amongst the responses from similar features. Could this be potentially exploited to speed up training?
- Speeding up the training process could allow you to add extra feature types to your set of weak classifiers. You could test if widening your feature pool improves your classification rate. And you could also apply these features to not just the grayscale intensity image but also the gradient magnitude image. For some inspiration check out the paper by Dollar et al. [2009].
- You could perhaps use some of the functions, coded in VLFeat, to compute different image patch descriptors such as SIFT, HOG and build weak classifiers based on these descriptors.
- What would be really fun would be to investigate if you could either integrate the OverFeat deep convolutional neural network that is available for download into your face detector or just compare its performance to the performance of your detector. At the OverFeat website you can download both parameters for a huge convolutional neural network as well as the code to run it on images. The details of what information you can extract via the code is summarized on the webpage, as is information on how the network was trained. I should just say that it is not entirely obvious to me how to integrate the features

extracted from **OverFeat** in an efficient way into our sliding window regime because we train on 19×19 image patches and **OverFeat** was trained for image patches of size 220×220 . But you guys are probably much smarter and more creative than me....

Please note I don't expect you to investigate all of the issues in the list or those you do at any great length. To have an *initial* investigation into one of these issues would be interesting to hear about at the poster session. At the very least run your learnt detector on a range of images containing faces and look and record its output. And, of course, this list is by no means exhaustive. You could potentially come up with your own ideas.

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