KTH Computer Science
and Communication

Communication Complexity: Problem Set 1

Due: September 14, 2012. Submit as a PDF file by e-mail to `lauria at kth dot se` with the subject line `Problem set 1:` ⟨`your name`⟩. Name the PDF file `PS1_`⟨`YourName`⟩`.pdf` (with your name coded in ASCII without national characters). Solutions should be written in LaTeX or some other math-aware typesetting system. Please try to be precise and to the point in your solutions and refrain from vague statements. In addition to what is stated below, the general rules stated on the course webpage always apply.

Collaboration: Discussions of ideas in groups of two to three people are allowed, but you should write down your own solution individually and understand all aspects of it fully. For each problem, state at the beginning of your solution with whom you have been collaborating.

Reference material: For some of the problems, it might be easy to find solutions on the Internet, in textbooks or in research papers. It is not allowed to use such material in any way unless explicitly stated otherwise. You can refer without proof to anything said during the lectures on in the lecture notes, except in the obvious case when you are specifically asked to show something that we claimed without proof in class. It is hard to pin down 100% formal rules on what all this means—when in doubt, ask the lecturer.

About the problems: Some of the problems in the problem sets are meant to be quite challenging and you are not necessarily expected to solve all of them. A total score of around 40 points should be enough for grade E, 60 points for grade C, and 80 points for grade A on this problem set. Any corrections or clarifications will be posted on the course webpage `www.csc.kth.se/utbildning/kth/kurser/DD2441/semteo12/`.

1 (10 p) For $x, y \in \{0,1\}^n$, let $\mathsf{GT}_n(x, y)$ be the function that evaluates to 1 if $x > y$ interpreted as n-bit numbers and to 0 otherwise. Use fooling sets to prove an exact bound on the deterministic communication complexity $D(\mathsf{GT}_n)$.

2 (20 p) Recall that for $x, y \subseteq \{1, 2, \ldots, n\}$, $\mathsf{DISJ}_n(x, y)$ is the function that evaluates to 1 if $x \cap y = \emptyset$ and to 0 otherwise. Use the rank lower bound method to prove an exact bound on the deterministic communication complexity $D(\mathsf{DISJ}_n)$.

3 (20 p) There are n teams in Sweden's top elite ice hockey league *Elitserien* (which, for the purposes of this problem, can be assumed to have bit strings of length $\log n$ as names). Yesterday there was a full round with all teams playing. Alice started watching a game between teams x and y but missed the end and would really like to know who won. For some (unexplained) reason, Bob happened to hear the name of the team that won this particular match, but does not know which other team the winning team was playing. For some (even more unexplained) reason, Alice knows that Bob knows which team won, and Bob knows that Alice wants to know the winner. How efficient a communication protocol can you find that allows Bob to convey this information to Alice?

4 (20 p) Let $R_\epsilon^{pub}(f)$ denote the public-coin randomized communication complexity of the function f and $R_\epsilon^{priv}(f)$ denote the private-coin randomized communication complexity (for some fixed but arbitrary $\epsilon < 1/2$). Does it always hold for any f that $R_\epsilon^{pub}(f) \leq R_\epsilon^{priv}(f)$? Or does it always hold that $R_\epsilon^{priv}(f) \leq R_\epsilon^{pub}(f)$? Or can this vary depending on what function f we are considering?

5 (30 p) As discussed in class, in the context of randomized communication complexity there are two natural definitions of the cost of a protocol on input (x, y), namely the worst-case cost over all random coin flips or the average-case cost. Prove that in the public-coin two-sided error communication model that was our main focus, the choice of worst-case or average-case cost does not matter much. More specifically, show that if there is a protocol that makes an error with probability at most ϵ and has cost at most c according to one of these definitions, then there is a protocol that errs with probability at most $\mathrm{O}(\epsilon)$ and has cost at most $\mathrm{O}(c/\epsilon)$ according to the other definition.