Lecture 4

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Feistel Networks

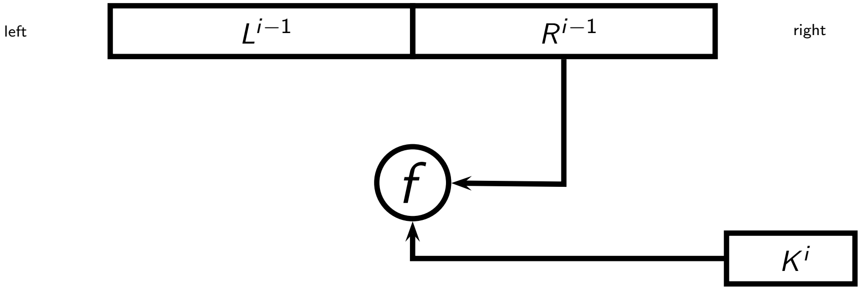
- Identical rounds are iterated, but with different round keys.
- The input to the ith round is divided in a left and right part, denoted L^{i-1} and R^{i-1}.
- f is a function for which it is somewhat hard to find pre-images, but f typically not invertible!
- One round is defined by:

$$L^i = R^{i-1}$$
$$R^i = L^{i-1} \oplus f(R^{i-1}, K^i)$$

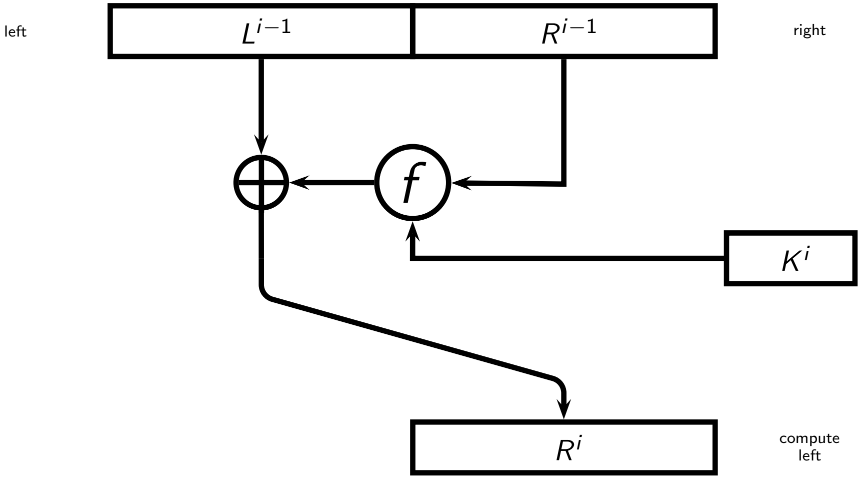
where K^i is the ith round key.

Feistel Round

Feistel Round



Feistel Round



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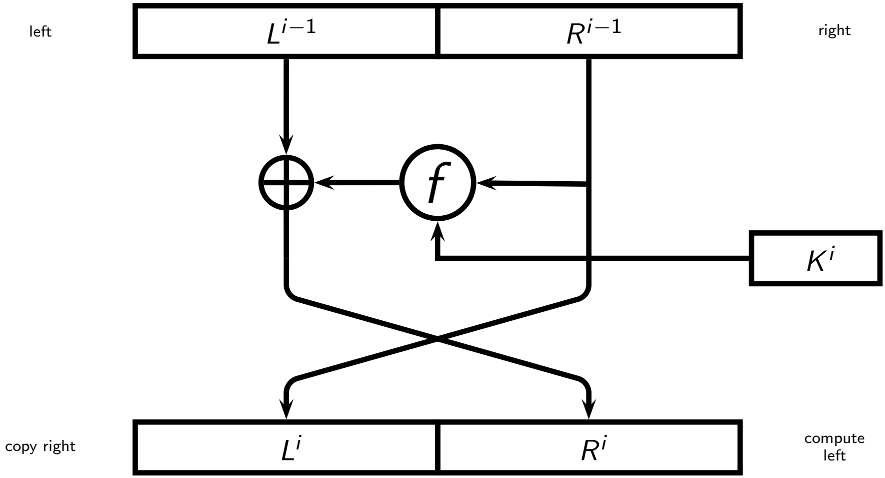
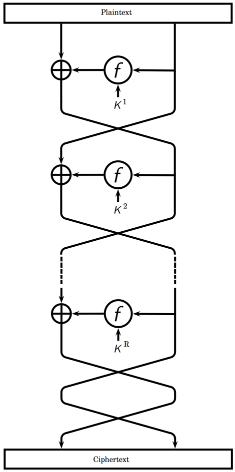


Feistel Cipher



Inverse Feistel Round

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Inverse Feistel Round.

$$L^{i-1} = R^i \oplus f(L^i, K^i)$$
$$R^{i-1} = L^i$$

Reverse direction and swap left and right!

Idealized Four-Round Feistel Network

Definition. Feistel round (H for "Horst Feistel").

$$H_{F_K}(L, R) = (R, L \oplus F(R, K))$$

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Theorem. (Luby and Rackoff) If F is a pseudo-random family of functions, then

$$H_{F_{k_1},F_{k_2},F_{k_3},F_{k_4}}(x) = H_{F_{k_4}}(H_{F_{k_3}}(H_{F_{k_2}}(H_{F_{k_1}}(x))))$$

(and its inverse) is a pseudo-random family of permutations.

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Why do we need four rounds?

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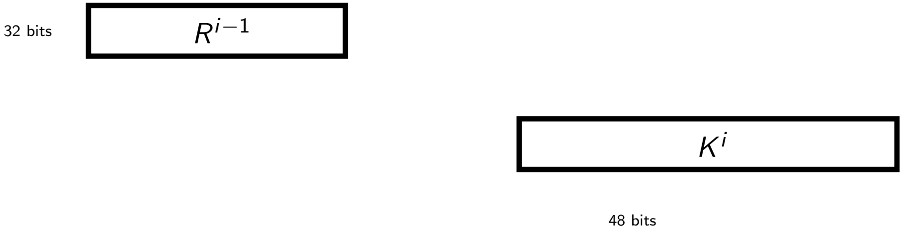
- 16-round Feistel network.
- Key schedule derives permuted bits for each round key from a 56-bit key. Supposedly not 64-bit due to parity bits.
- Let us look a little at the Feistel-function f.

DES's f-Function



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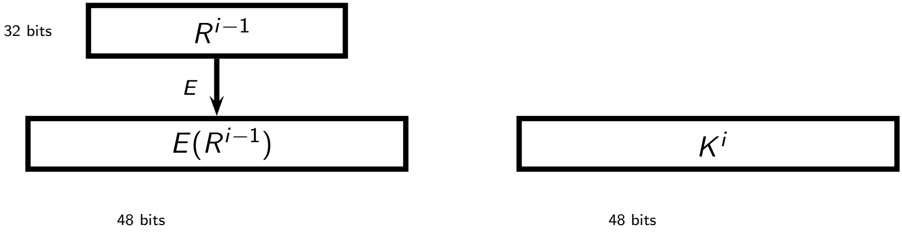

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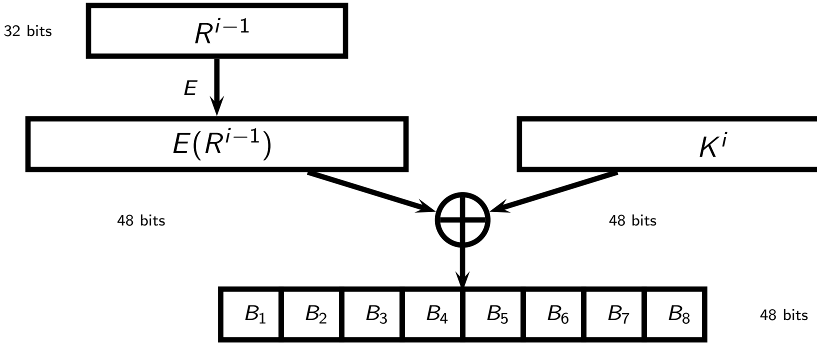

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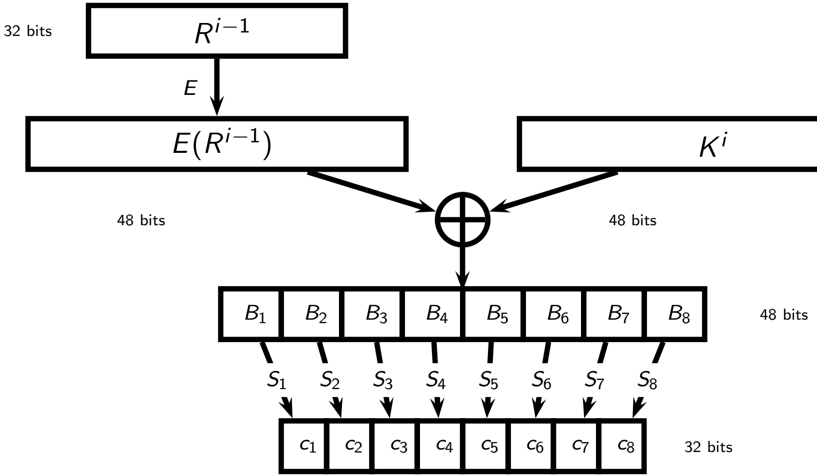

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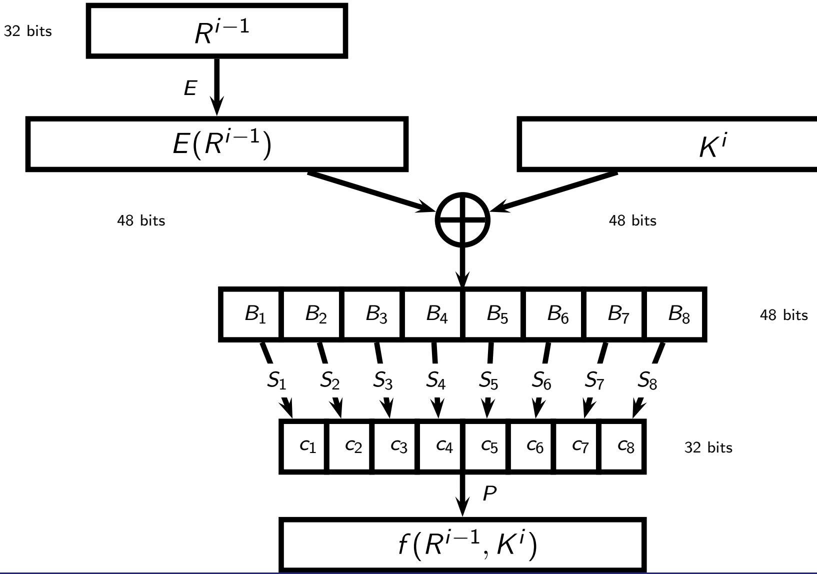

Security of DES

- **Brute Force.** Try all 2^{56} keys. Done in practice with special chip by Electronic Frontier Foundation, 1998. Likely much earlier by NSA and others.
- **Differential Cryptanalysis.** 2^{47} chosen plaintexts, Biham and Shamir, 1991. (approach: late 80'ies). Known earlier by IBM and NSA. DES is surprisingly resistant!
- **Linear Cryptanalysis.** 2^{43} known plaintexts, Matsui, 1993. Probably **not** known by IBM and NSA!

Double DES

We have seen that the key space of DES is too small. One way to increase it is to use DES twice, so called "double DES".

$$\mathrm{2DES}_{k_1,k_2}(x) = \mathrm{DES}_{k_2}(\mathrm{DES}_{k_1}(x))$$

Is this more secure than DES?

Meet-In-the-Middle Attack

- Get hold of a plaintext-ciphertext pair (m, c)
- Compute $X = \{x \mid k_1 \in \mathcal{K}_{\mathrm{DES}} \wedge x = \mathsf{E}_{k_1}(m)\}$.
- For $k_2 \in \mathcal{K}_{\mathrm{DES}}$ check if $\mathsf{E}_{k_2}^{-1}(c) = \mathsf{E}_{k_1}(m)$ using the table X. If so, then (k_1, k_2) is a good candidate.
- Repeat with (m', c'), starting from the set of candidate keys to identify correct key.

Triple DES

What about triple DES?

$$\mathrm{3DES}_{k_1,k_2,k_3}(x) = \mathrm{DES}_{k_3}(\mathrm{DES}_{k_2}(\mathrm{DES}_{k_1}(x)))$$

- Seemingly 112 bit "effective" key size.
- 3 times as slow as DES. DES is slow in software, and this is even worse. One of the motivations of AES.

Modes of Operation

- Electronic codebook mode (ECB mode).
- Cipher feedback mode (CFB mode).
- Cipher block chaining mode (CBC mode).
- Output feedback mode (OFB mode).
- Counter mode (CTR mode).

ECB Mode

Electronic codebook mode

Encrypt each block independently:

$$c_i = \mathsf{E}_k(m_i)$$

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- Identical plaintext blocks give identical ciphertext blocks.
- How can we avoid this?

CFB Mode

Cipher feedback mode

xor plaintext block with previous ciphertext block **after** encryption:

$$c_0 = \text{initialization vector}$$
$$c_i = m_i \oplus \mathsf{E}_k(c_{i-1})$$

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- Self-synchronizing.

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- Sequential encryption and parallel decryption.
- Self-synchronizing.
- How do we pick the initialization vector?

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Cipher block chaining mode

xor plaintext block with previous ciphertext block **before** encryption:

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OFB Mode

Output feedback mode

Generate stream, xor plaintexts with stream (emulate "one-time pad"):

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\begin{aligned}
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- Malleable!

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