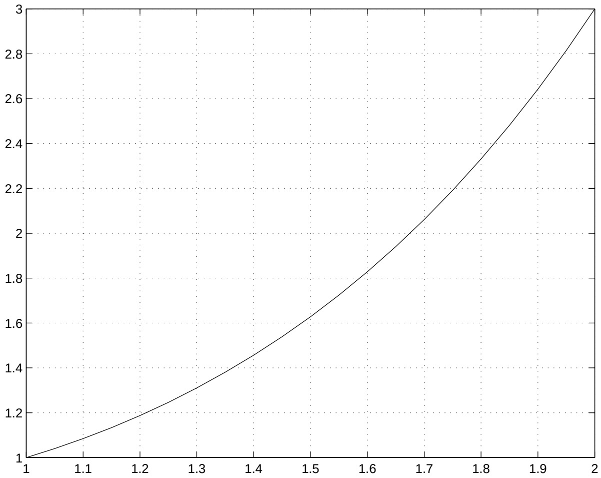
Differensmetod för randvärdesproblem DN1215 ME, Fö 5

Icke-linjärt problem

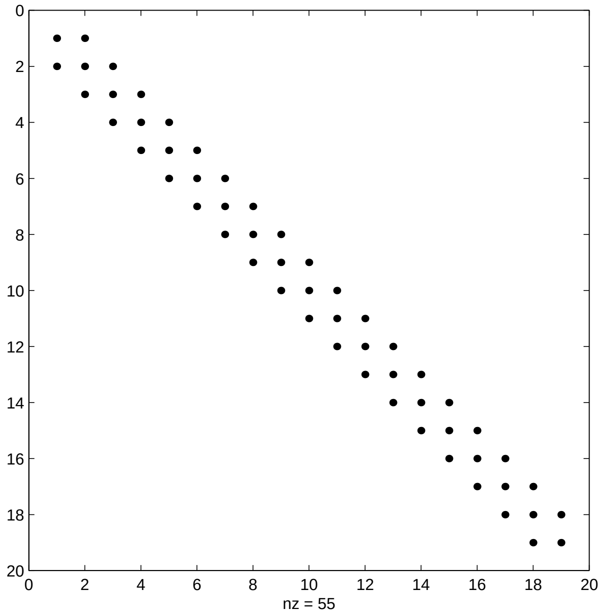
$$y'' = \sqrt{x^2 + y^2 + (dy/dx)^2}$$
$$y(1) = 1; \quad y(2) = 3$$

```
-------------------------------------------------------------------------
function G=nonlinbd(z);
global x h yL yR
N=length(z); h2=h^2; hh=2*h;
y=[yL; z; yR];
G=( y(3:N+2)-2*y(2:N+1)+y(1:N) )/h2- ...
    sqrt(x.^2+y(2:N+1).^2 +( ( y(3:N+2)-y(1:N) )/hh ).^2);
-------------------------------------------------------------------------
function jac=minjac(Fcn,z);
  NR=length(z); F=feval(Fcn,z); jac=[]; stegtol=1.E-8;
  for i=1:NR,
   z0=z;
   st=z0(i)*stegtol; if st==0, st=1.E-10; end
   z0(i)=z0(i)+st;
   jac=[jac ( feval(Fcn,z0)-F )/st];
  end
-------------------------------------------------------------------------
clear,clf,hold off
global x h yL yR
format short e, format compact
N=19; a=1; b=2; yL=1; yR=3;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
z=yL+2*x; %startgissning
dz=z;
while norm(dz,inf) >1.E-12*norm(z,inf),
  F=nonlinbd(z); J=minjac('nonlinbd',z);
  dz=-J\F; z=z+dz;
  disp(norm(dz,inf))
end
xp=[a;x;b]; yp=[yL;z;yR];
plot(xp,yp); grid; spy(J)
--------------------------------------------------------------------------
Med stegtol=1.E-8;   stegtol=1.E-3;
       2.3372e+00    2.3198e+00
       3.9825e-02    5.6220e-02
       4.8788e-05    2.8812e-04
       1.0984e-10    9.8061e-07
       1.1979e-16    3.1840e-09
                     1.0278e-11
                     3.3213e-14
```

Figurer:
Lösningskurva.



Jakobianmatrisens struktur av icke-nollor.



Bandmatrismetod för linjärt randvärdesproblem

$$y'' = 10\sqrt{1 + e^{x^3}}$$
$$y(1) = 1; \quad y(2) = 3$$

```
clf,hold off
format short e, format compact
a=1; b=2; yL=1; yR=3;
N=19; Ns=5;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
A=sparse(N,N); %lagrar A glest;ger snabbare lösare A\F
for i=1:N,
  A(i,i)=-2;
  if i>1, A(i,i-1)=1; end
  if i<N, A(i,i+1)=1; end
end
F=h^2*10*sqrt(1+exp(x.^3));
F(1)=F(1)-yL; F(N)=F(N)-yR;
y=A\F;
xp=[a;x;b]; yp=[yL;y;yR];
plot(xp,yp);
xt=xp(1:Ns:N+2); yt=yp(1:Ns:N+2);
[xt yt]

------------------------------------------------------------------------------

>> N=19; Ns=5;
   1.0000e+00   1.0000e+00
   1.2500e+00  -4.3671e+00
   1.5000e+00  -7.8801e+00
   1.7500e+00  -7.6680e+00
   2.0000e+00   3.0000e+00

>> N=39; Ns=10;
   1.0000e+00   1.0000e+00
   1.2500e+00  -4.3862e+00
   1.5000e+00  -7.9154e+00
   1.7500e+00  -7.7096e+00
   2.0000e+00   3.0000e+00

>> N=79; Ns=20;
   1.0000e+00   1.0000e+00
   1.2500e+00  -4.3909e+00
   1.5000e+00  -7.9243e+00
   1.7500e+00  -7.7201e+00
   2.0000e+00   3.0000e+00
```

Samma linjära randvärdesproblem med differensmetod, men utan matrisformulering. Jämför även med det ickelinjära problemet!

```
----------------------------------------------------
function G=lindb(z);
global x h yL yR
N=length(z); h2=h^2; hh=2*h;
y=[yL; z; yR];
G=( y(3:N+2)-2*y(2:N+1)+y(1:N) )/h2- ...
    10*sqrt(1+exp(x.^3));
----------------------------------------------------
clear,clf,hold off
global x h yL yR
format short e, format compact
N=19;Ns=5; a=1; b=2; yL=1; yR=3;
h=(b-a)/(N+1); x=[a+h:h:b-h]';
z=yL+2*x; %startgissning
dz=z;
while norm(dz,inf) >1.E-12*norm(z,inf),
  F=lindb(z); J=minjac('lindb',z);
  dz=-J\F; z=z+dz;
  disp(norm(dz,inf))
end
xp=[a;x;b]; yp=[yL;z;yR];
plot(xp,yp); grid;
xt=xp(1:Ns:N+2); yt=yp(1:Ns:N+2);
[xt yt]
--------------------------------------------------
   1.2735e+01
   1.1866e-07
   2.8266e-15
--------------------------------------------------
```

Tabellerna med utskrifter blir för övrigt desamma som för bandmatrismetoden!