

Comsol Lab: Ohm's Law, Resistors and Diodes.

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We shall model the potential field of a conductive layer in a plane. Ohm's law says that the current density \mathbf{J} , a vector with units [A/m] (in 2D), is related to the electrical potential $V(x, y)$ by

$$\mathbf{J} = -\mathbf{S} \cdot \nabla V = - \begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \quad (1)$$

where \mathbf{S} is the conductivity-tensor for orthotropic materials with conductivity S_x along the x -axis and S_y along the y -axis. An isotropic material has $S_x = S_y$. The conservation law for electric charge is then

$$\nabla \cdot \mathbf{J} = \frac{\partial}{\partial x} \left(S_x \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(S_y \frac{\partial V}{\partial y} \right) = 0 \quad (2)$$

which for an isotropic material with constant conductivity becomes Laplace's equation. Associated boundary conditions are that the potential is zero ("earth"), or any given potential, or that the current through the edge vanishes: $\mathbf{J} \cdot \mathbf{n} = 0$

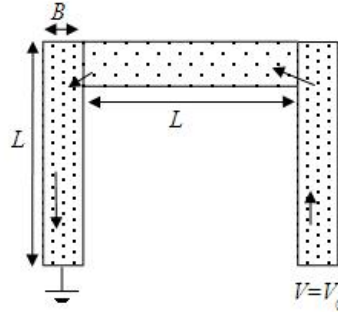


Figure 1: 2D geometry

1 A1 :Resistance

Analytical: Determine the resistance of the figure 1 if the layer thickness is d and the material has isotropic conductivity S (Ωm)⁻¹. Calculate the resistance of a $B \times L \times d$ "brick" with the potential difference V between the two $B \times d$ end surfaces, and from that the desired resistance. With COMSOL: Calculate the total current I as a line integral over the grounded edge,

$$I = \int_D \mathbf{J} \cdot \mathbf{n} ds$$

Then the resistance $R = V_0/I$.

1. Plot the current density. It shows that the solution is not smooth at the internal corners. See the four clover - example in `IntroPDEComsol-documentation`).
2. Refine the mesh repeatedly and show the convergence. How many elements are needed for 1% error?
3. Try then repeated local refinement of the problem-corners; Experiment! How many elements are needed to obtain 1% error?

2 A2. Half wave rectifier

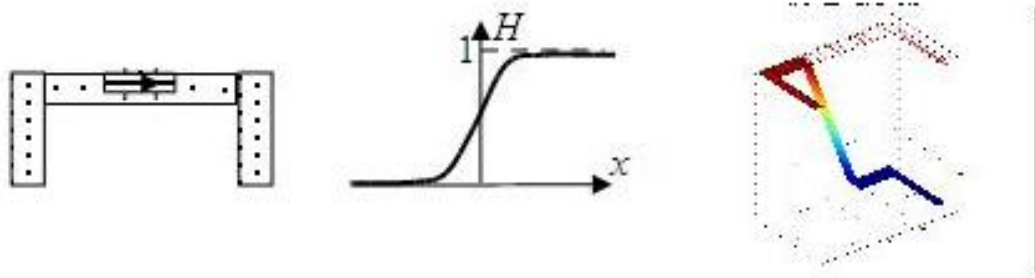


Figure 2: Geometry, smooth Heaviside-function and plot of potential.

Replace the middle part of the horizontal strip by an orthotropic material that conducts electricity only in the x -direction, and whose conductivity depends on field strength:

$$S_y = 0, S_x = s_{Lo} + H(V_x, \epsilon)(s_{Hi} - s_{Lo})$$

where H is a smoothed Heaviside function. Obviously, if $s_{Lo} < s_{Hi}$ it conducts current less easily when $\frac{dV}{dx} < 0$. When we take $s_{Lo} \ll s_{Hi}$ it becomes a diode that conducts right.

- Use the `parameter-solver` to solve the problem
 $V_0 = \sin(2\pi t), 0 < t < 3$
 and plot I as a function of t . Note that it is now a non-linear problem!
- Plot the potential with color, height and current density as arrows and animate. Is the diode on or off here in the plot ???)

3 B. The full-wave rectifier

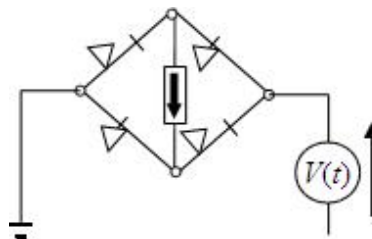


Figure 3: geometry,function and plot

Build a full-wave rectified (four diodes) with a load, calculate the current through the load, and plot I as a function of t so as to show that it works. It is customary to draw as shown in figure ??, but perhaps easier to orient the diodes horizontally in the COMSOL model. But think about how the S-tensor looks for 45° orthotropic diode material!

4 COMSOL Modeling

Use the `Conductive Media DC` - module. A: Constants under Options:

- conductivity of the various sub-domains; Current density components J (J_x , J_y) are predefined, check the name (probably Jx_emcd and Jy_emdc) under Physics / Equation System / subdomain Settings / Variables
- Also define a variable as $Itot$ under Options / Integration Coupling Variables / Boundary variable which calculates the current passing through selected boundaries.

For A2 and B also

- $V0$ as $\sin(2\pi \cdot tt)$ and a constant tt (*NOT* t , which is predefined as a time-variable)
- $Siglo$, $sigHi$ and sc for the diode and a Global Expression $sigdiod = Siglo + flsmhs(Vx, sc) * (sigHi - Siglo)$

Check the documentation how the smoothed Heaviside function is defined $flsmhs$. Use the parametric solver (selected in Solver Parameters) with $tt = -3 \dots 3$ with an appropriate step. (Try first to solve for only one value of $V0$)