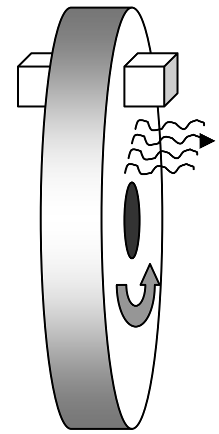
Heat conduction in a disc-brake



A disc brake may look like this. We neglect heat transport through outer cylindrical edges and can make a 2D model of one half, using the symmetry plane. The disk is make of carbon steel, $2d$ = 1 cm thick, with $2Ro$ = 0.3 m diameter, axle hole $2Ri$ = 0.05 m diameter, and the pad surface area A is a x b = 8 x 5 cm^2 placed c = 5 cm from the axle hole. The density is ρ = 7800 kg/m^3, heat conductivity k = 50 [W/K/m^2], specific heat C = 434 [J/kg/K].

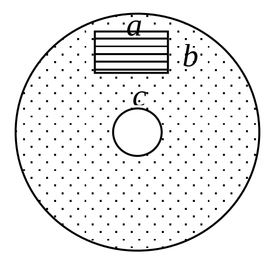
Heating and cooling

The pads press against the disk with force F. With coefficient of friction μ (e.g. = 0.8) the power density [W/m^2] becomes

$$Q = \frac{\mu F}{A} \cdot \omega r, r = \sqrt{x^2 + y^2} \text{ (polar coord. system)} \qquad (1)$$

in each point (x,y) in the pad contact area. The total power [W] is

$$P(\omega) = \omega \frac{\mu F}{A} \int_A r dx dy = \alpha F \omega, \alpha = \mu \cdot r_{av.}, r_{av.} = \frac{1}{A} \int_A r dx dy$$



where r_{av} is the arithmetic mean over the pad area A. Now, simplify the model a little by using instead a power density Q_{av}. which is *constant* over the pad area,

$$Q_{av} = P(\omega)/A = \alpha F \omega / A$$

The disk is cooled by the air by power density $h(T - T_{ext})$ [W/m^2] over the rest $S - A$ of the disk area where h = 100 W/m^2/K, T_{ext} = 300K.

The braking process to simulate:

Suppose that during time t_0 you brake a car by four equally loaded disk brakes with constant F. Its mass is m, and velocity v_0 m/s. At speed v the disks spin at $\omega = v/D$ rad/sec (wheel radius D). The brake power becomes $8P$ and

$$1/2 m dv^2/dt = -8F\alpha(v/D) \text{ (enligt Newton)} \qquad (2)$$

Now we can determine F and how ω and the power Q depend on t. The temperature distribution in the spinning disk is described by

$$d\rho C\left(\frac{\partial T}{\partial t} + \mathbf{u}(x,y,t) \cdot \nabla T\right) = d\nabla \cdot (k \nabla T) + \begin{cases} -h(T - T_{ext}) \text{ i S - A} \\ Q_{av}(t) = \dfrac{\alpha F}{A} \omega(t) \text{ i A} \end{cases}$$

$$k \nabla T \cdot \mathbf{n} = 0 \text{ on the boundaries } r = Ri \text{ and } r = Ro$$

$$T(x,y,0) = T_{ext}$$

where the velocity field in the disk, spinning as a rigid body, is

$$\mathbf{u} = \omega(t)(-y,x) \qquad (3)$$

in the car-fixed (x,y)-system. The task is to make a movie of the temperature field during the braking and a while after, $0 < t < 5t_0$

Read the whole lab-text! The equation to implement is on the next page and has fewer parameters, computed from those given here. Work in the lab session on getting that equation –

with guessed values for ω, C1, C2 – to run. Later with paper and pen you can do the analysis, put the formulas into the comsol-model – use `options/constants` and `options/expressions` – and run the simulations, mesh refinements, etc., asked for..

"Preparatory" analysis, can be done after first lab session

1. Explain (1), (2) and (3) and solve (2) – also for $t > t_0$!

2. We must compute α. As one soon notices, the formulas become formidable (Try!), so we do it with Comsol at the lab session. r_{av} is easily estimated. Use the estimate to check the number from Comsol.

3. Introduction of non-dimensional variables ("primed") for temp. and lengths

$$x' = x/L, y' = y/L, T' = (T - T_{ext})/T_0$$

and proper choice of scales L (length) and T_0 (temp.) reduces the model to

$$\frac{\partial T'}{\partial t} - \omega y' T'_{x'} + \omega x' T'_{y'} - C_1 \Delta' T' = \begin{cases} -C_2 T' \text{ outside the pad} \\ \omega \text{ "under" the pad} \end{cases}$$

$$\partial T' / \partial n' = 0 \text{ on boundaries } r = Ri/L \text{ and } r = 1$$

$$T'(x', y', 0) = 0$$

where the length scale L has already been chosen = Ro. Choose temp-scale T_0 and compute C_1, C_2**,** expressed in the original parameters.

4. Question 4 below needs analytical treatment Here the equation is integrated over the whole of S + A. Motivate

a) by Gauss' sats how it comes to be so,

b) why the terms marked disappear.

You need for the *)-term to use the divergence theorem and that div **u** = 0, so show that first. The area element *dxdy* is called dΩ here.

$$d\rho C(\underbrace{\frac{\partial}{\partial t}\int_S T d\Omega}_{\to 0, t\to\infty} + \underbrace{\int_S \mathbf{u}(x,y,t)\cdot\nabla T d\Omega}_{=0^*)}) = \underbrace{d\int_{\partial S} k\nabla T\cdot\mathbf{n} ds}_{=0} - h(\int_{S-A} T d\Omega - (S-A)T_{ext}) + \int_S Q d\Omega$$

$$0 = -h(\int_{S-A} T d\Omega - (S-A)T_{ext}) + \underbrace{\int_S Q d\Omega}_{P}$$

In the lab-session

Model: 2D plane, module `Heat Conduction`, transient, and the pre-selected Lagrange–P2–elements.
Geometry: Two circles E1 and E2 and a rectangle R1 give E1–E2+R1 as computational domain. Keep the `interior boundaries`, lest the brake pad disappear. Two sub-domains, one for the pad (A) and one for the disk outside (S - A).
PDE: Define all the parameters and C1 and C2 as constants (`options/constants`). It is possible to write expressions for a constant in terms of other constants (practical for C1 and C2). Under `physics/subdomain settings,` enter coefficients, velocities, initial data. For a time-variable ω you may need an entry in `options/expressions/global expressions`.

Boundary conditions: Under `physics/boundary settings`: Along the outer boundary insulation, there is a shortcut checkbox for this.

Questions 1-5 are for a steady state where the driver brakes and keeps her foot on the accelerator so the speed stays constant.
1. Make initial mesh, refine the triangulation at least twice and record the max. temperature for the three computations. What is the order of convergence/accuracy? Is this a good way to determine the order of accuracy? Are there singularities in the solutiion (temperature field)?

2. Change to P1-element ("Lagrange-Linear"). What is the order of accuracy now?

3. Compute α with `Post processing/Subdomain integration`.

4. What is the mean temperature outside A, $\frac{1}{S-A}\int_{S-A} T dx dy$? Compute with COMSOL and check with an analytical approximation.

5. Find parameter values to give qualitatively different looks to the steady temperature field.
a) almost axi-symmetric, hottest in a ring halfway between inner and outer edges.
b) a "comet" with head under the pad and curved tail
c) almost constant temperature.

6 For a transient process, select `transient` under `solver/solver parameters`
Try ultra-fast braking and slow, find a braking time which gives a "comet" temperature immediately after the car is brought to a standstill.