Comsol Laboration: The Triode
2010-09-23, JO

In the later Stone age, vacuum tube amplifiers were used in computers, radio- and TV-appliances. The active components were often triodes.

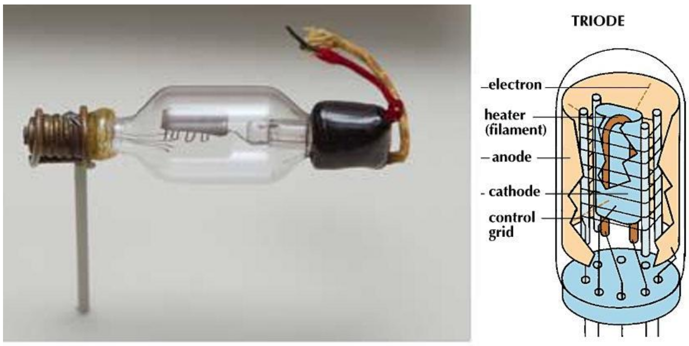

Figure 1: Lee De Forest's Audion 1908(L)/Triode(R)

Figure 1 shows Lee De Forest's Audion from 1908. It was improved around 1912 with indirect heating of the cathode and was called a triode, which is used in an amplifier circuit as shown in figure 2.

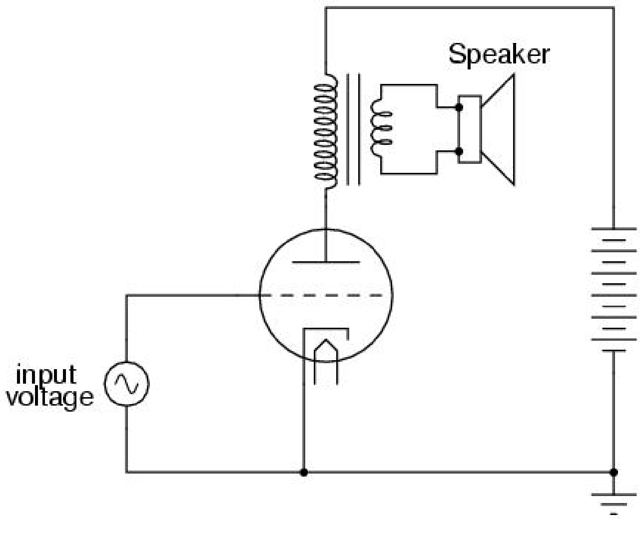

Figure 2: Amplifier Circuit

Your task is to build a triode in COMSOL and calculate its amplification factor,

$$\mu = \left(\frac{\partial V_A}{\partial V_G} \right)_{I_A} \tag{1}$$

where V_G is the input voltage on the grid, V_A is the potential at the anode and I_A is the current flowing through the anode. One can show that, if the effects of the space charge (the electron cloud around the cathode) can be neglected, the amplification factor can be written

$$\mu = \frac{C_{GK}}{C_{AK}} \tag{2}$$

where C_{XY} is the "partial capacitance" (see below) between X and Y. If we neglect the influence of the electron cloud on the electrical field, the Laplace equation holds,
$\Delta V = 0$
$V = 0$ at cathode
$V = V_A$ at anode
$V = V_G$ at the grid

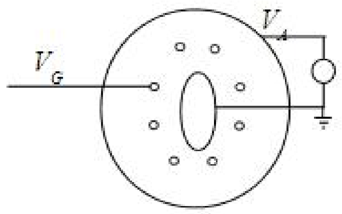

Figure 3: Voltage at anode, cathode and the grid

In our 2D model, the cathode and the grid are completely surrounded by the anode. In the figure the grid wires are the small circles, the cathode is the center ellipse and the anode is the outer circle.

1 Calculation of the partial capacitances

A number of charged bodies $B_i = 1, ..., N$ in vacuum have the potentials V_i respectively. The charge on a body is

$$Q_i = \int_{\partial B_i} \nabla V \cdot n dS \tag{3}$$

and from the linearity of the Laplace equation follows, that

$$Q_i = \Sigma_{j=1}^{N} c_{ij} V_j \tag{4}$$

where the coefficient c_{ij} is called the capacitance coefficient between body i and j $(c_{ij} = c_{ji}), i, j = A, G, K$, The partial capacitance C_{ij} is defined by

$$Q_i = C_{ii} V_i + \Sigma_{j \neq i}^{N} C_{ij} (V_i - V_j) \tag{5}$$

which gives a simple relation between C_{ij} and c_{ij}.

1.1 Investigations

1. Show that $Q_A + Q_K + Q_G = 0$ by the divergence theorem, $\int_{\Omega} \nabla \cdot \nabla V d\Omega = \int_{\partial\Omega} \nabla V \cdot nds$
2. Show that $\mu = \frac{c_{GK}}{c_{GA}}$

With COMSOL you get the partial capacitance by integration of the equation 3 over the boundaries with different V_G.
Since $V_A = Constant$, there holds $\frac{dQ_k}{dV_G} = c_{KG}$ and for $V_G = 0$ we get $Q_K = c_{KA} \cdot V_A$. So

$$\frac{c_{GK}}{c_{AK}} = \frac{V_A \cdot \frac{dQ_K}{dV_G}}{Q_K(V_G = 0)} \tag{6}$$

2 Modelling

2.1 mode

Use 2D COMSOL `Multiphysics>Electromagnatics>Electrostatics`

2.2 Geometry

In the figure 4 the dimensions are in meter. The anode has a radius of 2 cm, the cathode is elliptic with minor axis of 5 mm and major axis of 10 mm. The 8 grid wires have 1 mm diameter.

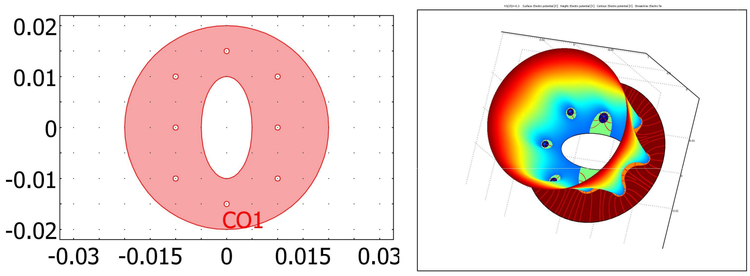

Figure 4: Triode Geometry(L) and Surface Data Plot(R)

Choose if you wish a different geometry, preferably with thinner and greater number of grid wires.

2.3 Constants and Expressions

- Create $V_A$and V_G as constants under `options/constants`
- The boundary condition is Dirichlet on all boundaries , $V = 0$ at the cathode (ground) , V_G on the grid and $V_A(= 1V?)$ at the anode. Try first with some initial values at V_G and solve and check that the boundary conditions are fulfilled.

3 A-Task

Use solutions to check the COMSOL calculation, with regard to

- linearity (Q_i as functions of the different V_j)
- the sum of the charges
- convergence under mesh refinement. How many elements do you need to get 1% accuracy in the charge calculation of the grid?. Use the global refinement (triangular icon with red shade)

4 B-Task

- use the parametric solver under `solver>solver parameter`

  and vary V_G from -1 to 1.

- Visualize the potential field as colour code and z values.
`Post-processing/plot parameters/Surface/Surface Data` and `Height data`
Draw the electron orbits as the field lines
`Post-processing/plot parameters/Streamlines` from the cathode, such as the image on the right.

- Make an animation

- Use
`Options / Integration Coupling Variables / Boundary Variables`
with a `global destination` for calculation of Q_A, Q_G and Q_K in the calculations. The E-field normal component becomes $\nabla V \cdot (n_x, n_y) =$ `Vx*nx + Vy*ny` Type in this expression under `expression` and choose the boundaries to be included in the integral. Make sure the `Global Destination` is selected!

- Plot all three with
`Post-processing>global variables plot`.

- Read off $\frac{dQ_K}{dV_G}$ and $Q_K(V_G = 0)$ from the plot and calculate the amplification factor