Comsol Laboration: Heat Conduction in a Chip

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A chip on a circuit board is heated inside and cooled by convection by the surrounding fluid.

- What is the steady state temperature distribution, and how long does it take to reach steady state ?
- How do the chip dimensions influence the max. temperature?

1 Model

The mathematical model is an initial-boundary value problem for heat equation, to determine $T(x, y, z, t)$ for $t > 0$ when

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q \text{ in } \Omega$$

$$k \frac{\partial T}{\partial n} = h\,(T_{ext} - T) \text{ on } \partial\Omega$$

with initial data $T(x, y, z, 0) = f(x, y, z)$

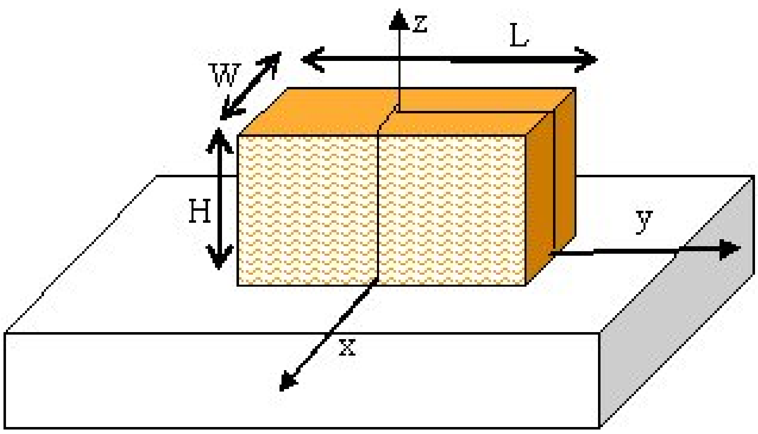

Figure 1: Chip geometry

ρ is the mass density [kg/m^3], specific heat C [J /(kg K)], k thermal conductivity [W/(mK)] and Q the heat source [W/m^3]. T_{ext} is ambient temperature and h the convection heat transfer coefficient [W /(m^2K)] between the chip surface and surroundings.
We make some simplifying assumptions:

1. No heat transfer to the card
2. Constant heat transfer coefficient h on all the other sides.
3. Uniform heat source Q
4. The chip is thin and long, $H << W << L$, and we shall study the impact of thinness below. In the future, we use symmetry to compute on a H $\times$(W / 2) $\times$ (L / 2) part of the chip.

Data for silicon: $\rho = 2300[kg/m^3]$, C = $380[J/KgK]$, $k = 3.6[W/mK]$. Total power is P [W], ie. $Q = \frac{P}{V}[W/m^3]$ where V is the volume of the chip. The heat transfer coefficient h when the chip is cooled by a weak fan is h_{coef} (E.g. 100) W/m^2K and $T_{ext} = 20^oC$.

2 Preliminary analysis

- a) Symmetry makes it sufficient to simulate 1 / 4^{th} of the chip.

  Proof: Suppose that the solution is unique, say $T(x, y, z, t)$. Show that $T(-x, y, z, t)$ and $T(x, -y, z, t)$ are also solutions.

- b) An important characteristic quantity is the *Biot number* $Bi = \frac{h}{k/d}$, the ratio of thermal resistance $1/h$ for convection and d/k for the chip itself, where d is a characteristic length for heat transport through the chip. Here H is the smallest dimension. When Bi is large, the surface temperature is close to T_{ext}, and when Bi is small, the temperature will be nearly uniform in the body.

- c) What is the temperature when Bi (ie h) is small?

- d) Assume that the chip, after being on for a long time, is turned off at $t = 0$ so $Q = 0$ for $t > 0$. The Newton cooling law, often used in crime fiction to determine time of death, is

  $$\tau \frac{dT}{dt} = T_{ext} - T$$

  which is the limiting case of vanishing Bi.

- e) What is the time-scale τ for the chip? Tip: Start from the PDE 1, integrate over the entire chip and use Gauss's theorem. Use that T is almost constant through the chip.

- f) Derive a similar model for heating when the chip is turned back on.

3 In the Lab-session

- **Model:** Use 3D, `Classical PDES / Heat Equation`. It is worthwhile to define the material parameters as `Constant` under `Options`. Then you can easily change values etc, and the equations get to be in understandable terms.

- **Geometry:** A brick ! give the right dimensions under `properties`

- **PDE:** Set the value of k and Q (named `f` in the GUI menu)

- **Boundary conditions:** Neumann-conditions : insulation on the bottom and symmetry-boundaries is obtained by `q = g = 0`. On the cooled surfaces, with `q` and source term `g` with h_{coef} and $h_{coef} \times T_{ext}$ these may first be defined as the `Constant` under `Options`.

- **Solver :** `Stationary linear`, except for transient runs (e) conducted with `Time Dependent`. Choose a finish time based on the calculated time-scale.

1. (A) Calculate the maximum temperature when

   P = ... W . . . yy

   h_{coef} = 20 W /m K . . . dd

W = 5+ mm / 2 . . . mm

where the analyst's birth date is yy mm dd.

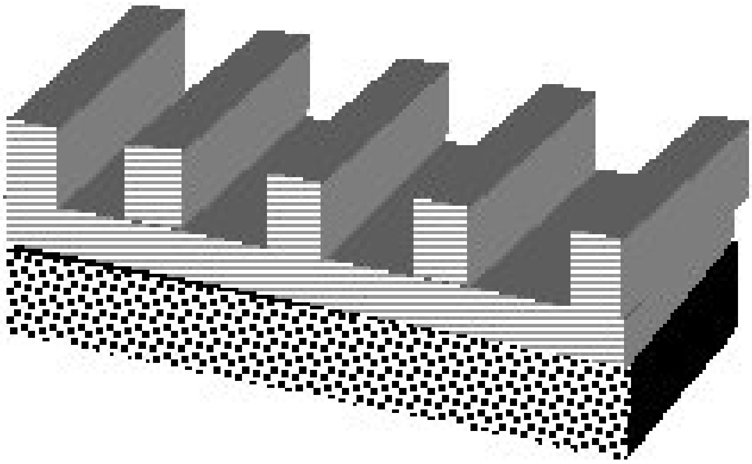

Figure 2: Chip with cooling fins

2. (B) for W = 10 mm, do the nätförfinings study and try the P2 and P1 - elements.
3. (C) Plot the temperature profiles and find maximum temperatures for W $= 10.1, 0.2$ mm
4. (D) Derive for $h = \infty$ the approximate value $T_{max} = H^2Q/(2k)$. Is this perhaps an upper or lower limits? Tip: The differential equation can be simplified to

$$0 = k\frac{d^2T}{dz^2} + Q, \frac{dT}{dz}|_{z=0} = 0, T(h) = T_{ext}$$

5. (E) The question "Is it as fast to heat up as to cool down?" is imprecise. Please specify, observe and justify/ motivate . It is easiest to run a simulation starting with $T = T_{ext}$, turn on Q at $t = 0$ and shut off after so long time that the temperature has become (almost) stationary: $Q = (t < t_{end}) \times Q_0$
6. (F) The temperature can be reduced by cooling. Let the chip-octant be $10x10x2mm^3$. Place a 1 mm thick aluminum cap on the chip, then 5 pcs. 1 mm thick and 1 cm high cooling fins:

   Data for aluminum: ρ = 1600 [kg/m^3]C = 380[J / (kg K)], k = 60 [W / (m K)]

How much is the maximum temperature reduced by the cooling fins?

Is it worthwhile to make the flanges higher and / or thinner and more numerous?

A thought: Total surface area exposed to air is an important parameter in the model (see (a) above). Then, maybe we could do with just a little Al, by making many densely packed thin flanges. But that is not how the actual chip cooler looks . Why? What is wrong with our model? *Geometry:* First one creates the top as a brick and places it by the filling the right geometry data under `Draw /Object Properties:`
Then make an outer flange, place it, then copy it and give the right translation in the dialog box (four times). Create thew `Composite Object` from the union of all the flanges and the lid

without `interior borders`. Finally, make an object composed of top, flanges, and chip, with retained `interior borders`. That's it!

PDE: Fill `materials data` and `initial data` in the two `Subdomains` (Al and Si).

Boundary conditions: Select `Neumann` for *all* boundaries, with heat transfer coefficient q and source term g like h_{coef} and $h_{coef} * T_{ext}$ - these may first be defined as the `Constant` under `Options`. Then change the (few) symmetry-boundaries to insulation.