Comsol Laboration

Design of Bourgogne Organ

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1 Background

The task is to determine the frequency of the sound obtained by blowing across the mouth of a partially filled standard Bourgogne bottle, and to select the liquid levels to make a properly tuned bottle organ. The rotationally symmetric geometry is defined by the bottom radius R, also the radius of the spherical shoulder segment, the height H, the neck radius r and length h, the liquid level is x.

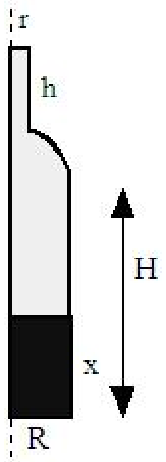

Figure 1: 2D Axis Symmetric Geometry of the Bourgogne bottle

2 Helmholtz Resonator Model

In the Swedish High-school national physics contest 1994 one learns that the contraption is a Helmholtz resonator, where the air in the bottleneck acts as a rigid body, and the (large) volume in the bottle itself acts as a spring. The test suggests the use of $pV = const.$, i.e. assuming that the process is isothermal. This is erroneous and gives about 20% error, see below, so one should use instead γ =ratio of specific heats for air $= 1.4$,

$$\frac{dp}{dV} = -\gamma \frac{p}{V} \tag{1}$$

i.e. $pV^{\gamma} = const.$ One can deduce from this the lowest resonance frequency.

3 Analytical and Preparation Tasks

1. Q.1 You may assume that $H = 3R$, $R = 3r$, and volume = $3/4$ litres, the temperature is $20°$, and standard air pressure of 10^5 Pa, at which conditions the density of air is$\rho = 1.273$ Kg/m^3. Plot the frequency as function of x, $0 \leq$ x $\leq$H
   Hint: The air plug has mass $m = (\ldots)$Kg and is accelerated by the pressure difference

acting on its cross section area A. If it has velocity v, positive outward, then the volume rate of change of the air in the large volume V is $\frac{dV}{dt} = Av$ But $\frac{dp}{dt} = \frac{\partial p}{\partial V}\frac{dV}{dt}$ so we now have two first order equations for p and V from which the resonance frequency can be obtained (by linearization).

4 COMSOL Calculations

4.1 Eigenvalue computation

Now let us do this by eigenvalue computation in COMSOL. It follows from thermodynamics that air pressure deviations p from ambient pressure P are described by the wave equation,

$$p_{tt} = c^2 \Delta p \tag{2}$$

$$\frac{\partial p}{\partial n} = 0 \text{ at solid boundaries}$$
$$p = 0 \text{ at the mouth}$$

c is the adiabatic speed of sound, 340 m/s at the conditions above.

1. Q2 Plot the frequency as function of x, $0 \leq$ x$\leq$H, in the same diagram as the curve in 1.
2. Q3 Comment on the differences between the Helmholtz resonator results and the(probably more accurate) COMSOL results.
   READ QUESTION 3 BEFORE DOING THE CALCULATIONS!

4.2 Variation in x

The variation of x is conceptually most easily treated by using the `parametrized geometry` Module. An option is to include the pressure waves in the liquid so the liquid level becomes a parameter which can be changed without changing the geometry.
Here is how:

4.2.1 Liquid

The liquid is almost incompressible and the correct model is
$(E)^{-1}p_{tt} = \nabla \cdot (\frac{1}{\rho}\nabla p)$
where E is the elastic modulus (N/m^2) and ρ is the density (1000 kg/m^3). It is known that the sound speed in water is about 1500 m/s.

- Q.4 Deduce the value of E from this. Hint: With constant ρ and E we obtain
  $p_{tt} = E/\rho\Delta p$

4.2.2 Gas

The equation for the gas is

$$(c^2\rho)^{-1}p_{tt} = \nabla \cdot (\frac{1}{\rho}\nabla p) \tag{3}$$

4.2.3 Interface conditions

For the whole model, p and $\frac{\frac{\partial p}{\partial n}}{\rho}$ must be continuous across the interface. This comes automatically with the FE formulation. Define ρ as

$$\rho(r,z) = Hs(z-x)\rho_{Gas} + (1 - Hs(z-x))\rho_{Liq}$$

where Hs(s) is a smoothed Heaviside function jumping from 0 to 1 at s $= 0$. A similar definition is made of the coefficient of p_{tt} (called e_a in COMSOL),

$$e_a(r,z) = Hs(z-x)\frac{1}{\rho_{Gas}c^2} + (1 - Hs(z-x))\frac{1}{E_{Liq}}$$

4.3 Heaviside Function

Hs jumps over a distance d, a parameter to the FLH2S function built into COMSOL. One suspects that d must be O(element size) so the jump occurs over a few elements lest the quadratures become inaccurate.

- Q5 Experiment with small d. Look at the pressure gradient near the interface. If it looks spiky d is too small.
- Q6 Run the computations. Compare a few with results obtained by changing the geometry.
- Q7 So armed with the tables, select the eight liquid levels.

5 COMSOL implementation

5.1 Mode

5.2 Geometry

5.3 Constants

5.4 Expressions

5.5 Solver Settings

5.6 Convergence Study

6 Appendix

Adiabatic relation between pressure p and Volume V

Assume that the process is adiabatic, i.e. no heat exchange with the outside. Then, for the volume in the bottle
0= added heat = $dQ = dE + pdV$
where p = m/V R T, R = universal gas constant Assuming the gas to be calorically perfect, its internal energy per unit mass e depends only on temperature:

E = me = m C_v T = p C_v V/R,

so

0 = C_v/R d(pV) + p dV = (C_v/R + 1) p dV + C_v/R V dp;

$$\frac{dp}{dV}_{dQ=0} = -(C_v + R)/C_v p/V = -\gamma p/V$$

where γ is $7/5 = 1.4$, a value which can be deduced from the kinetic theory of gases for twoatomic gases such as air. The isothermal process has $\frac{dp}{dV}_{dT=0} = -p/V$ so the error is 40% which translates into an error of 20% in wave speed. Use the correct relation in problem 1.