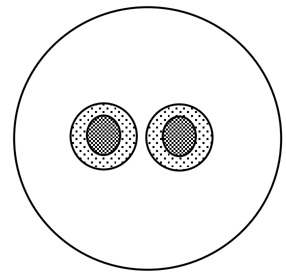
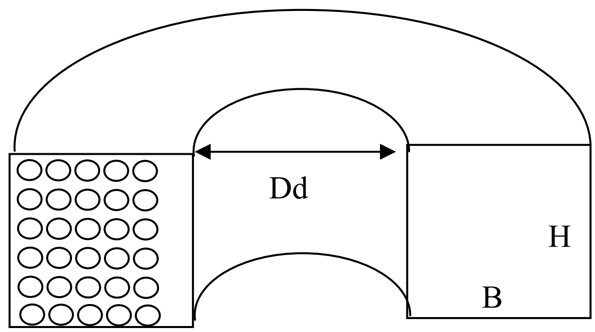
Heat conduction in cables and reels

This is a cross section of a RVOE-twin conductor-cable: Two layers of plastic, the inner t (≈0.85mm) thick; The area of the copper conductors A is 1.5 mm^2 each and the outer diameter of the cable is Dy (9.4 mm). How much warmer than the ambient, and where is it warmest, if it connects a (U =) 230V-power outlet to a P (= 2000W) heating element ? We assume a purely resistive load.



Two cases:
1: The cable hangs in free air and is cooled by it with a heat transfer coefficient of h [W/m^2/K]

2: The cable, L m long, is very carefully wound into a reel, picture right.



Data that may be useful: Copper has conductivity σ_{Cu} of 5.8 10^7 $[\Omega m]^{-1}$, density ρ_{Cu} 8930 [kg/m^3] specific heat c_{Cu} 390 J/kg/K, heat conductivity k_{Cu} 380 W/m/K.
Plastic: electric conductivity 0, density 1300 [], specific heat 920 [], heat conductivity 0.2 []. The two kinds of plastic are assumed identical (except for color ...), so no boundary between them is needed. Values for t, Dy and h can be taken from the student's birthdate: yy mm dd

h = dd [W/m2/K]; t = yy/100 [mm]; Dy = 5 + mm/6 [mm]

Determine the steady (d/dt = 0) temperature distribution with Comsol Multiphysics. The differential equation is (**n** is outward unit normal)

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot \left(k \nabla T\right) + Q$$

$$k\nabla T \cdot \mathbf{n} = h(T_{ext} - T) \text{ on outer boundaries;}$$

$$h = 0 \text{ on symmetry boundaries, take } T_{ext} = 0.$$

"Preparatory" analysis, can be done after first lab session.

1. Show that the heating power Q W/m^3 in the copper kopparen is $Q = \left(\frac{P}{UA}\right)^2 \frac{1}{\sigma_{Cu}}$.

a) what is the total current to give total power P?
b) what is the resistance in l m of the Cu-conductor?
c) what is the potential drop? and the power?
2. The boundary Γ has mean temperature

$$\overline{T} = \frac{1}{L}\int_{\Gamma} T(x,y)d\Gamma = 2AQ/Lh \text{ where } L \text{ is the length of } \Gamma \text{. Why is it so?}$$

3. Where will the temperature gradients be largest, in Cu or Plastic?

4. Suppose that the two Cu-regions can be replaced by *one* central circular disk Cu with area 2A, radius R_i. Then, the equation for T in Pl becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0, T(D_y/2)=\overline{T}$$

$$\Rightarrow T(r)=C_1 \ln r + C_2 \quad (2)$$

$$k_{Pl}\cdot 2\pi R_i \cdot \left.\frac{dT}{dr}\right|_{r=R_i} = 2AQ \quad (3)$$

a) Show (2) and explain (3).

b) Determine C_1, C_2 and the temperature at the Cu-Pl interface

In the lab-session

1. Free cable

Model: Use 2D, mode **`Multiphysics/Heat Transfer/Conduction/Steady-state`**, and the pre-selected `Lagrange–P2`–elements.
Geometry: If you want to use the symmetry to save on elements; The union of two circles, intersected with a rectangle gives the desired quadrant as computational domain. If not, one large and two small circles will do. Two (three) sub-domains, one (two) for Cu and one for Pl..
PDE: Define *P*, *U*, *A*, sigCu, ..., *Q* as constants. It is possible to write expressions, like for *P*, enter the expressions under `subdomain`.
Boundary conditions: Along *xy* = 0 there is symmetry, along the outer boundary a heat transfer coefficient.

1. Make an initial mesh, refine the triangulation (at least) twice and note the max-temperatures for the three calculations. What is the order of accuracy/convergence? Is the use of max-temp. a good way to assess order of accuracy? Are there singularities in the solution (temperature field)?
2. Change to P1-elemens. What is the order now?
3. Check for a calculation by `boundary integration` the mean temperature formula above.

B-task
The temperature in the copper has very small gradients compared to the plastic..

- Quantify this statement by considering the heat flux through the Cu-plastic interface.

A model where the Cu has constant (but unknown) temperature as boundary condition for a model for only the plastic should be quite accurate:

- Make one ! The “constant” temperature in Cu must be determined so the correct total heat flows through the plastic. How is that done?
  Hint: The Cu-temperature is an affine function of external temperature and the power:
  $T_{Cu} = T_{external} + K\,Q$ (*)
  You can determine *K* by a solution with some assumed T_{Cu}-värde.
  Verify (*)
  1. *empirically* by using the COMSOL parametric solver (under **`Solve/Solver Parameters`**) with T_{Cu} as parameter, and
  2. *theoretically* by using the differential equation.

2. Cable on a reel

Model: Use `Axisymmetric (2D), mode` **`Multiphysics/Heat Transfer/Conduction/Steady-state`.** We model one half (symmetry) of a cross section of the reel with 5 x 3 x (2 Cu-circles) packed so the distance between the 2-groups becomes 10 mm. The whole pattern is placed in a rectangle whose smallest distance to the circles is 2 mm everywhere. Then fill the space between the Cu-circles with plastic, and cool the outer boundaries like above.

Geometry: There is an “array”-constructor (icon with four small squares) which makes n x m copies i an n x m pattern. **Or manually:** First make one Cu-circle, then its neighbor in the two-group form from them a composite object, say G. Copy and translate to make a row. Make a composite object from these 5 G, say `R`. Copy and translate `R` to make three rows. Make a composite object, say `Cu`, from the three `R`.

Create the rectangle `R1`, form the union `R1+Cu` – *Note: keep interior borders*!

PDE: There are 31 subdomains. But 30 of them have the same data, so select *all* 31 and give them Cu-properties and heat source. Then change the data in the (single) plastic domain.

Boundary conditions: Easy, the three outerhave the same transfer coefficient, and the symmetry plane is insulated.

4. Compute the temperature field and show as 3D plot and color with streamlines to visualize the heat flux.

Where are the temperature gradients strongest? Plot `grad T` to show. Where should the element mesh be refined?

Try adaptive refinement (select under `solver parameters`) and see if you agree with the algorithm.